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SCHOLAR SERIES

FOR FEDERAL & KPK BOARD

11

A Conceptual Approach to

PHYSICS

SUBJECTIVE

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Preface

ALLAH Almighty be thanked a million times for showering his special blessing a work of such magnitude. Those who have under taken such a Heculean task can fully understand the obstacles, barriers and problems which need to be overcome.

This book is unique in its kind and texture as it caters for the need of the students and teachers alike. It covers a wide range of textual styles on one hand and on the other tends to develop the knowledge, skills and appetite for attaining practical knowledge for future fears.

We also owe our deepest sense of gratitude to **Scholar Publications** for inculcating in us a self belief which proved out to be our greatest source of strength.

We medge that any suggestions from our worthy teaching fraternity shall be highly appreciated for the improvement and a mention about any mistake.

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Every Chapter gives you

- Learning Objects
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- Past Boards MCQ's, Short Questions Long Questions
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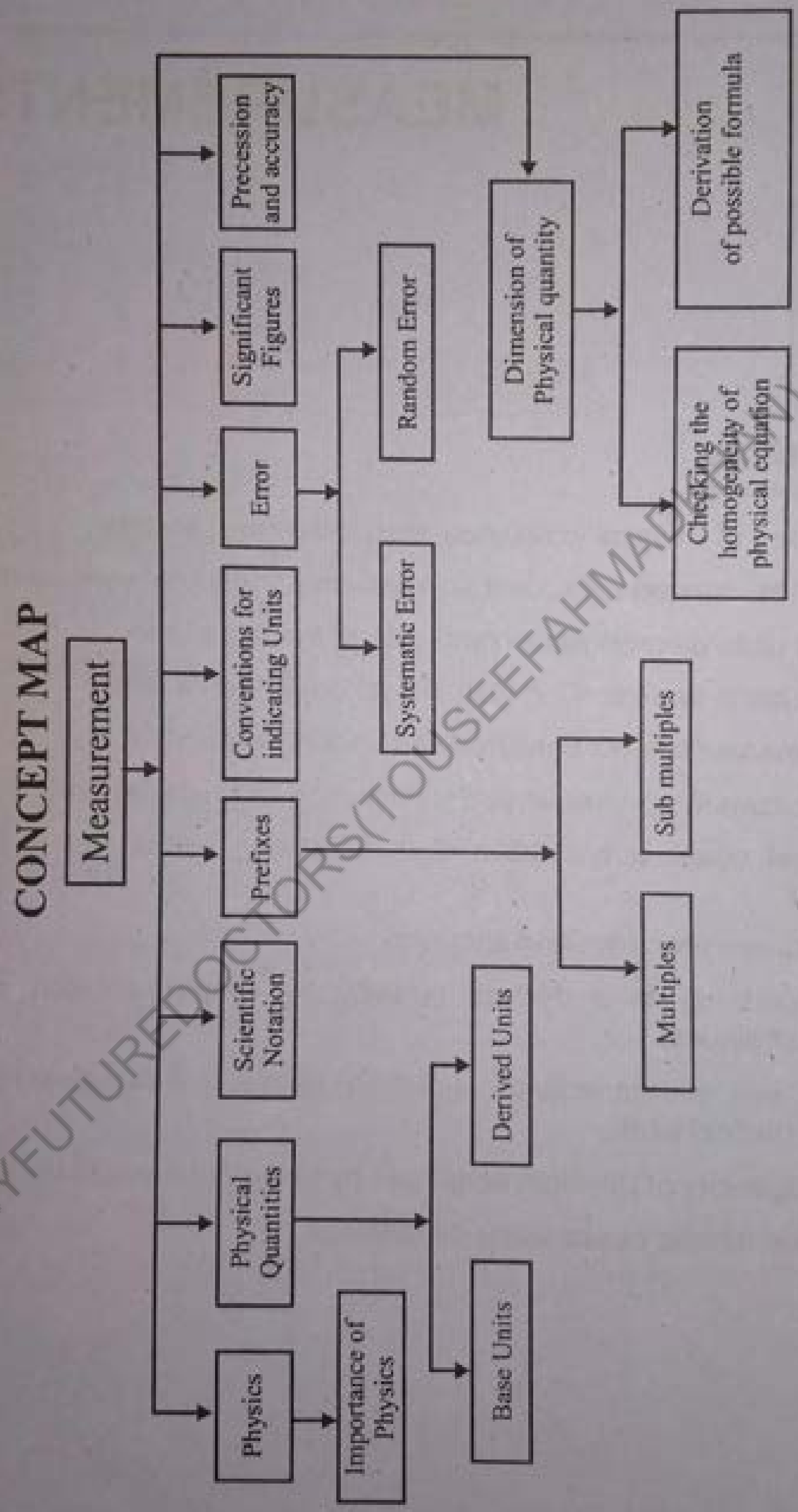
CHAPTER 1

MEASUREMENTS

Learning Objectives

- Describe the scope of Physics in science, technology and society.
- State SI base units, derived units, and supplementary units for various measurements.
- Express derived units as products or quotients of the base units.
- State the conventions for indicating units as set out in the SI units.
- Explain why all measurements contain some uncertainty.
- DISTINGUISH between systematic errors (including zero errors) and random errors.
- Identify that least count or resolution of a measuring instrument is the smallest increment measurable by it.
- Differentiate between precision and accuracy.
- Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
- Quote answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
- Check the homogeneity of physical equations by using dimensionality and base units.
- Derive formulae in simple cases using dimensions.

Chapter No. 1


CONCEPT MAP
Measurement
Physics
Importance of Physics
Physical Quantities
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Prefixes
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Conventions for indicating Units
Error
Systematic Error
Random Error
Significant Figures
Precession and accuracy
Dimension of Physical quantity
Checking the homogeneity of physical equation
Derivation of possible formula

Q.1 Define Physics? Explain the scope and importance of physics in science, technology and society?

Ans: PHYSICS AND ITS SCOPE

"Physics is the branch of science in which we study about properties of matter and energy; and their mutual relationship."

OR

"Physics is the branch of science that involves the study of the physical world in specific and physical universe in general: energy, matter, and how they are related."

Scope and Importance of Physics

- In physics modeling of the natural world is made with theory, and is usually expressed quantitatively with mathematical description.
- Even if we do not study PHYSICS as a subject, we depend on it for nearly everything.
- From walking to driving a car, from cooking to using a gadget, from cutting a tree to building a new house everything involves physics.
- Even as we read this sentence, physics is at work. Physicists investigate the motions of electrons and rockets, the energy in sound waves and electric circuits, the structure of the proton and that of the universe.
- Physics is also called 'the fundamental science' because the subject of study of all branches of natural science like chemistry, astronomy, geology, and biology are constrained by laws of physics. We can say 'All other natural sciences stem from physics. Chemistry is essentially applied physics and biology is applied chemistry'.
- For technologies to develop physics is essential.
- Physics generates fundamental knowledge needed for the future technological advances that will continue to drive the economic engines of the world.
- So many pivotal discoveries of the 20^{th} century – including the laser, television, radio, computer technology plus internet, DNA and nuclear weapons are all credited to advancement in physics.
- Physics contributes to the technological infrastructure and provides trained personnel needed to take advantage of scientific advances and discoveries. It is important for improvement in health, telecommunication, transport and design of our future.
- Nearly all consumer goods we use at home have been developed from *research in physics.*

Q.2 What is system of units? In SI what is meant by base, derived and supplementary units?

Ans: SYSTEM INTERNATIONAL (SI)

A complete set of units for all physical quantities is called system of units.

However, to form a system we do not need to define every quantity. We take only a few quantities (called base quantities) and base units to agree on accessible and invariable standards for measurement such that all other quantities and units are expressed in terms of those quantities.

The **International System of Units** (abbreviated SI from **systeme internationale**, the French version of the name) is a scientific method of expressing the magnitudes or quantities of important natural phenomena.

Science require that quantities must be defined and measured. Things that cannot be measured like beauty, love, hate, are all not science.

On the other hand quantities like length, time, density, temperature, electric fluxes can be measured therefore they are called physical quantities.

"The quantities which can be measured are called physical quantities."

Different systems of quantities:

In earlier times scientists around the world were using different systems of units for their liking. Three such systems, the MKS, the CGS and the FPS (or British) system were in use extensively till recently.

In 1960 an international committee agreed on a single system for whole world, the system's official name is the Systems International, or SI, meaning International System.

We can use other systems and its units (fahrenheit, pounds, and miles) for our convenience but in science we must always use SI.

Base Units: In SI, SEVEN physical quantities chosen arbitrarily as base quantities and their corresponding units are called base units.

Table 1.1: SI BASE QUANTITIES AND BASE UNITS

Base Quantity		SI Base Unit	
Name	**Symbol**	**Name**	**Symbol**
Length	l, x, r etc.	meter	m
Mass	m	kilogram	Kg
Time, duration	t	second	s
Electric current	I	ampere	A
Thermodynamic temperature	T	kelvin	K
Amount of substance	n	mole	mol
Luminous intensity	I_v	candela	cd

Derived Units: A quantity and its unit obtained and developed from base quantities and their respective units without giving any consideration to the directional properties are called derived quantities and its units.

Some of the derived quantities are given by:

TABLE 1.2: SI DERIVED QUANTITIES AND DERIVED UNITS

Derived Quantity		SI Coherent Derived Unit	
Name	**Symbol**	**Name**	**Symbol**
Area	A	Square meter	m^2
Volume	V	Cubic meter	m^3
Speed, velocity	V, $\vec{V}$	Meter per second	ms^{-1}
Acceleration	a	Meter per second squared	ms^{-2}

Supplementary Units: These are such units which can neither be placed as base units nor in derive units.

Pure geometrical units (radian and the steradian) were classified by the System International (SI) as supplementary units.

(But this designation was abrogated in 20th CGPM (french words Conférence générale des poids et mesures abbreviated from General Conference on Weights and Measures) in 1995 and the units were grouped as derived units).

There are two supplementary units i.e. radian and steradian.

Q.3 Differentiate between radian and steradian. Discuss in detail

Ans: Radian: radian is the unit of plane angle.

"One radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle."

Formula:

Mathematically,

$$\textit{Number of radians } (\theta) = \frac{\textit{Arc Length}}{\textit{Radius of same circle}} = \frac{S}{r}$$

Relation radian measurement and degree measurement:

Number of degrees in one revolution = 360° ________ (1)

Where as the number of radians in one revolution = $\frac{\text{circumsference of Circle}}{\text{Radius of same circle}}$

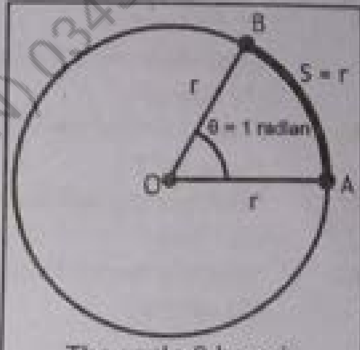
Number of radians in one revolution = $\frac{2\pi r}{r}$ = 2π radians ________ (2)

Comparing Eq (1) & (2)

As number of degrees in one revolution = Number of radians in one revolution

Therefore, 2π rad = 360°

OR $1 \text{ rad} = \frac{360°}{2\pi} = \frac{360°}{2\times3.14} = 57.3°$



The angle θ here is approximately 1 radian.

An angle of approximately 57° corresponds to 1 radian.

In one rotation,

$\theta = 2\pi$ radians = 2×3.14 radians = 6.28 radians

And $360° = 2\pi\, rad$

- $1° = \frac{2\pi}{360}\, rad = \frac{2\times3.14}{360}\, rad = 0.01745\, rad$

Steradian: Steradian is the unit for solid angle.

"Steradian is defined as the solid angle subtended at the center of sphere by an area of its surface equal to the square of radius of that sphere".

Formula:

Mathematically,

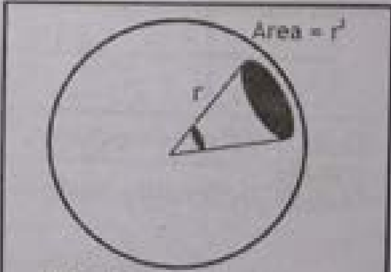
Number of steradians in sphere = $\frac{\text{Area of Sphere}}{r^2}$

Since

Surface area of closed sphere of radius r is $4\pi r^2$.

Therefore

Number of steradians in sphere = $\frac{4\pi r^2}{r^2}$



When the area on the surface of sphere become equal to the radius squared, then the angle subtended at the center is one steradian.

Sphere (or any closed surface) subtends 4π (12.56 sr).

Assignment 1.1:

A pulley of radius 0.9 m is used to lift a bucket from the well. If it took 3.6 rotations for the pulley to take water out of the well, how deep is water in the well?

Solution: r = 0.9 m

θ = 3.6 rotations = 3.6 × 2π radian = 3.6 × 2 × 3.14 radian = 22.6 radians

Depth of well = distance covered by pulley in 3.6 rotations = S = ?

Since

S = r θ

⇨ S = 0.9 × 22.6

⇨ S = 20.3 m = 20 m

MCQ's

1. **Silicon is obtained from:**

(A) Water (B) Metals (C) Wood (D) Sand

2. **Mass is a________ quantity:**
(A) Derived (B) Base (C) Both derived and base (D) None of these

3. **The SI unit of solid angle is:**
(A) steradian (B) Degree (C) Revolution (D) Radian

4. **Solid angle subtended at the centre of sphere of radius 'r' in steradian is:**
(A) 2π (B) 4π (C) 6π (D) 8π

5. An example of derived unit is
(A) candela (B) ampere (C) coulomb (D) mole

6. **Which is not a base unit in SI units?**
(A) kilogram (B) joule (C) ampere (D) kelvin

7. **The unit of work in base units is __________:**
(A) $kg\ m\ s^{-2}$ (B) $kg\ m\ s^{2}$ (C) $kg\ m^{2}\ s^{-2}$ (D) $kg\ m^{-1}s^{-2}$

8. **The angle made by Ice Cone at its edge is a:**
(A) Plane angle (B) Solid angle (C) Critical angle (D) Abtuse angle

9. **Candela is the S.I unit of:**
(A) Charge (B) Luminous intensity (C) Power (D) Refractive index

10. **The S.I unit of power in terms of base unit are:**
(A) $kg\ m^{-1}s^{-2}$ (B) $kg\ m^{-1}s^{-3}$ (C) $kg\ m\ s^{-2}$ (D) $kg\ m^{2}s^{-3}$

11. **Which of the following is a derived quantity?**
(A) Force (B) Mass (C) Length (D) Time

12. **Which is derived unit?**
(A) candela (B) ampere (C) kelvin (D) newton

13. Unit used for the factor $\sqrt{\frac{\ell}{g}}$ may be:
(A) meter (B) second (C) kilogram (D) radian

Answers Key

1. D	2. B	3. A	4. B	5. C	6. B	7. C	8. B	9. B	10. D	11. A	12. D
13. B											

Q.4 What is scientific notation?

Ans: Scientific Notation

"Numbers are expressed in standard form called scientific notation, which employs power of ten."

OR

"Writing a number in powers of 10 or standard form **M x 10^P** *is called scientific notation."*

It is the product of a number greater than 1 and less than 10 (called the mantissa) and a power (or exponent) of 10:

$$\text{number} = \text{mantissa} \times 10^{\text{Power}}$$

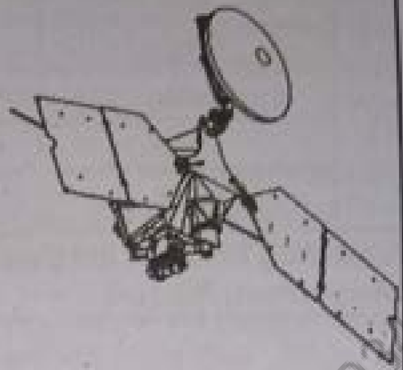
- For example, if someone writes that human body contains approximately 7,000,000,000,000,000,000,000,000,000 atoms = 7×10^{27}, where 7 is the mantissa and 27 is the exponent.
- Another advantage of scientific notation is that it makes it easy to add, subtract, multiply and divide large numbers, For example, to multiply two numbers in scientific notation, we multiply their mantissas and then add their exponents.
- If we wanted to estimate, as how many atoms are contained in the bodies of all the people on Earth, we could do this calculation easily. The population of earth is approximately 7 billion (or 7×10^{9}). To find our answer we have to multiply 7×10^{27} by 7×10^{9}. We do this by multiplying the two mantissas and adding their exponents:

$$(7 \times 10^{27}) \times (7 \times 10^{9}) = (7 \times 7) \times 10^{27+9} = 49 \times 10^{36} = 4.9 \times 10^{37}$$

POINT TO PONDER

Why is it important to have a standard system of units that is used by all scientists and engineers?

In December 1998, the NASA launched the Mars Climate Orbiter on a scientific mission to collect climate data from Mars. Nine months later, on September 23, 1999, the Orbiter disappeared while approaching Mars. The investigation showed that the orbital calculations were incorrect due to an error in the transfer of information between two teams working on the project. One team used English units such as feet and pounds, while the other group assumed the result of the calculation was being reported in SI.



Q.5 What are prefixes? Draw their table.

Ans: Prefixes:

These are letters or symbols written before unit of the measurement to represent certain specific powers of ten. Some prefixes have powers of ten positive (multiples) and some have negative power of ten (Sub multiples). OR

A mechanism through which a term in scientific notation is expressed by giving a proper name to its power of ten is called prefix to the power of ten.

For example,

- The length of a housefly, 5×10^{-3}m, is equivalent to 5 millimeters (mm)
- The distance of a satellite 8.25×10^{5} m from Earth's surface can be expressed as 825 kilometers (km).

PREFIXES

Prefix	Decimal Multiplier	Symbol
Yotta	10^{24}	Y
zetta	10^{21}	Z
Exa	10^{18}	E
Peta	10^{15}	P
Tera	10^{12}	T
Giga	10^{9}	G
Mega	10^{6}	M
Kilo	10^{3}	k
Hector	10^{2}	h
Deca	10^{1}	da
Deci	10^{-1}	d
Centi	10^{-2}	c
milli	10^{-3}	m

micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
femto	10^{-18}	a
zepto	10^{-21}	z
yocto	10^{-24}	y

Q.6 Define the following units of lengths which are mostly used in measurements. Light Year, Angstrom, Micron.

Ans: Prefixes:

i. **Light Year:**
A light-year (ly), is a non-SI unit of length.
"It is defined as the distance that light travels in a vacuum in one year."
1 light year = 9.4607×10^{15}m.
For example
a. Spiral galaxies like our own Milky Way measure approximately 2×10^5 light-years in diameter.
b. Our nearest-neighbor galaxy is the great spiral galaxy Andromeda, which has been determined to be approximately 2.5 million ly away.

ii. **Angstrom:** The angstrom or ångstrom (Å) is also a non-SI of length.
"It is unit of length equal to 1×10^{-10} meters (m) or 0.1 nanometer (nm)."
For example,
a. A helium atom has a size of about 1 Ångström,
b. Nucleus of helium is only 1 femtometer (10^{-15} meters) in diameter.

iii. **Micron:** A micron (μ) is an obsolete name of a micrometer,
It is equal to 1×10^{-6} meters.
For examples,
a. Red Blood Cells (RBCs) are approximately 10 microns in diameter.
b. Human hair is between 10 and 100 microns in *diameter*.

MCQ's

1. One Peta is equal to ________:
(A) 10^{18} (B) 10^{15} (C) 10^{12} (D) 10^{12}
2. The ratio of 1 nanometre to 1 attometre is ________:
(A) 10^{9} (B) 10^{8} (C) 10^{-9} (D) 10^{-12}
3. The prefix pico is equal to ________:
(A) 10^{-12} (B) 10^{-15} (C) 10^{-9} (D) 10^{12}
4. One tera is equal to ________:
(A) 10^{10} (B) 10^{9} (C) 10^{12} (D) 10^{11}
5. Which of the following is least multiple ________:
(A) pico (B) femto (C) nano (D) atto
6. The prefix atto stands for ________:
(A) 10^{9} (B) 10^{18} (C) 10^{-9} (D) 10^{-18}
7. How many nanometers are in a meter?
(A) 10^{-19} (B) 10^{19} (C) 10^{9} (D) 10^{-9}
8. One femto is equal to ________:
(A) 10^{-12} (B) 10^{-13} (C) 10^{-14} (D) 10^{-15}
9. One giga is equal to ________:
(A) 10^{9} (B) 10^{-9} (C) 10^{12} (D) 10^{-12}

Answers Key

1. B	2. A	3. A	4. C	5. D	6. D	7. C	8. D	9. A

DO YOU KNOW

There are some non - SI related units for distances expensively used around the world. Some of these units are discussed below.

A. Light Year: A light-year (ly), is a non-SI unit of length. It is defined as the distance that light travels in a vacuum in one year, which is 9.4607×10^{15} m. For example spiral galaxies like our own Milky Way measure approximately 2×10^{5} light-years in diameter. Our nearest-neighbor galaxy is the great spiral galaxy Andromeda, which has been determined to be approximately 2.5 million ly away.

B. Angstrom: The angstrom or ångstrom (Å) is also a non-SI and internationally recognized unit of length equal to 1×10^{-10} meters (m) or 0.1 nanometer (nm). For example, a helium atom has a size of about 1 Ångström, while its nucleus is only 1 femtometer (10^{-15} meters) in diameter.

C. Micron: A micron (μ) is an obsolete name of a micrometer, which is a decimal fraction of the meter about 1×10^{-6} meters. For examples Red Blood Cells (RBCs) are approximately 10 microns in diameter. Human hair is between 10 and 100 microns in diameter.

Q.7 Write down the conventions for indicating the units. What are scientific notation?

Ans: **Conventions for Indicating the Units (WRITING UNIT SYMBOLS AND NAMES)**

Unit symbols:

- Unit symbols are printed in roman (upright) type. For example m for metre, s for second and Pa for pascal.
  e.g. when writing pico meter, p m is wrong, correct is pm.
- Compound prefixes are never used. For example, nm, not mμm (milli micro meter) or pm not μμm (micro micro meter).
- To multiply or divide the two units, the normal rules of algebraic (multiplication or division) apply.
  - ⇨ Multiplication must be indicated by a space or a half-high (centred) dot (·),
  - ⇨ Division is indicated by a horizontal line, by a solidus (oblique stroke, /) or by negative exponents.

  For example, N m or N · m, for a newton metre and m/s or m s^{-1} for metre per second.
- It is not allowed to use abbreviations for unit symbols or unit names, such as sec (for either s or second), sq. mm (for either mm^2 or square millimetre), cc (for either cm^3 or cubic centimetre).
- When multiple of unit is raised to the power, the power applies to the whole multiple not just the unit.

Unit names:

- Full name of the unit does not begin with a capital letter even if named after scientist e.g., newton.
- The symbol of units named after a scientist has initial capital such as N for newton.
- A multiple or sub-multiple prefix is part of the unit and is written before the unit symbol without any space.

Q.8 Define error. What are its different types?

Ans: **ERRORS:**

"The difference between measured value and acceptable value in any measurement is called an error."

OR

"Error is the doubt that exists about the result of any measurement. For every measurement (even the most careful) there is always a margin of doubt which is called error."

Types of Errors:

The errors in measurement can be broadly classified as (A) systematic errors and (B) random errors.

A. Systematic Errors: The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are:

(a) Instrumental Errors: This error arises due to fault in the instruments.

Causes:

It may be due to imperfect design of measuring instrument

Example:

i. Incorrect calibration of the measuring instrument
ii. Zero error in the instrument

(b) Personal errors: The error due to carelessness or improper knowledge about an instrument or incorrect reading of a scale by an experimenter, is called personal error.

Causes:

It may be due to

i. An individual's bias
ii. Lack of proper setting of the apparatus
iii. Individual's carelessness in taking observations without observing proper precautions

Reduction in Systematic Error:

Systematic errors can be minimized by

i. Improving experimental techniques,
ii. Selecting better instruments and removing personal bias as far as possible.
iii. By gaining the experience after performing the experiment again and again

B. Random Error / Accidental Error / Statistical Error

Random error is said to take place when repeated measurements of the same quantity, gives **different values** under the same conditions.

OR

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size.

Causes:

i. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. changes in temperature, humidity, wind velocity, etc.),
ii. Personal (unbiased) errors by the observer taking readings, etc. For example, when the same person repeats the same observation, it is very likely that he may get different readings every time.

Minimization:

The random error can be reduced by taking several readings of same quantity and then taking their mean value.

Least Count Error:

"The smallest value that can be measured by the measuring instrument is called its least count." All the readings or measured values are good only up to this value. The least count error is the error associated with the resolution of the instrument.

For example, a vernier callipers has the least count as 0.01 cm; a spherometer may have a least count of 0.001 cm. Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors. We can reduce the least count error by using instruments with higher resolution, improving experimental techniques, etc.

Q.9 Define error. What are its different types? How uncertainty is calculated in addition, subtraction, multiplication, division and power factor measurements?

Ans: UNCERTAINTIES

The estimate of the possible range of an error in any measurement is called uncertainty.

Or

The quantification or magnitude of error or doubt in measurement is called uncertainty.

- Uncertainty estimate how small or large the error is.

➤ As there is uncertainty to some extent in every measurement, therefore every measurement need to be written in the form

$$\textit{measurement} = \textit{best estimate} \pm \textit{uncertainty}$$

Example:

For example a measurement of (5.07 ± 0.02)g means that the experimenter is confident that the actual value for the quantity being measured lies between 5.05 g i.e. (5.07 - 0.02)g and 5.09 g i.e. (5.07 + 0.02)g.

Types of Uncertainties:

Absolute Uncertainty or Uncertainty = Least count of measuring instrument
(It is denoted by the symbol 'Δ' and has the same units as the quantity.)

i. **Fractional Uncertainty or Relative Uncertainty** = $\frac{\text{Absolute Uncertainty}}{\textit{Measurement}}$
(It is denoted by the symbol 'ε' and has no units.)

ii. **Percentage Uncertainty** = Fractional Uncertainty x 100%
$= \frac{\text{Absolute Uncertainty}}{\textit{Measurement}} \times 100\%$

For example, length of a cylinder measured with Vernier calipers of least count 0.01 cm is 2.35cm, so

Measurement of length = 2.35cm
Uncertainty = L.C. of Vernier calipers= ±0.01 cm

So its length will be written as:

L = 2.35 ± 0.01 cm

Indicating Uncertainty in Calculation:

A numeric measure of confidence in a measurement or result is known as uncertainty. A lower uncertainty indicates greater confidence. Uncertainties are usually expressed by using statistical methods.

A. Sum or difference: Absolute uncertainties are added.

➤ Suppose two physical quantities A and B have measured values
A ± ΔA and B ± ΔB respectively
where ΔA and ΔB are their absolute uncertainties. The following steps are followed for the result Z = Z ± ΔZ in their sum and difference.

Sum: Let Z = A + B and the measured values of A and B are A ± ΔA and B ± ΔB. We have by addition, Z ± ΔZ = (A ± ΔA) + (B ± ΔB). Z ± ΔZ = (A +B) ± (ΔA + ΔB). The maximum possible uncertainty is Z ± ΔZ = ΔA + ΔB	**Difference:** Let Z = A - B and the measured values of A and B are A ± ΔA and B ± ΔB. We have Z ± Δ Z = (A ± ΔA) – (B ± ΔB) Z ± ΔZ = (A – B) ± ΔA + ΔB The maximum value of the uncertainty ΔZ is again ΔZ = ΔA + ΔB.

B. Product or quotient: Fractional uncertainties are converted into percentage uncertainties which are added.

➤ Suppose two physical quantities A and B have measured values A ± ΔA, B ± ΔB respectively where ΔA and ΔB are their absolute uncertainties and ΔA% and ΔB% are their percentage uncertainties. The following steps are followed for the result Z ± ΔZ in their product and quotient.

Product: Suppose Z = AB and the measured values of A and B are A ± ΔA and B ± ΔB. Then Z ± ΔZ = (A ± ΔA) (B ± ΔB) Convert fractional uncertainty to percentage uncertainty Z ± ΔZ = (A ± ΔA%) (B ± ΔB%)	**Quotient:** Suppose Z = A/B and the measured values of A and B are A ± ΔA and B ± ΔB. Then Z ± ΔZ = (A ± ΔA) / (B ± ΔB) Convert fractional uncertainty to percentage uncertainty Z ± ΔZ = (A ± ΔA%) / (B ± ΔB%)

Multiply the product and add percentage uncertainties $Z \pm \Delta Z = AB \pm (\Delta A\% + \Delta B\%)$ $Z \pm \Delta Z = AB \pm (\Delta A + \Delta B)\%$ Convert back to absolute fractional uncertainty $Z \pm \Delta Z = AB \pm (\Delta Z)$	Divide the ratios and add percentage uncertainties $Z \pm \Delta Z = A / B \pm (\Delta A\% + \Delta B\%)$ $Z \pm \Delta Z = A / B \pm (\Delta A + \Delta B)\%$ Convert back to absolute uncertainty $Z \pm \Delta Z = A / B \pm (\Delta Z)$

C. Power: Percent uncertainty is multiplied by power.

The result is converted back into fractional uncertainty and then into absolute uncertainty.

- Suppose $Z = A^n$ and the measured values of A are $A \pm \Delta A$.

Then

$$Z \pm \Delta Z = (A \pm \Delta A)^n$$

Convert fractional uncertainty to percentage uncertainty

$$Z \pm \Delta Z = (A \pm \Delta A\%)^n$$

$$Z \pm \Delta Z = A^n \pm n \times \Delta A\%$$

$$Z \pm \Delta Z = A^n \pm (n\Delta A)\%$$

Convert back to absolute uncertainty

$$Z \pm \Delta Z = A^n \pm (\Delta Z)$$

DO YOU KNOW:

Technical specification data

Manufacturers of scientific instruments often supply data sheets that specify how accurate the instrument is. The instruments have been tested against very accurate standard instruments and the results are shown on the data sheet.

As you might expect, instruments that are guaranteed to be more accurate are usually more expensive.

Assignment 1.2:

A physicist calculated the wall width of half brick thickness (the brick is laid in a flat position, lengthwise called stretcher position), as (13.6 ± 0.1) cm. And one brick thickness (the brick is placed in flat position, lengthwise orthogonal to wall, called header position), as (23.6 ± 0.1)cm. Calculate the difference in width of walls with uncertainty in it.

Solution:

Given Data: $W_1 = 13.6 \pm 0.1$ cm

$W_2 = 23.6 \pm 0.1$ cm

To Find: Difference in widths = W = ?

$\Rightarrow W = (W_2 - W_1) \pm (\Delta W_1 + \Delta W_2)$

$\Rightarrow W = (23.6 - 13.6) \pm (0.1 + 0.1)$

$\Rightarrow W = 10.6 \pm 0.2$ cm

Assignment 1.3:

The voltage 'V (V ± ΔV)' is measured as 7.3 V ± 0.1 V and current 'I (I ± ΔI)' is measured as 2.73 A ± 0.05 A. Calculate the resistance 'R' by using Ohm's Law as R = V/I.

Solution

Given Data:

$V = 7.3 \pm 0.1$ V

$I = 2.73 \pm 0.05$ A

According to ohm's law

$$V = IR \quad \Rightarrow \quad R = \frac{V}{I}$$

$$\Rightarrow \quad R = \frac{V}{I} = \frac{7.3}{2.73} = 2.7\ \Omega$$

The percentage uncertainty in voltage V is:

$$\text{Percentage uncertainty in V} = \frac{\Delta V}{V} \times 100\% = \frac{0.1}{7.3} \times 100\%$$

$$= 1.37\% = 1\%$$

and percentage uncertainty in current is

POINT TO PONDER:

Error versus uncertainty: It is important not to confuse the terms 'error' and 'uncertainty'. Error is the difference between the measured value and the true value of the thing being measured. Uncertainty is a quantification of the doubt about the measurement result. Whenever possible we try to correct for any known errors: for example, by applying corrections from calibration certificates. But any error whose value we do not know is a source of uncertainty.

Percentage uncertainty in $I = \frac{\Delta I}{I} \times 100\% = \frac{0.05}{2.73} \times 100\%$

$= 1.83\% = 2\%$

Thus total uncertainly in R = 1% + 2% = 3%

Absolute uncertainty in R = ±(3/100) x 2.7Ω = ± 0.08Ω

So result is: R = (2.7 ± 0.08) Ω

Assignment 1.4:

The radius of sphere 'r' is measured with vernier callipers as (r ± Δr)= (2.25 ± 0.01) cm. Calculate the volume of sphere. **(47.7 ± 0.6) cm³)**

Solution:

Given Data: Radius of sphere = r = 2.25 cm

Absolute uncertainty (uncertainty) in radius = Δr = ±0.01 cm

So, r = 2.25 ±0.01 cm

To Prove: Volume = V = ?

Uncertainty in V = ?

So, volume of sphere is:

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (2.25)^3$$

$$= 47.689\ cm^3 = 47.7\ cm^3$$

Absolute uncertainty in r (Δr) = Least count of Vernier calipers = ±0.01 cm

%age uncertainty in r = % (Δr) = (0.01/2.25) x 100% = 0.4%

Total percentage uncertainty in V = 3 x % uncertainty in r = 3 x 0.4% = 1.2%

Absolute Uncertainty in $V = \pm (\frac{1.2}{100} \times 47.7\ cm^3) = \pm 0.6\ cm^3$

So, $V = (47.7 \pm 0.6)\ cm^3$

MCQ's

1. The percentage uncertainty in mass and velocity of an object are 2% and 3% respectively. Which of the following is the maximum uncertainty in the measurement of its kinetic energy?
(A) 11% (B) 8% (C) 6% (D) 1%

2. If x_1 = 10.5 ± 0.1 cm and x_2 = 26.8 cm. ± 0.1cm $x_2 - x_1$ is given by:
(A) 16.3 ± 0.1cm (B) 16.3 ± 0.2cm (C) 16.1 ± 0cm (D) 16.3 ± 0.4cm

3. The percentage uncertainty in radius of a sphere is 2%. Which of the following is the total percentage uncertainty in the volume of a sphere?
(A) 2% (B) 4% (C) 6% (D) 8%

Answers Key

1. B	2. B	3. C

Q.10 What are significant figures? How can we estimate the number of significant figure in the physical measurement?

Ans: **Significant Figures**

In any measurement, the accurately known digits and the first doubtful digit are called significant figures.

OR

In other words, a significant figure is the one which is known to be reasonably reliable.

Importance

The concept of significant figures is very important and useful. It shows the extent or limit to which readings are reliable.

Rules for Finding Significant Figures

1. NON ZERO digits are always significant. That is all the digits from 1 to 9 are significant, e.g. the number of significant figures in 47.872 is 5.
2. ZERO in between two significant digits is always significant, e.g. the number of significant figures in 301.5006 is 7.
3. ZEROs to the left of significant figures are not significant, e.g. the number of significant figures in 0.000538 is 3.
4. ZEROs to the right of the significant figure may or may not be significant.
   i. In decimal fractions zero to the right of a decimal fraction are significant, e.g. in 5.200 there are 4 significant figures.
   ii. However if the number is an integer, number of significant figures depends upon the least count of the measuring instrument.
   Example: consider the measurement 8000 kg,
   If the least count is 1 kg then there are four significant figures and the measured value will be expressed as 8.000×10^3 kg.
   Similarly if the least count is 10 kg and the correct measurement in appropriate significant figures is 8.00×10^3 kg, then there are three significant figures.
5. In scientific notation or standard form, the figures other than power of ten are significant, e.g. in number 2.1000×10^4, there are 5 significant figures.

Significant Figures in Calculation:

(a) Addition and Subtraction:

Rule:

When two or more quantities are added or subtracted, the result is as precise as the least precise of the quantities. After adding or subtracting, round the result by keeping only as many decimal places as are in the figure containing least decimal places of the quantities that were added or subtracted.

For example, $44.56005 + 0.0698 + 1103.2 = 1147.82985$.

We do not want to write all of those digits in the answer. Answer should be written upto one decimal place, because data has least one decimal place.

the sum is written = 1147.8.

(b) Multiplication and Division:

Rule:

When quantities are multiplied or divided, the result has the same number of significant figures as the quantity with the smallest number of significant figures.

For example, A calculator gives $45.26 \times 2.41 = 109.0766$.

Since the answer should have only three significant figures, we round the answer to

$$45.26 \times 2.41 = 109.2$$

In scientific notation, we write 1.09×10^2.

TIP

In a series of calculations, rounding to the correct number of significant figures should be done only at the end, not at each step. Rounding at each step would increase the chance that roundoff error could snowball and have an adverse effect on the accuracy of the final answer. It's a good idea to keep at least two extra significant figures in calculations, then round at the end.

Assignment 1.5:

Calculate the answers to the appropriate number of significant figures.

(a) 0.31 + 0.1 **(b) 658.0 + 23.5478 + 1345.29** **(c) 8 × 7**

(d) 0.9935 × 10.48 × 13.4 **(e) 5.5 / 1.1** **(f)** $\frac{73.2 + 18.72 \times 6.1}{3.4}$

Solution:

(a) $0.31 + 0.1 = 0.41 = 0.4$ (data has least 1 decimal place)

(b) $658.0 + 23.5478 + 1345.29 = 2026.8378 = 2026.8$ (data has least 1 decimal place)

(c) $8 \times 7 = 56 = 60$ (data has least one significant figure)

(d) $0.9935 \times 10.48 \times 13.4 = 139.519192 = 1.39519192 \times 10^2 = 1.40 \times 10^2$ (data has least 3 significant figure)

(e) $5.5 / 1.1 = 5 = 5.0$ (data has 2 significant figures)

(f) $\frac{73.2 + 18.72 \times 6.1}{3.4} = 55.1152941176 = 55$ (data has 2 significant figures)

MCQ's

1. **How many Significant figures are in 0.00876?**
(A) 3 (B) 4 (C) 5 (D) 6

2. **What is the sum of three numbers 2.7543, 4.10, 1.273 up to correct decimal place?**
(A) 8.12 (B) 8.13 (C) 8.1273 (D) 8.127

3. **Which is correct for the diameter of wire when measured by a screw gauge of least count 0.001 cm?**
(A) 2.3cm (B) 2.31cm (C) 2.312cm (D) 2.3124cm

4. **In scientific notation the number 0.0001 may be written as ________:**
(A) 10^{-2} (B) 10^{-3} (C) 1.0×10^{-4} (D) 10×10^{4}

5. **A student added three figures 72.1, 3.32 and 0.003. What is the correct answer regarding the rules of the addition of the significant figures?**
(A) 75.423 (B) 75.42 (C) 75.4 (D) 75.5

6. **Number of significant figures in 01. 020 mm are ________:**
(A) 2 (B) 3 (C) 4 (D) 5

7. **275.00 has the significant digits ________:**
(A) 2 (B) 3 (C) 4 (D) 5

8. **Significant figures in 0.0010 are ________:**
(A) 1 (B) 2 (C) 3 (D) 4

9. **Which one of the following is the correct record for the diameter of a ball when measured with a vernier calipers of least count of 0.01 cm:**
(A) 5.3 cm (B) 5.31 cm (C) 5.312 cm (D) 5.3124 cm

10. **The significant figures in 0.04060 are ________:**
(A) 2 (B) 4 (C) 5 (D) 6

Answers Key

1. A	2. B	3. C	4. C	5. C	6. C	7. D	8. B	9. B	10. B

Q.11 Differentiate between precision and accuracy in measurement.

Ans: **PRECISION AND ACCURACY**

When a value is measured, two parameters precision and accuracy affect the quality of the measurement. Therefore it is important to clearly distinguish between them.

Precision:

i. *In measurements the term precision describes the degree of exactness with which a measurement is made and stated (that is, the position of the last significant digit i.e. last digit of least count).*

For example, precision of the measurement 293,000 km is 1000 km. (The position of the last significant digit is in the thousands place).

Similarly the precision of the measurement 0.0210 s is 0.0001 s. (The position of the last significant digit is in the ten thousandths place).

ii. *A precise measurement is the one which has less* ***absolute uncertainty.***

iii. *The precision of a measurement is associated with the least count of the measuring instrument.*

iv. The precision of a measurement is determined by the **instrument** or device being used.

v. Smaller the least count of the instrument, measurement will be more precise and vice versa.

vi. Precision means how close the measured values are to each other.

Accuracy:

i. *In measurement the accuracy describes the closeness of a measured value to the actual value of the measured quantity.*

OR

The accuracy of a measurement is the difference between your measurement and the accepted correct answer. The bigger the difference, the less accurate your measurement.

ii. *An accurate measurement is one which has less fractional or percentage error (i.e. percentage uncertainty).* The accuracy of a measurement depends on the fractional or relative error (uncertainty) or percentage uncertainty in that measurement.

iii. Accuracy is related with the measurement.

iv. Smaller the fractional or percentage uncertainty, measurement will be more accurate and vice versa.

v. The accuracy of a measurement depends upon the number of significant digits. The greater the number of significant digits given in a measurement, the better is the accuracy, and vice versa. For example the accuracy of the measurement 0.025 cm is indicated by two significant digits.

vi. Accuracy means how close a measured value (result) is to the actual (true) value.

TABLE : PRECISION AND ACCURACY

Measurement	Precision	Accuracy (Significant digits)
2642 m	1 m	4
2050 m	10 m	3
34,000 km	1000 km	2
203.05 kg	0.01 kg	5
0.000285 kg	0.000001 kg	3
75 N	1 N	2
4.050 μs	0.001 μs	4
100.050 km	0.001 km	6

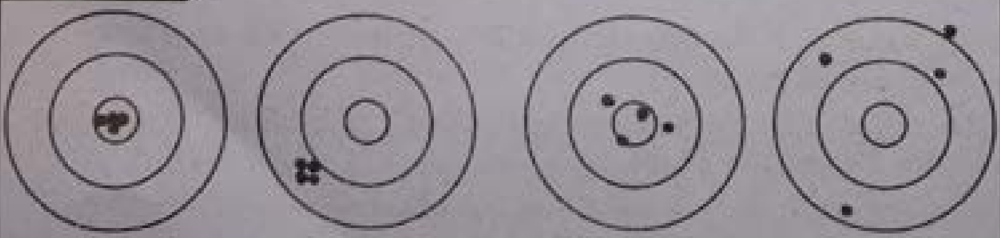
Accuracy shows how well the results of a measured value agree with the actual value (that is the accepted value as measured by competent experimenters). As it is difficult to know the actual (true) value; it is only predicted theoretically, and then is accepted based on the results of repeated experiments.
For example, the accepted value of acceleration due to gravity is 9.80 m/s.

Understanding Precision and Accuracy Using Game of Dots

Consider a dart game with bulls-eye at the center as shown in the Figure.

- If the darts land close to the bulls-eye and close together, there is both accuracy and precision as in Figure (a).
- If all of the darts land very close together, but far from the bulls-eye, there is precision, but not accuracy as in Figure (b).
- If the darts are spread around the bulls-eye there is mathematical accuracy because the average of the darts is in the bulls-eye as in Figure (c).
- If the darts are neither close to the bulls-eye, nor close to each other, there is neither accuracy, nor precision as in Figure (d).

FIGURE 1.3



(a) both accurate and precise | (b) precise but not accurate | (c) accurate but not precise | (d) neither accurate nor precise

$$[R.H.S] = [LT^{-1}] + [LT^{-1}]$$
$$= 2[LT^{-1}]$$

As 2 has no dimension being a number, so

$$[R.H.S.] = [LT^{-1}]$$

Thus $[R.H.S.] = [L.H.S]$

Hence the equation is dimensionally correct.

(b) $[L.H.S.] = [S] = [L]$

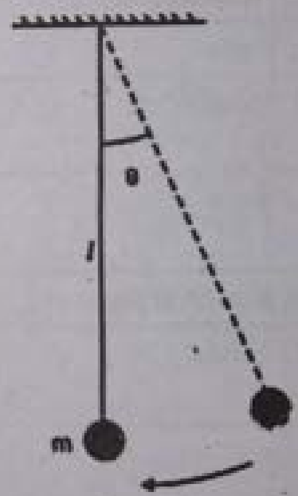
$$[R.H.S.] = [V_i][t] + \frac{1}{2}[a][t]^2$$
$$= [LT^{-1}][T] + [LT^{-2}][T]^2$$
$$= [LT^{-1+1}] + [LT^{-2+2}]$$
$$= [LT^{0}] + [LT^{0}]$$
$$= [L] + [L] = 2[L]$$
$$= [L]$$

$\Rightarrow$ $[L.H.S.] = [R.H.S.]$

So, equation is dimensionally consistent.

Assignment 1.7:

Find an expression for the time period 'T' of a simple pendulum. The time period 'T' may depend upon (i) mass 'm' of the bob of the pendulum, (ii) length 'l' of pendulum, (iii) acceleration due to gravity 'g' at the place where the pendulum is suspended.



Solution:

To Find:

Relation for the time period of simple pendulum = T = ?

Calculations:

The relation for the time period 'T' will be of the form.

$$T \propto m^a \times l^b \times \theta^c \times g^d$$

or $T = \text{constant } m^a\, l^b\, \theta^c\, g^d$ ——— (1)

Taking dimensions on both sides, we get

As $S = r\theta$

Or $\theta = \frac{S}{r}$

Thus $\theta = [LL^{-1}]$

$$[T] = \text{constant } [M]^a [L]^b [LL^{-1}]^c [LT^{-2}]^d$$

Comparing the dimensions on both sides

$$[T]^1 = [T]^{-2d}$$
$$[M]^0 = [M]^a$$
$$[L]^0 = [L]^{b+d+c-c}$$

Equating powers on both sides, we get

$-2d = 1$ or $d = -\frac{1}{2}$

$a = 1$

and $b + d = 0$

Or $b = -d$ or $b = \frac{1}{2}$

and $\theta = [LL^{-1}]^c = [L^0]^c = 1$

Substituting the values of a, b, c and d in equ.(1)

$$T = \text{constant} \times m^0 \times \ell^{\frac{1}{2}} \times 1 \times g^{-\frac{1}{2}}$$

or $$T = \text{constant} \times \ell^{\frac{1}{2}} \times g^{-\frac{1}{2}}$$

or $$T = \text{constant}\sqrt{\frac{\ell}{g}}$$

DIMENSIONAL FORMULA (DIMENSIONS) OF PHYSICAL QUANTITIES

Quantity	Formula	Dimensions	SI Unit	In terms of base unit
1. Displacement	Distance in a particular direction	[L]	m	m
2. Area	$[\text{Length}]^2$	$[L^2]$	m^2	m^2
3. Volume	$(\text{length})^3$	$[L^3]$	m^3	m^3
4. Density	Mass/volume	$[ML^{-3}]$	Kg/m^3	$Kg\,m^{-3}$
5. Speed	Distance/time	$[LT^{-1}]$	m/s	$m\,s^{-1}$
6. Acceleration	$a = \Delta v/\Delta t$	$[LT^{-2}]$	$m\,s^{-2}$	$m\,s^{-2}$
7. Force	F = ma	$= [MLT^{-2}]$	newton (N)	$Kg\,m\,s^{-2}$
8. Momentum	P = mv	$= [MLT^{-1}]$	N s	$Kg\,m\,s^{-1}$
9. Work	W = F d	$[MLT^{-2}] \times (L)$ $= [ML^2T^{-2}]$	joule (J)	$Kg\,m^2\,s^{-2}$
10. Kinetic energy	$K.E = 1/2\,mv^2$	$[ML^2T^{-2}]$	joule (J)	$Kg\,m^2\,s^{-2}$
11. Torque	τ = Force × moment arm	$[MLT^{-2}] \times (L)$ $= [ML^2T^{-2}]$	N m	$Kg\,m^2\,s^{-2}$
12. Power	$P = \frac{Work}{Time}$	$= \frac{[ML^2T^2]}{(T)}$ $= [ML^2T^{-3}]$	watt (W)	$Kg\,m^2\,s^{-3}$
13. Pressure	$P = \frac{Force}{Area}$	$= \frac{[MLT^{-2}]}{(L^2)}$ $= [ML^{-1}T^{-2}]$	Nm^{-2} and Pascal (Pa)	$Kg\,m^{-1}\,s^{-2}$
14. Stress	=Force/area	$[ML^{-1}T^{-2}]$	Nm^{-2}	$Kg\,m^{-1}\,s^{-2}$
15. Modulus of elasticity	$= \frac{Stress}{Strain}$	$= [ML^{-1}T^{-2}]$	Nm^{-2}	$Kg\,m^{-1}\,s^{-2}$
16. Gravitational constant G	$= \frac{Force \times (Distance)^2}{(Mass)^2}$	$[M^{-1}L^3T^{-2}]$	$Nm^2\,kg^{-2}$	$Kg^{-1}m^3\,s^{-2}$

17. Co-efficient of viscosity	$=\frac{Force}{Velocity \times Distance}$	$=\frac{[MLT^{-2}]}{[L^2]\times[LT^{-1}]}$ $=[ML^{-1}T^{-1}]$	Nsm^{-2}	$Kg\,m^{-1}s^{-1}$
18. Time period	$T=\frac{total\ time}{No.of\ Vibrations}$	[T]	s	s
19. Frequency	$=\frac{Number\ of\ vibrations}{Time}$	$=[T^{-1}]$	Hz	
20. Force constant (OR) Spring constant	$K=\frac{force}{extension}$	$=\frac{[MLT^{-2}]}{[L]}$ $=[MT^{-2}]$	$N\,m^{-1}$	$kg\,s^{-2}$
21. Angle	$\theta = \frac{Arc\ Length}{Radius}$	$=\frac{[L]}{[L]}$ No dimensions	radian (rad)	
22. Angular velocity	$\omega=\frac{Angle}{Time}$	$[T^{-1}]$	$rad\,s^{-1}$	
23. Angular acceleration	$\alpha=\frac{change\ in\ angular\ velocity}{time}$	$=[T^{-2}]$	$rad\,s^{-2}$	
24. Angular momentum	L = mvr	$[M \times LT^{-1} \times L]$ $=[ML^2T^{-1}]$	$kg\,m^2\,s^{-1}$ (J s)	$Kg\,m^2\,s^{-1}$

QUANTITIES HAVING SAME DIMENSIONS

Quantities	DIMENSIONS
1. Length, distance, displacement, width, height, radius, diameter, thickness, depth ,Light year, Wavelength, Range of projectile.	[L]
2. Momentum, Impulse	$[MLT^{-1}]$
3. Angular momentum, Planck's constant	$[ML^2T^{-1}]$
4. Work, Energy, Moment of force (Torque),	$[ML^2T^{-2}]$
5. Pressure, Stress, Elastic constant, Energy density	$[ML^{-1}T^{-2}]$
6. Frequency, , Angular velocity, Angular frequency	$[T^{-1}]$
7. Force , weight, tension in string	$[MLT^{-2}]$

MCQ's

1. **Which pair has the same dimension?**
   (A) Work and Power (B) Force and Torque (C) Torque and Power (D) None
2. **Which of the following are the dimensions of work?**
   (A) $[MLT^{1}]$ (B) $[MLT^{2}]$ (C) $[ML^2T^{-2}]$ (D) [MLT]
3. **Which of the following are the dimensions of light year?**
   (A) $[LT^{-1}]$ (B) [T] (C) $[ML^2T^2]$ (D) [L]
4. **Which of the following pair has the same dimensions?**
   (A) Work and Power (B) Momentum and Energy (C) Power and Pressure (D) Work and Torque

5. The dimensions $[ML^2T^{-2}]$ belongs to:
(A) Pressure (B) Momentum (C) Power (D) Heat energy

6. Which of the following are the dimensions of density?
(A) $[ML^{-2}]$ (B) $[M^2L^{-2}]$ (C) $[ML^{-3}]$ (D) $[M^{-1}L^{-1}]$

7. $[M^0L^0T^{-1}]$ refers to:
(A) Velocity (B) Time period (C) Frequency (D) Force

8. Which of the following are the dimensions of torque?
(A) $[ML^{-1}T]$ (B) $[ML^2T^1]$ (C) $[ML^2T^{-2}]$ (D) $[ML^{-1}T^2]$

9. Which of the following are the dimensions of acceleration due to gravity?
(A) $[MLT^{-2}]$ (B) $[MLT]$ (C) $[LT^{-2}]$ (D) $[M^0L^2]$

10. Which of the following are the dimensions of angular velocity?
(A) $[LT^{-1}]$ (B) $[LT^{-2}]$ (C) $[L^{-1}T]$ (D) $[T^{-1}]$

11. Which of the following pair have same dimensions?
(A) Work and power (B) Momentum and impulse (C) Force and torque (D) Torque and power

12 The dimension of $\sqrt{\frac{\ell}{g}}$ is same as that of:
(A) Time (B) Energy (C) Velocity (D) Force

13. The dimensions of the relation $\sqrt{\frac{F \times l}{m}}$ are equal to the dimensions of:
(A) Force (B) Momentum (C) Acceleration (D) Velocity

14. Which of the following are the dimensions of Power?
(A) $[ML^{-2}T^{-3}]$ (B) $[MLT^{-2}]$ (C) $[ML^2T^{-3}]$ (D) $[M^2L^2T^{-1}]$

15. Which of the following are the dimension of coefficient of viscosity?
(A) $[ML^{-1}T^{-1}]$ (B) $[MLT^{-2}]$ (C) $[MLT^{-1}]$ (D) $[ML^{-3}T^{-5}]$

Answers Key

1. D	2. C	3. D	4. D	5. D	6. C	7. C	8. C	9. C	10. D	11. B	12. A
13. D	14. C	15. A									

General Information

1. Which of the following is equal to the numerical value of one light year?
(A) 9.46×10^{5} mm (B) 9.46×10^{15} cm (C) 9.46×10^{15} m (D) 9.46×10^{15} km

2. How many number of colors used in the process of color printing to produce the entire range of colors:
(A) 4 (B) 5 (C) 6 (D) 7

3. Light year is the unit of _______:
(A) Length (B) Mass (C) Time (D) Speed

Answers Key

1. C	2. A	3. A

Key Points

- **Physics:** The study of the physical word in specific and physical universe in general.
- **System International (SI):** System of units is adopted specifically by the science community for measurement of physical quantities. The SI units consists of seven fundamental units from which all the units for other physical quantities developed called derived units.
- **Least count or resolution:** The smallest increment measurable by measuring instrument.
- **Error:** The doubt that exists about the result of any measurement.
- **Uncertainty:** The quantification or magnitude of error or doubt in measurement.
- **Precision:** The degree of exactness with which a measurement is made and stated.
- **Accuracy:** The closeness of a measured value to the actual value of the measured quantity.
- **Significant figures:** In any measurement the number of accurately known digits and first doubtful digit.
- **Dimension:** Expressing a physical quantity in terms of base physical quantities (by using special symbols). Dimensions of a physical quantity help to understand its relation with base physical quantities.

Solved Examples

Example 1.1: **Two connected gears are rotating. The smaller gear has a radius of 0.4 m and the larger gear's radius is 0.7 m. What is the angle through which the larger gear has rotated when the smaller gear has made one complete rotation?**

Given Data: Larger gear's radius 'r_L' = 0.7 m
Smaller gear's radius 'r_s' = 0.4 m

Required: Angle of rotation for larger gear 'θ_L' = ?

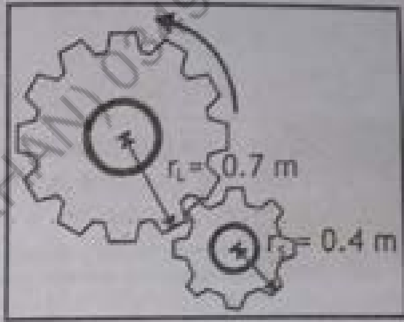
Solution:

The smaller gear performs one complete rotation ($\theta = 2\pi$), the length of the S arc traveled is: $S = r_s \times \theta_s$

$$S = r_s \times \theta_s$$

Putting values $S = 0.4\text{m} \times 2\pi$

$$S = 0.8\pi\text{m}$$



So, an 8π arc length on the larger circle would form an angle as follows:

$$\theta_L = \frac{S}{r_L}$$

Putting values $\theta_L = \frac{0.8\pi\text{m}}{0.7\text{m}}$

$\theta_L = 3.6$ radians **Ans**

Extension Exercise:

What is the angle in degrees through which the larger gear has rotated?

$\theta_L = 3.6 \text{ radians} = 3.6 \times 57.3^\circ = 206.3^\circ$

Example 1.2: **If d ± Δd = (101.41 ± 0.05) mm represents the internal diameter of the metal pipe and d ± Δd = (102.79 ± 0.05) mm represents the external diameter of the metal pipe, then find the thickness of the metal part of the pipe including uncertainty in it.**

Given Data: Internal diameter $d \pm \Delta d = (101.41 \pm 0.05)$ mm
External diameter $d \pm \Delta d = (102.79 \pm 0.05)$ mm

Required: Thickness of Pipe 't' = ?

Solution:

To find the thickness of the pipe we would subtract the internal diameter from the external diameter however we will add its fractional uncertainties as

$$d = d_2 - d_1 \pm (\Delta d_1 + \Delta d_2)$$

Putting values $d = (102.79 - 101.41)\text{ mm} \pm (0.05 + 0.05)\text{ mm}$

$$d = (1.38 \pm 0.10)\text{mm}$$

Be careful not to subtract uncertainties when subtracting measurements uncertainty ALWAYS gets worse as more measurements are combined.

Since the difference in the radius is required for the thickness 't' therefore both the diameter and the uncertainty must be divided by 2 (as the percentage uncertainty remains the same), Hence

$$t = \frac{d}{2} = \left(\frac{1.38}{2} \pm \frac{0.10}{2}\right)\text{mm}$$

$\Rightarrow$ $t = (0.69 \pm 0.05)\text{mm}$

Example 1.3: **The length and width of a rectangular room are measured to be l = (l ± Δl) = (3.955 ± 0.005) m and w = (w ± Δw) = (3.050 ± 0.005) m. Calculate the area A = (A ± ΔA) of the room and its uncertainty.**

Given: length $l = (4.050 \pm 0.005)$ m
width $w = (2.955 \pm 0.005)$ m

Required: Area $A = (A \pm \Delta A) = ?$

Solution:

For the product percentage uncertainties are added. The length 'l' and width 'w' in percentage uncertainties are

$$l = 4.050\ m \pm \frac{0.005m}{4.050m} \times 100\% = 4.050\ m \pm 0.12\%$$

and $$w = 2.955m \pm \frac{0.005m}{2.955m} \times 100\% = 2.955\ m \pm 0.17\%$$

Since the area o fa rectangle is the product of length and width

$$A = l \times w \quad \text{or} \quad A = (4.050m \pm 0.12\%) \times (2.955m \pm 0.17\%)$$

In multiplication the percentage uncertainties are added

$$A = (4.050m \times 2.995m) \pm (0.12\% + 0.17\%) \quad \text{or} \quad A = 12.20m^2 \pm 0.29\%$$

To convert it back to fractional uncertainty, we have

$$A = (12.20 \pm \frac{0.29}{100} \times 12.20)m^2$$

Or $$A = (12.20 \pm 0.035)m^2$$

Example 1.4:	**A ball drops from rest from an unknown height 'h'. The time 't' it takes for the ball to hit the ground is measured to be (t ± Δt) = (1.3 ± 0.2) s. The height is related to this time by the equation h = ½ gt² (where g = 9.81m/s²). Assume that the value for 'g' carries no uncertainty and calculate the height 'h' including its uncertainty.**

Given: time t $(t \pm \Delta t) = (1.3 \pm 0.2)$ s

acceleration due to gravity 'g' = $9.81 m/s^2$

Required: height 'h' = $(h \pm \Delta h) = ?$

Solution:

For the power percentage uncertainties is multiplied with power. The percentage uncertainty in time 't' is

$$t = 1.3s \pm \frac{0.2s}{1.3s} \times 100\% = 1.3s \pm 15.4\%$$

Since the Height 'h' is given by $h = \frac{1}{2} gt^2$

Putting values $h = \frac{1}{2} \times 9.81ms^{-2} \times (1.3s \pm 15.4\%)^2$

For the power percentage uncertainties is multiplied with power, therefore

$$h = \frac{1}{2} \times 9.81ms^{-2} \times 1.69s^2 \pm 2 \times 15.4\%$$

Or $$h = 8.30m \pm 30.8\%$$

To convert it back to absolute uncertainty, we have

$$h = (8.30 \pm \frac{30.8}{100} \times 8.30)m$$

Or $$h = (8.3 \pm 2.6)m$$

Example 1.5:	**Calculate the answers to the appropriate number of significant figures.** **(a) 0.35 – 0.1** **(b) 32.567 + 135.0 + 1.4567** **(c) 420.03 + 299.270 + 99.068** **(d) 14 × 8** **(e) (2400) (3.45) (16.21)** **(f)** $\frac{32.09 + 1.2 - 17.035}{19.8}$

Solution:

(a) Calculating

$$\begin{array}{r} 0.35 \\ -\underline{0.1} \\ \underline{0.25} \end{array}$$

Not worrying about significant figures, the result of 0.35 – 0.1 = 0.25.

But, according to the rules of significant figures, the result should have the same number of decimal places as the input with the fewest number of decimal places. The result of our calculation should be rounded to the tenths place — so, $0.35 - 0.1 = 0.2$.

Or $0.35 - 0.1 = 0.2$ **Answers.**

(b) Calculating:

$$\begin{array}{r} 32.567 \\ 135.0 \\ +\ \ 1.4567 \\ \hline \mathbf{169.0237} \end{array}$$

But, since according to the rules of significant figures, the result should be rounded to the lowest number of decimal places as in the input given numbers. The result of our calculation should be rounded to one decimal place.

So

Or $32.567 + 135.0 + 1.4567 = 169.0$ **Answers.**

(c) Calculating:

$$\begin{array}{r} 420.03 \\ 299.270 \\ +99.068 \\ \hline \mathbf{818.368} \end{array}$$

But, since according to the rules of significant figures, the result should be rounded to the lowest number of decimal places as in the input given numbers. The result of our calculation should be rounded to two decimal place. Therefore

Or $420.03 + 299.270 + 99.068 = 818.37$ **Answers.**

(d) Calculating:

$$\begin{array}{r} 14 \\ \times\ \ 8 \\ \hline \mathbf{112} \end{array}$$

However, according to the rules of significant figures, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 14 has two significant digits and number 8 has one significant digit. Therefore, the result of our calculation should be rounded to only one significant digit. Therefore

Or $14 \times 8 = 100 = 1 \times 10^2$ **Answers.**

(e) Calculating:

$$\begin{array}{r} 2400 \\ 3.45 \\ \times\ \ 16.21 \\ \hline \mathbf{134,218.8} \end{array}$$

However, according to the rules of significant figures for multiplication, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 2400 has two significant digits, number 3.45 has three significant digits and number 16.21 has four significant digits. Therefore, the result of our calculation should be rounded to two significant digits. Hence

Or $(2400)(3.45)(16.21) = 130,000$ **Answers.**

(f) Calculating:

$$\begin{array}{r} 32.09 \\ +\ \ 1.2 \\ \hline \mathbf{33.29} \end{array}$$

Although the answer should have one decimal place, but we will keep both and proceed.

$$\begin{array}{r} 33.29 \\ -\underline{17.035} \\ 16.255 \end{array}$$

Again the answer should have one decimal place, but we will keep all digits.

$$\frac{16.255}{19.8} = 0.820959596$$

However, according to the rules of significant figures for division, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 16.2 has five significant digits and number 19.8 has three significant digits. Therefore, the result of our calculatio should be rounded to three significant digits. Hence

Or $\frac{32.09 + 1.2 - 17.035}{19.8} = 0.821$ **Answers.**

Example 1.7: **When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η (having dimensions η [$M^1 L^{-1} T^{-1}$]) of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess formula for F using the method of dimensions.**

Given:
Dimensions of coefficient of viscosity 'η' = $[M^1L^{-1}T^{-1}]$
Dimensions of radius 'r' = $[L^1]$
Dimensions of velocity 'v' = $[L^1T^{-1}]$
Dimensions of force 'F' = $[M^1L^1T^{-2}]$

Required: Possible formula for drag force 'F' = ?

Solution:

Let the drag force depends upon coefficient of viscosity 'η', radius 'r' and velocity 'v' by the following equation

$FD \propto \eta^a r^b v^c$ or $F_D = k\eta^a r^b v^c$ ______ (1)

Where k is constant of proportionality, now putting appropriate dimensions

$$[M^1L^1T^{-2}] = [M^1L^{-1}T^{-1}]^a [L^1]^b [L^1T^{-1}]^c$$
$$[M]^1[L]^1[T]^{-2} = [M]^a[L]^{-a+b+c}[T]^{-a-c}$$

Comparing the powers of similar physical quantities

For M	For T	For L
$[M]^1 = [M]^a$	$[T]^{-2} = [T]^{-a-c}$	$[L]^1 = [L]^{-a+b+c}$
a = 1	-a -c = -2	-a + b + c = 1
	as a = 1	as a = 1 and c = 1
	-1 - c = -2	-1 + b + 1 = 1
	-c = -2 + 1	b = 1
	-c = -1	
	c = 1	

Putting values of a, b and c in equation 1 we get

$FD = k\eta^1 r^1 v^1$ $F_D = k\eta rv$

Dimension analysis does not give information about the value of constant, however from experiment we know that this constant is 6π.

Or $FD = 6\pi\eta rv$ **Answer.**

Text Book Exercises

Q.1 Select the correct answer of the following questions.

1. What is the radian measure between the arms of watch at 5.00 pm?
A. 1 radian B. 2 radian C. 3 radian D. 4 radian

2. 1° = ________
A. 0.01745 radian B. 1 radian C. 3.14 radian D. 2π radian

3. The metric prefix for 0.000001 is:
A. hecto B. micro C. deca D. nano

4. Which of the following is the CORRECT way of writing units?
A. 71 Newton B. 12 mμ s C. 8 Kg D. 43 kg m^{-3}

5. A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as:
A. (49.8 ± 0.2) cm. B. (49.8 ± 0.4) cm. C. (50.0 ± 0.2) cm. D. (50.0 ± 0.4) cm.

6. The percent uncertainty in the measurement of (3.76 ± 0.25) m is:
A. 4% B. 6.6% C. 25% D. 376%

7. The temperatures of two bodies measured by a thermometer are t_1 = (20 ± 0.5) °C and t_2 = (50 ± 0.5) °C. The temperature difference and the error therein is:
A. (30 ± 0.0) °C B. (30 ± 0.5) °C C. (30 ± 1) °C D. (30 ± 1.5) °C

8. (5.0m ± 4.0%) × (3.0s ± 3.3%) =:
A. 15.0 ms ± 13.2% B. 15.0 ms ± 7.3% C. 15.0 ms ± 0.7% D. 15.0 ms ± 15.3%

9. $(2.0 \text{ m} \pm 2.0\%)^3$ =
A. 8.0m^3 ± 1.0% B. 8.0m^3 ± 2.0% C. 8.0m^3 ± 5.0% D. 8.0m^3 ± 6.0%

10. The number of significant figures in measurement of 0.00708600 cm are:
A. 3 B. 4 C. 6 D. 9

11. How many significant figures does 1.362 + 25.2 have?
A. 2 B. 3 C. 5 D. 8

12. Compute the result to correct number of significant digits 1.513 m + 27.3 m =:
A. 29 m B. 28.8 m C. 28.81 m D. 28.813 m

13. If 7.635 and 4.81 are two significant numbers. Their multiplication in significant digits is:
A. 36.72435 B. 36.724 C. 36.72 D. 36.7

14. The precision of the measurement 385,000 km is:
A. 10 km B. 100 km C. 1000 km D. 1000000 km

15. [M°L°T°] are dimension of:
A. strain B. refractive index C. magnification D. all of these

15. [M°L°T°] are dimension of:
A. strain B. refractive index C. magnification D. all of these

16. The dimensions of torque are
A. [MLT] B. [M^{2}L^{2}T] C. [ML^{2}T^2] D. [ML^{2}T^2]

No.	Option	ANSWER	EXPLANATION
1.	C	3 rad	In one rotation, $\theta = 360^o$ Hours hand is divided in 12 parts, angle covered when hours hand covers one hour is: $\theta = \frac{360^o}{12} = 30^o$ So, total angle covered in 5 hours is $\theta = 30^o \times 5 = 150^o$ $\Rightarrow \theta = 150 \times \frac{2\pi}{360}$ rad

			$\Rightarrow \theta = 150 \times \frac{2\times3.14}{360}\ rad$ $\Rightarrow$ 2.61 rad = 3 rad
2.	A	0.01745 rad	$360^\circ = 2\pi rad \Rightarrow 1^\circ = \frac{2\pi}{360} rad = \frac{2\times3.14}{360} rad = 0.01745\ rad$
3.	B	Micro	$0.000001 = 1 \times 10^{-6}$
4.	D	43 kg m^{-3}	
5.	C	(50.0 ± 0.2) cm.	It is average of given maximum and minimum values
6.	C	6.6%	$\%\Delta = \frac{0.25}{3.76} \times 100\% = 6.6\%$
7.	C	(30 ± 1) °C.	$t_2 - t_1 = (50-20) \pm (0.5+0.5) = 30 \pm 1\ °C$
8.	B	15.0 ms ± 7.3%	$(5.0 \times 3.0) \pm (4\% + 3.3\%) = 15 \pm 7.7\%\ ms$
9.	D	$8.0 m^3 \pm 6.0\%$	$(2.0\ m \pm 2.0\%)^3 = (2.0\ m)^3 \pm 3(2.0\%) = 8.0\ m^3 \pm 6.0\ \%$
10.	C	6	In (0.00708600), digits 708600 are significant, zeros on left side of 7 are not significant.
11.	B	3	1.362 + 25.2 = 26.562 = 26.6 As answer should be up to least decimal places in the given data, 25.2 has one decimal place, so answer will be 26.6 after rounding off which contains 3 significant figures.
12.	B	28.8 m	1.513 m + 27.3 m = 28.813 m = 28.8 m As answer will be up to one decimal place
13.	D	36.7	7.635 × 4.81 = 36.72435 = 36.7 In multiplication, answer should be upto least significant figures in the data.
14.	C	1000km	Since least count of measurement is 1000km as predicted by measurement.
15.	D	All of these	All are dimensionless quantities.
16.	C	ML^2T^{-2}	[Torque] = [(Force)(moment arm) = $[MLT^{-2}][L] = [ML^2T^{-2}]$

Short Answers of the Exercise

Q.2 Write short answers of the following questions.

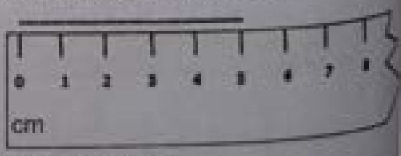
Q.1 For an answer to be complete, the units need to be specified. Why?

Ans: Explanation:

It is meaningless for the number without a unit and a unit does not mean anything without a number

For example:

Let us consider, drawing a line using scale



- **The measurement given is 5 [meaningless]**

When there is only a number, it will lead to a confusion whether to draw a line for cm or m.

Hence, the above-mentioned statement is meaningless.

Since the measurement is 5 cm. This is making complete sense.

Q.2 What are the advantages of using International System of Units (SI)?

Ans: Advantages System of Units (SI)

i. It consists of only seven base quantities, most of the derived quantities are obtained by multiplying and dividing the base units.
ii. Each quantity has only one standard unit, so we donot need to specify a quantity with different units.
iii. There are no numerical definitions or constants for students to memorize. For example, the quantity power is defined as energy per unit time ($Power = \frac{work}{time}$). Therefore, the SI unit of power (watt), is defined as the unit of energy per the unit of time i.e. joule/second.
iv. SI units are coherently derived as the simple algebraic division or multiplication of a few base units, using the same equation as the quantity being measured.
v. SI uses decimals exclusively, eliminating difficult fractions and mixed numbers.

Q.3 How many radians account for circumference of a circle? How many steradians account for circumference of a sphere?

Ans: Radians in One Circle

Mathematically,

$$\text{Number of radians } (\theta) = \frac{\text{ArcLength}}{\text{Radius of same circle}} = \frac{S}{r}$$

So, the number of radians in one revolution $= \frac{\text{circumsference of Circle}}{\text{Radius of same circle}}$

$$\text{Number of radians in one revolution} = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

Steradians in One Sphere

Mathematically,

$$\text{Number of steradians in sphere} = \frac{\text{Area of Sphere}}{r^2}$$

Since

Surface area of closed sphere of radius r is $4\pi r^2$.

Therefore

$$\text{Number of steradians in sphere} = \frac{4\pi r^2}{r^2} = 4\pi \text{ sr}$$

Sphere subtends 4π sr (12.56 sr).

Q.4 What is least count error? How can least count error be reduced?

Ans: Least Count Error:

"The smallest value that can be measured by the measuring instrument is called its least count."

- All the readings or measured values are good only up to this value.
- For example, a vernier callipers has the least count as 0.01 cm; a spherometer may have a least count of 0.001 cm.
- Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors.

Reduction

We can reduce the least count error by using instruments with higher resolution, improving experimental techniques, etc.

Q.5 Why including more digits in answers, does not make it more accurate?

Ans: Including more digits in answers, does not make it more accurate but it can make answer more precise.

Explanation: Accurate answer is the one which is closest to actual or acceptable value. For example, actual length of a wire is 10.6 cm. If student A measures its value as 10.5 cm and student B measures it as 10.15cm. We see that Student A measurement is closer to actual value, so his measurement is more accurate than measurement of student B, although student B reading has more digits.

Q.6 What determines the precision of a measurement?

Ans: ***Precision means how close the measured values are to each other.*** Smaller the least count of the instrument, measurement will be more precise and vice versa.

Example: Consider length of a wire is measured with Vernier Calipers is:

$$L = 10.5 \pm 0.1 \text{ mm}$$

This means that its true length lies between 10.4 mm to 10.6 mm.

Then the length of the same wire is measured by screw gauge which is:

$$L = 10.50 \pm 0.01 \text{ mm}$$

This means that its length lies between 10.49 mm to 10.51mm.

We see that measurement taken by the screw gauge is more precise than that of Vernier calipers, because they are closer to each other.

Q.7 If two quantities have different dimensions, is it possible to multiply and/or divide. Can we add and/or subtract them?

Ans: YES, we can multiply and divide quantities with different dimensions and after multiplication/ division, they will give us dimensions of any derived quantity.

For Example: As force= mass × acceleration

Dimension of force= dimension of mass × dimension of acceleration

$$[F] = [M][LT^{-2}]$$

$$[F] = [MLT^{-2}]$$

Remember, only those quantities can be added or subtracted which have same dimensions.

Q.8 The human pulse and the swing of a pendulum are possible time units. Why are they not often used?

Ans: A quantity can be taken as time standard if it is accessible and **invariable** (i.e. constant).

Human Pulse

It can not taken as time standard because pulse rate varies as a person rests, walks and runs etc.

Swinging Pendulum:

As the time period of the simple pendulum can be expressed as;

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

But this time period cannot serve as reasonable time slandered due to several reasons.

Roason:

(i) Length of pendulum may change due to ***change in temperature*** therefore time period may change.

(ii) Time period of the simple pendulum ***varies with g*** and g varies with altitude.

(iii) ***Frictional force*** of air and support may affect the time period.

Q.9 If an equation is dimensionally correct, is that equation a right equation?

Ans: If an equation is dimensionally correct, then that equation may or may not be a right equation.

For Example;

Consider the equation of newton's 2nd law $F = ma$

We know that this equation is correct equation in any sense.

But if we write $F = 2\pi\,(ma)$ where 2π is dimensionless constant, so by applying principle of homogeneity, dimensions on L.H.S. and R.H.S. will be same, therefore, it will be dimensionally consistent but it is not correct equation.

Comprehensive Questions

Q3. Give a short response to the following questions.

1. **Define Physics? Explain the scope and importance of physics in science, technology and society?**

Ans: See Q # 1

2. **What is system of units? In SI what is meant by base, derived and supplementary units?**

Ans: See Q # 2

3. **What conventions are used in SI to indicate units?**

Ans: See Q # 6

4. **What are errors? Differentiate between systematic and random errors?**

Ans: See Q # 7

5. **What is uncertainty in measurement? explain the propagation of uncertainty in addition, subtraction, multiplication and division?**

Ans: See Q # 8

6. **What are significant figures? What are the rules for determining significant figures in the final result after addition, subtraction, multiplication and division?**

Ans: See Q # 9

7. **Differentiate between precision and accuracy in the measurement.**

Ans: See Q # 10

8. **What is meant by dimensions of physical quantities? What are limitations and applications of dimensional analysis?**

Ans: See Q # 11

NUMERICAL QUESTIONS

1. **A circular pizza into 3 equal parts, one piece of pizza is taken out. Estimate the degree measure of the single piece of pizza and convert the measure into radians. What is the radian measure of the angle of the remaining part of pizza?**

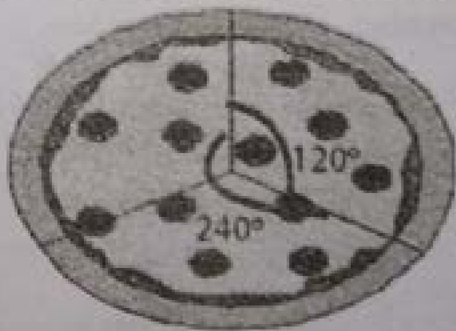
Solution:

Giveen Data:

Number of pieces of circular pizza = 3

To Find: Angle subtended by one piece = ?

Angle subtended by remaining part pizza = ?



Solution: Angle subtended by circle i.e. pizza is: $\theta = 2\pi \; radian$

Angle subtended by one of the three pieces is

$$\theta = \frac{2\pi}{3} \; radian$$

$$\theta = \frac{2(3.14)}{3} \; radian = 2.09 \; rad$$

Angle subtended by the remaining two pieces of pizza is:

$$\theta = 2 \times 2.09 \; rad = 4.18 \; rad$$

2. The length of a pendulum is (1.5 ± 0.01) m and the acceleration due to gravity is taken into account as (9.8 ± 0.1) m s^{-2}. Calculate the time period of the pendulum with uncertainty in it.

Given Data: Length $= L = 1.5 \pm 0.01$ cm

Acceleration $= g = 9.8 \pm 0.1$ ms^{-2}

To Find:

%age uncertainty in T = % ΔT = ?

Solution:

Calculation of Time Period:

∵ Formula of time period is: $T = 2\pi\sqrt{\frac{\ell}{g}}$

⇒ $T = 2 \times 3.14\sqrt{\frac{1.5}{9.8}} = 2.5\ s$

%age uncertainty in $L = \frac{\Delta L}{L} \times 100\%$

$= \frac{0.01}{1.5} \times 100\% = 0.67\%$

%age uncertainty in $g = \frac{\Delta g}{g} \times 100\%$

$= \frac{0.1}{9.8} \times 100\% = 1.020\%$

∵ Formula of time period is: $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T = 2\pi\left(\frac{\ell}{g}\right)^{\frac{1}{2}}$

%age uncertainty in $T = \frac{1}{2} \times$ (%age uncertainty in ℓ + % age uncertainty in g)

$= \frac{1}{2} \times (0.67\ \% + 1.020\%)$

$= \frac{1}{2} \times (1.69\%) = 0.84\% = 0.8\%$

⇒ $\%\ \Delta T = \pm 0.8\%$

⇒ $T = 2.5 \pm 0.8\%$

3. Determine the area of a rectangular sheet with length ($l \pm \Delta l$) = (1.50 ± 0.02)m and width ($w \pm \Delta w$) = (0.20 ± 0.01) m. Calculate the area ($A \pm \Delta A$).

Given Data: $l \pm \Delta l = 1.50 \pm 0.02$ cm

$w \pm \Delta w = 0.20 \pm 0.01$ cm

To Prove: $A \pm \Delta A = ?$

Solution: Calculation of Area

$A = L \times W$

$A = 1.50 \times 0.20$

$A = 0.30$ cm^2

%age Uncertainty in $L = \frac{\Delta L}{L} \times 100\%$

$= \frac{0.02}{1.50} \times 100\% = 1.33\%$

%age uncertainty in $W = \frac{\Delta W}{W} \times 100\%$

$= \frac{0.01}{0.20} \times 100\% = 5\%$

⇒ Total %age uncertainty in A = %age uncertainty in L + %age uncertainty in W

$= 1.33\% + 5\%$

$= 6.33\%$

Absolute uncertainty in Area (ΔA) is calculated as:

$$\Delta A = \pm\frac{6.33}{100} \times 0.30 = 0.018\ cm^2 = 0.02\ cm^2$$

$\Rightarrow$ Area of rectangular sheet $= A \pm \Delta A = 0.30 \pm 0.02\ cm^2$

4. Calculate the answer up to appropriate numbers of significant digits

(a) 246.24 + 238.278 + 98.3 **(b) 1.4 × 2.639 + 117.25**

(c) $(2.66 \times 10^4) - (1.03 \times 10^3)$ **(d) (112 × 0.456) / (3.2 × 120)**

(e) 168.99 × 9 **(f) 1023 + 8.5489**

Solution:

(a) $246.24 + 238.278 + 98.3$
$= 582.818$
$= 582.8$ (As data has least 1 decimal place)

(b) $1.4 \times 2.639 + 117.25$
$= 3.6946 + 117.25$
$= 120.9446$
$= 120.9$ (As data has least 1 decimal place)

(c) $(2.66 \times 10^4) - (1.03 \times 10^3)$
$= 26600 - 1030$
$= 25570$
$= 2.5570 \times 10^4$
$= 2.56 \times 10^4$ (As data has at least 3 significant figures)

(d) $(112 \times 0.456) / (3.2 \times 120)$
$\frac{51.072}{384} = 0.133 = 0.13$ (As data has at least 2 significant figures)

(e) 168.99×9
$= 1520.91$
$= 2000$
(As data has least one significant digit so 1520.91 is rounded off to 2000 which contains one significant digit)

(f) $1023 + 8.5489$
$= 1031.5489$
$= 1032$
(As in given data 1023 has no decimal place, so answer will also don't have any decimal place)

5. Calculate the answer up to appropriate numbers of significant digits

(a) The ratio of mass of proton 'm_p' to the mass of electron 'm_e'

$$\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}\,kg}{9.1096 \times 10^{-31}\,kg}$$

(b) The ratio of charge on electron 'q_e' to mass of electron 'm_e'

$$\frac{q_e}{m_e} = \frac{1.67 \times 10^{-19}\,C}{9.1096 \times 10^{-31}\,kg}$$

Solution:

(a) The ratio of mass of proton 'm_p' to the mass of electron 'm_e'

$$\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}\,kg}{9.10196 \times 10^{-31}\,kg}$$

$= 0.18332 \times 10^4$
$= 1.8332 \times 10^3$
$= 1.83 \times 10^3$

(Answer should be up-to 3 significant figures as these are least significant digits in the given data)

(b) The ratio of charge on electron 'q_e' to mass of electron 'm_e'

$$\frac{q_e}{m_e} = \frac{1.6 \times 10^{-19}}{9.1096 \times 10^{-31}}$$
$$= 0.1756388864 \times 10^{12}\ C\ kg^{-1}$$
$$= 1.756388864 \times 10^{11}\ C\ kg^{-1}$$
$$= 1.8 \times 10^{11}\ C\ kg^{-1}$$

(Answer should be up-to 2 significant figures as these are least significant digits in the given data)

6. Find the dimensions of

(a) Planck's constant 'h' from formula $E = hf$
Where E is the energy and f is frequency.

(b) gravitational constant 'G' from the formula $F = G\frac{m_1 m_2}{r^2}$

Where 'F' is force, 'm_1' and 'm_2' are masses of objects and 'r' is the distance between centers of objects.

Solution: (a) Dimensions of Planck's Constant

From equation E = h f

$$[h] = \frac{[E]}{[f]} = \frac{[M\,L^2T^{-2}]}{[T^{-1}]} = [M\,L^2T^{-1}]$$

(b) Formula for Gravitational force = $F = G\frac{m_1 m_2}{r^2}$

Or $$G = \frac{Fr^2}{m_1 m_2}$$

Dimension of $$G = \frac{[MLT^{-2}][L^2]}{[M][M]}$$
$$= \frac{ML^3T^{-2}}{[M^2]}$$
$$= [M^{-1}L^3T^{-2}]$$

7. Show that (a) $KE = \frac{1}{2}mv^2$ and (b) $PE_g = mgh$ are dimensionally correct.

Solution: (a) $K.E. = \frac{1}{2}mv^2$

[L.H.S.] $= [K.E.] = [W] = [F][d]$
$= [MLT^{-2}][L] = [ML^{1+1}T^{-2}] = [ML^2T^{-2}]$

[R.H.S.] $= \frac{1}{2}[m][v^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

Since dimensions of [L.H.S.] = [R.H.S.], so given equation is dimensionally correct.

(b) P.E. = m g h

[L.H.S.] $= [P.E.] = [W] = [F][d]$
$= [MLT^{-2}][L] = [ML^{1+1}T^{-2}] = [ML^2T^{-2}]$

[R.H.S.] $= [m]\,[g]\,[h] = [M]\,[LT^{-2}]\,[L] = [M\,L^2\,T^{-2}]$

Since dimensions of [L.H.S.] = [R.H.S.], so given equation is dimensionally correct.

Additional Conceptual Short Questions With Answers

1. What are the uses of dimensions?

Ans: The three main uses are:

(i) Checking the correctness of the possible formula (equation).
(ii) Deriving a relationship between different physical quantities in a physical problem.
(iii) Conversion of one system of units into another.

2. What are the limitations of dimensional analysis?

Ans: The limitations are:

(i) The numerical values of constant of proportionality in an equation cannot be determined with the help of dimensions. It can be determined experimentally or theoretically.
(ii) Physical relations involving logarithm, exponential or trigonometric functions (all dimensionless), cannot be derived and verified.
(iii) It does not tell whether the quantity is scalar or vector.

3. Name the physical quantities which have the same dimensions but different units?

Ans: The following quantities have the same dimensions but different units.

- Work and torque have same dimensions = $[ML^2T^{-2}]$
- S-I unit of work is joule (J) but torque is N m

4. Prove that followings equations are homogeneous w.r.t. dimensions:

$$a_c = \frac{v^2}{r}$$

Ans: For centripetal acceleration,

$$a_c = \frac{v^2}{r}$$

Unit of a_c = m s^{-2}
So $[a_c] = [LT^{-2}]$
Unit of v = m s^{-1}
So $[v] = [LT^{-1}]$

$[L.H.S.] = [a_c] = [LT^{-2}]$

$$[R.H.S.] = \left[\frac{v^2}{r}\right] = \frac{[LT^{-1}]^2}{[L]} = \frac{[L^2T^{-2}]}{[L]}$$

$$= [L^{2-1}T^{-2}] = [LT^{-2}]$$

Since dimensions of [L.H.S.] = [R.H.S.], so given equation is dimensionally correct.

5. What do you understand by "Assignable errors" and "Un-assignable errors"?

Ans: **Assignable Errors:** The errors to which we can assign a cause are called assignable errors. For example, zero error is an example of assignable error. The error also arising in measurement due to poor calibration of the instrument is also an assignable error.

Un-assignable Error: The errors in measurement to which we cannot assign a cause. These are due to unknown reasons. These are also called random errors.

6. Show that equations of motions
$V_f^2 = 2as + V_i^2$
are dimensionally consistent?

Ans: The given equation is

$V_f^2 = 2aS + V_i^2$

$[L.H.S.] = [V_f]^2 = [LT^{-1}]^2 = [L^2T^{-2}]$

$[R.H.S.] = 2[a][S] + [V_i]^2$
$= 2[LT^{-2}][L] + [LT^{-1}]^2$
$= 2[L^2T^{-2}] + [L^2T^{-2}]$
$= 3[L^2T^{-2}]$
$= [L^2T^{-2}]$

Since dimensions of [L.H.S.] = [R.H.S.], so given equations are dimensionally correct.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **Which of the following is not a unit of pressure?**
   A. $kgm^{-1}s^{-2}$ B. $kgm^{-3}s^{-3}$ C. Nm^{-2} D. pascal
2. **In colour printing, entire range of colours can be obtained by mixing ______ colours.**
   A. Seven B. Six C. Five D. Four
3. **Which of the following pair have same dimensions?**
   A. Power, Speed B. Force, Momentum C. Work, Torque D. Velocity, Acceleration
4. **Numbers of significant figures in 8.70 x 104 are ______.**
   A. 1 B. 3 C. 4 D. 7
5. **SI unit of time is ______.**
   A. 60 min B. Cesium second C. Krypton 86 D. 60 s
6. **Light year is unit of ______.**
   A. Distance B. time C. Light intensity D. Speed
7. **Solid angle of sphere is ______.**
   A. 12.57 sr B. 6.28 sr C. 3.14 sr D. 57.3 sr
8. **Base unit of linear momentum are**
   A. Ns^2 B. kgm/s C. kgm^2/s D. None
9. **Which of the following is dimensionless?**
   A. Stress B. Pressure C. Surface tension D. Strain
10. **Which of the following is not unit of work?**
    A. Joule B. Nm C. $kgms^{-1}$ D. Watt sec
11. **The dimensions of Power are:** (ANNUAL 2017)
    A. $[MLT^{-2}]$ B. $[MLT^{-3}]$ C. $[M^2L^2T^{-2}]$ D. $[ML^2T^{-3}]$
12. **The significant figures in 34.678 are:** (ANNUAL 2017)
    A. 4 B. 3 C. 5 D. 2
13. **Which of the following pairs of units are both derived units?** (ON- ANNUAL 2017)
    A. Kilogram, Angstrom B. Ampere, Degree C. Newton, Candela D. Joule, Watt
14. **The Prefix one peta is:** (ANNUAL 2018)
    A. 10^9 B. 10^{18} C. 10^{15} D. 10^{12}
15. **One year is equal to:** (ANNUAL 2018)
    A. $3.15 \times 10^7 s$ B. $5.4 \times 10^4 s$ C. $1.41 \times 10^{17} s$ D. $8.6 \times 10^4 s$
16. **Which of the following may be used as valid formula to calculate speed of ocean waves? (v = speed, g = acceleration due to gravity, λ = wavelength, ρ = density, h = depth)** (ANNUAL 2019)
    A. gh/λ B. $\sqrt{\lambda g}$ C. $\frac{\lambda}{gh}$ D. ρgh
17. **In a cricket match 500 spectators are counted one by one. How many significant figures will be there in the final result?** (ANNUAL 2019)
    A. 1 B. 2 C. 3 D. 0

Answers Key

1.	D	2.	D	3.	C	4.	B	5.	B
6.	A	7.	A	8.	B	9.	D	10.	C
11.	D	12.	C	13.	D	14.	C	15.	A
16.	B	17.	C						

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 **Choose the correct answer from the given options.** **(1 x 6 = 6)**

SECTION – A

1. **Solid angle subtended at the centre of sphere of radius 'r' in steradian is:**
   (A) 2π (B) 4π (C) 6π (D) 8π
2. **The S.I unit of power in terms of base unit are:**
   (A) $kg\ m^{-1}s^{-2}$ (B) $kg\ m^{-1}s^{-3}$ (C) $kg\ m\ s^{-2}$ (D) $kg\ m^2s^{-3}$
3. **How many nanometers are in a meter?**
   (A) 10^{-19} (B) 10^{19} (C) 10^{9} (D) 10^{-9}
4. **The percentage uncertainty in mass and velocity of an object are 2% and 3% respectively. Which of the following is the maximum uncertainty in the measurement of its kinetic energy?**
   (A) 11% (B) 8% (C) 6% (D) 1%
5. **Significant figures in 0.0010 are ____________;**
   (A) 1 (B) 2 (C) 3 (D) 4
6. **Which of the following pair have same dimensions?**
   (A) Momentum and impulse (B) Work and power
   (C) Force and torque (D) Torque and power

Question.No.2 **Give short answers of followings:** **(3 x 7 = 21)**

SECTION – B

(i) How many radians account for circumference of a circle? How many steradians account for circumference of a sphere?
(ii) What determines the precision of a measurement?
(iii) The human pulse and the swing of a pendulum are possible time units. Why are they not often used?
(iv) Show that (a) $KE = \frac{1}{2}mv^2$ and (b) $PE_g = mgh$ are dimensionally correct.
(v) Two connected gears are rotating. The smaller gear has a radius of 0.4 m and the larger gear's radius is 0.7 m. What is the angle through which the larger gear has rotated when the smaller gear has made one complete rotation?
(vi) Differentiate between the terms, 'precision' and 'accuracy'.
(vii) Write down any three sign conventions for writing measurements and their units.

Question.No.3 **Extensive Questions.** **(13)**

SECTION – C

(a) Find an expression for the time period 'T' of a simple pendulum. The time period 'T' may depend upon (i) mass 'm' of the bob of the pendulum, (ii) length 'l' of pendulum, (iii) acceleration due to gravity 'g' at the place where the pendulum is suspended. (05)
(b) The length of a pendulum is (1.5 ± 0.01) m and the acceleration due to gravity is taken into account as $(9.8 \pm 0.1)\ m\ s^{-2}$. Calculate the time period of the pendulum with uncertainty in it. (04)
(c) Write down any three advantages and three limitations of dimensional analysis. (04)

The End

CHAPTER 2

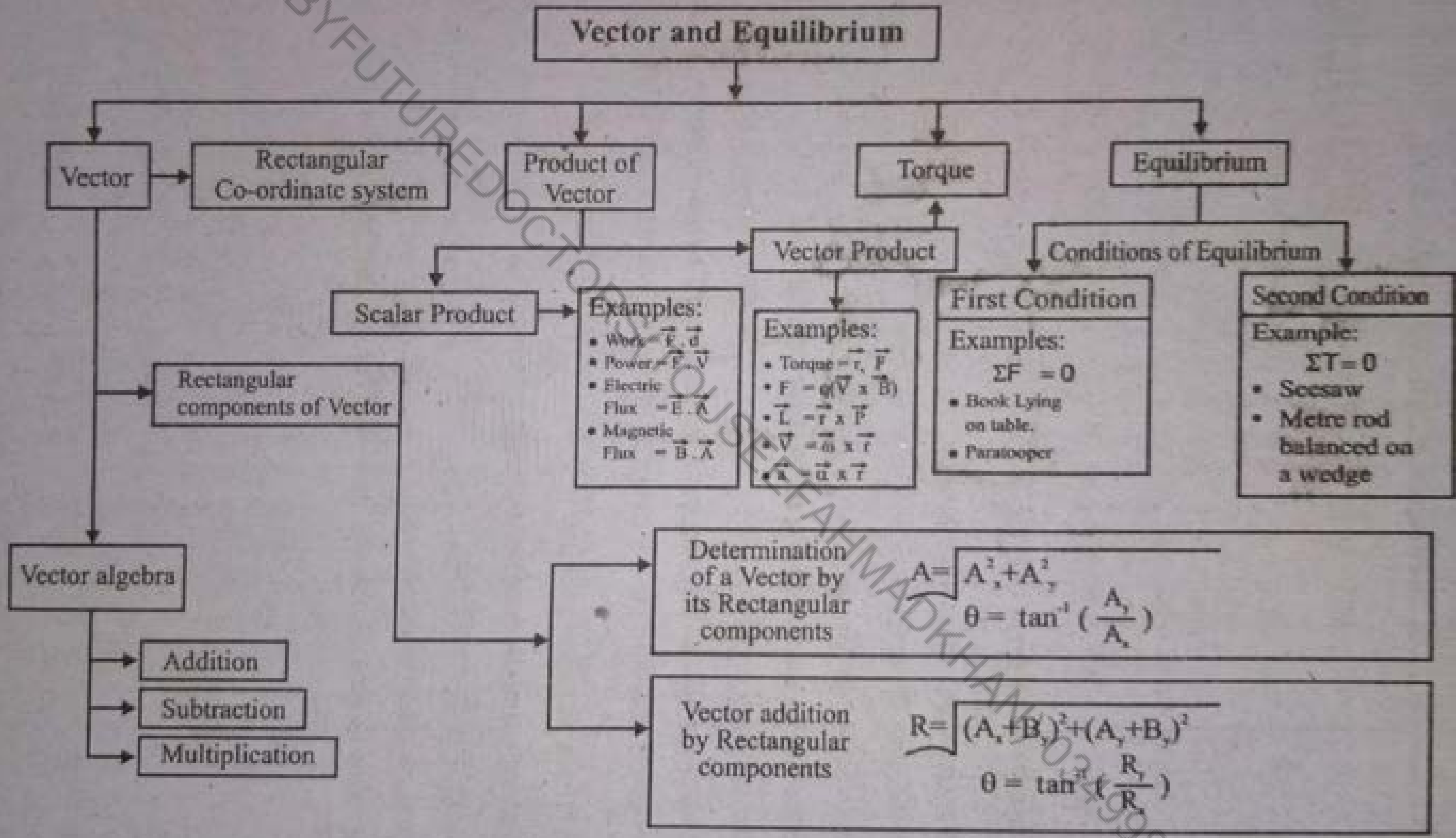
VECTORS & EQUILIBRIUM

Learning Objectives

- Describe the Cartesian coordinate system.
- Determine the sum of vectors using head to tail rule.
- Represent a vector into two perpendicular components.
- Determine the sum of vectors using rectangular components.
- Describe scalar product of two vectors in terms of angle between them.
- Describe vector product of two vectors in terms of angle between them.
- State the method of determine the direction of vector product of two vectors.
- Define the torque as vector product $\vec{r} \times \vec{F}$.
- List applications of torque or moment due to a force.
- State first condition of equilibrium.
- State second condition of equilibrium.
- Solve two dimensional problems involving forces (statics) using first and second conditions of equilibrium.

Chapter No. 2

CONCEPT MAP



Q.1 Define the following terms: Magnitude, Scalars, Vectors.

Ans: **Magnitude:** It is the numerical value of a physical quantity with suitable unit.

Scalars:

A physical quantity which can be completely described by ***magnitude only*** is called scalar quantity.

Examples: Time, distance, mass, temperature, speed, energy, work, volume, area, electric charge etc.

Note that scalars can be added, subtracted, multiplied etc like numbers using ordinary algebra.

Vector:

A physical quantity which can be completely described by both ***magnitude*** and ***direction*** is called vector quantity.

Note that the vector quantities also obey the laws of vector addition (i.e. they follow vector algebra).

Examples: Force, velocity, displacement, torque, momentum, acceleration, weight, angular velocity, electric intensity etc.

Q.2 How is a vector represented?

Ans: **Vector Representation**

A vector is represented in two ways.

(i) Symbolic representation (ii) Graphical representation

Symbolic Representation

It is represented by ***bold face*** letter such as **A**, **d**, **r** and **v** etc. It can be also be represented by a ***letter with an arrow*** placed above or below the letter such as $\vec{A}$ or $\underset{\rightarrow}{A}$

Graphical Representation

It is represented by a ***straight line*** with an ***arrow head*** at its one end. The length of line represents magnitude of vector (according to suitable scale). Arrow head represents the direction of vector.

The starting point is called origin or tail point and the end point of the vector is called head point.

Graphical representation of a vector V

Note: Representation of magnitude of vector

The magnitude of vector is represented by ***light face italic*** letter such as *A*, *d*, *r* and *v* etc. or by the taking modulus of a vector such as $|\vec{A}|, |\vec{v}|$ etc.

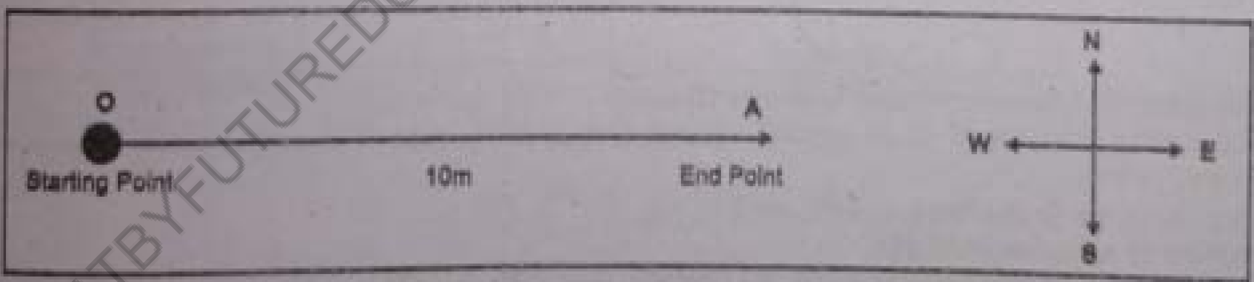


Fig: 2.1

Q.3 What are geometric vectors, Co-ordinate axes and algebraic vectors?

Ans:

I. **Geometric Vectors:** Geometric vectors are those vectors that are considered without reference to coordinate axes.

II. **Co-ordinate axes:** Any set of values that indicate the position of a point in a given reference system.

OR

Any system which helps us to locate position of a point w.r.t. to some reference point.

III. **Algebraic Vectors:** These are vectors drawn in a coordinate system (in plane or space).

Q.4 What is Cartesian coordinate system or rectangular coordinate system?

Ans: **Cartesian Co-ordinate System (Rectangular Coordinate System)**

The set of two or three mutually perpendicular lines intersecting at a point is called cartesian or rectangular coordinate system.

- The lines are called ***coordinate axes***.
- One of these lines is called *x-axis (or **horizontal axis**)*
- The other is called *y-axis or **vertical axis.***
- The line perpendicular to both x and y –axes is called *z – axis.*
- The point of intersection is called ***origin***.

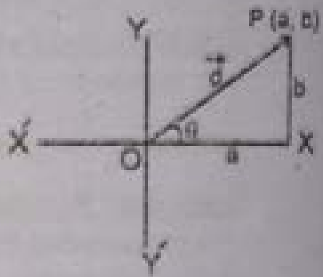
Two dimensional coordinate system (Plane)

If the system consists of *two* perpendicular lines then it is called two dimensional coordinate system.

The point where these lines intersect is called origin.

- Each point in plane has two co-ordinates P(a,b). Starting from the origin. The point P is 'a' unit along x-axis and 'b' unit along y-axis.

Direction of a vector in plane

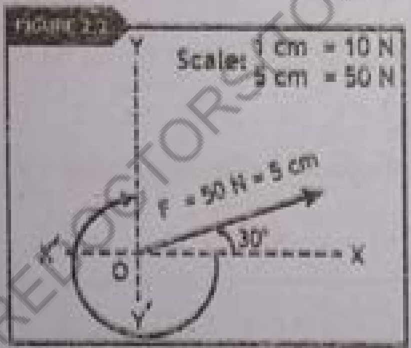


It is represented by the angle which the vector makes with *positive x-axis* in *anti-clock* wise direction.

Example of A Vector in plane:

Consider a force vector of 50 N making an angle of 30° with horizontal is drawn as shown.

The scale is 1 cm = 10 N, so its length 5 cm representing 50 N with 30° angle from x-axis.



> **For Your Information**
> In plane, only one angle is required to represent a vector while three angles are required to represent a vector in space.

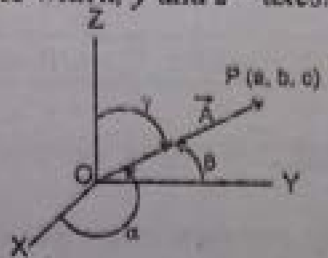
Three dimensional co-ordinate systems (Space)

If the system consists of *three* mutually perpendicular lines, then it is called three dimensional co-ordinate systems.

Each point in space has three co-ordinate P (a,b,c)

Direction of a vector in space

It is represented by *three* angles which the vector makes with *x, y* and *z – axes.*



Angle with X – axis is α
Angle with Y – axis is β
Angle with Z – axis is γ

Q.5 What are steps to represent a vector in cartesian coordinate system?

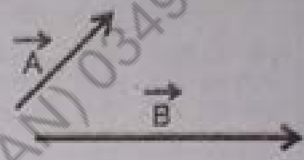
Ans: **STEPS TO REPRESENT A VECTOR IN CARTESIAN COORDINATE SYSTEM**

The following method is used to represent a vector

1. Draw a cartesian coordinate system.
2. Select a suitable scale.
3. Draw a line in the specified direction.
4. Cut the line equal to the magnitude of the vector according to the selected scale.
5. Put an arrow in the direction of the vector.

Q.6 How will you add vectors by graphical method?

Ans: **ADDITION OF VECTORS:**

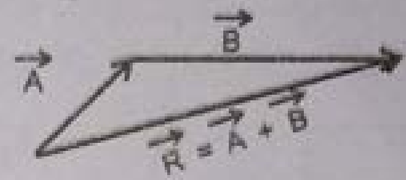


Vectors can be added by two different approaches, namely graphical (or geometrical method) and analytical method.

Graphical or Geometrical Method for Addition of Vectors:

In this method vectors are added by sketching scaled diagrams. For the graphical approach of addition of vectors we use Head-to-Tail Rule.

HEAD TO TAIL RULE:



Following steps must be followed to add vectors by head-to tail rule:

i. Select a suitable scale for the representation of given vectors.

ii. Sketch all the vectors according to the selected scale in its given direction.

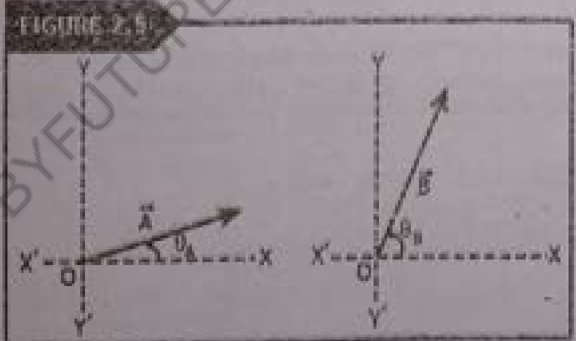
iii. Now join the tail of the 2nd vector to the head of the 1st vector and draw its representative line according to the selected scale in the given direction and continue this process till last vector is added.

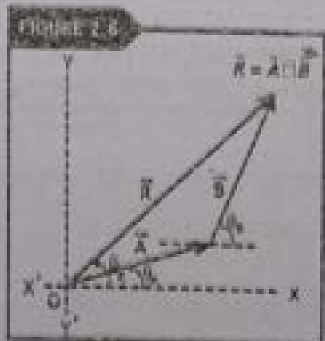
iv. To find resultant, join the tail of the first vector to the head of the last vector and put an arrow on this line pointing away from the origin. Label this vector as $\overline{R}$, means resultant vector.

vi. To determine the magnitude of $\overline{R}$, measure the length of R and convert it according to the selected scale.

vii. To determine the direction of R (θ), measure the angle of $\overline{R}$ with respect to + x-direction in anti-clockwise direction.

(**Resultant Vector:** Resultant vector is a single vector which has same effect as that of combined effect of all the vectors to be added.)





Example:

- Consider two vectors A and B, drawn to same scale making certain angles θ_A and θ_B with the x-axis respectively as shown in the Figure.
- To add these vectors, we redraw them to a common scale.
- Place them head to tail as in the steps mentioned above.
- Such that the tail of vector B is on head of vector A.
- Joining the tail of the first vector with the head of the last to get the resultant vector R as shown in Figure.

ADDITION OF MORE THAN TWO VECTORS:

We can add any number of vectors using head to tail rule.

Example:

- Consider four vectors **A, B, C** and **D** in xy- plane as shown in the Figure.
- To add these vectors draw them to a common scale.
- Place them head to tail. Such that the tail of each vector is on head of the previous vector.
- Draw the resultant vector **R** from tail of first vector to head of last vector. Measure the length of vector R and convert it into magnitude of the quantity according to given scale.
- To determine the direction measure the angle of resultant with x-axis.

FIGURE 2.8

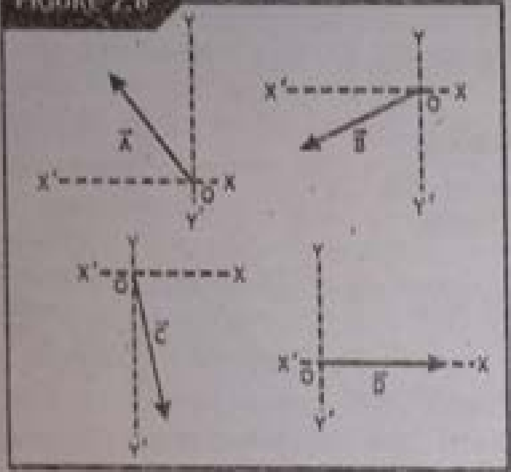
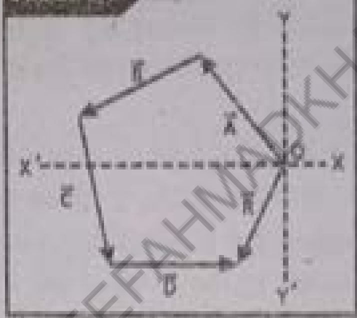


FIGURE 2.9



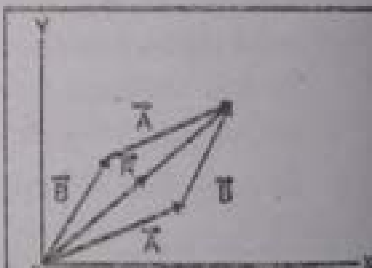
Q.7 Does vector addition follow commutative property?

Ans: **Commutative Property**

From figure, it is clear that either we add $\vec{A}$ to $\vec{B}$ or $\vec{B}$ to $\vec{A}$, the resultant is same. They will form parallelogram. i.e.

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

It means that when vectors are added, the result is the same for any order of addition. Hence, **vector addition is commutative.**



This diagram shows that $\vec{A}+\vec{B}=\vec{R}$ and $\vec{B}+\vec{A}=\vec{R}$

Fig 2.8

Assignment 2.1:

An airplane is moving at 120 m/s at an angle of 10 with x–axis, through a 30 m/s ocross wind blowing at angle of 260 with x–axis. Determine the resultant velocity of the airplane.

Solution:

Let $\vec{v}_1 = 120\text{m/sec}$ making an angle of 10o with x-axis and $\vec{v}_2 = 30\text{m/sec}$ making an angle of 260° with x-axis.

Required:

Resultant velocity of airplane = $\vec{v}$ = ?

Calculation:

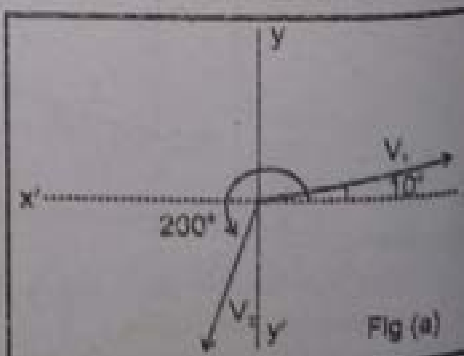


Fig (a)

The resultant velocity is given by,

$$v = \sqrt{v_x^2 + v_y^2}$$

First we resolve $\vec{v}_1$ and $\vec{v}_2$ into its horizontal and vertical components as shown in figure (b). Now we have,

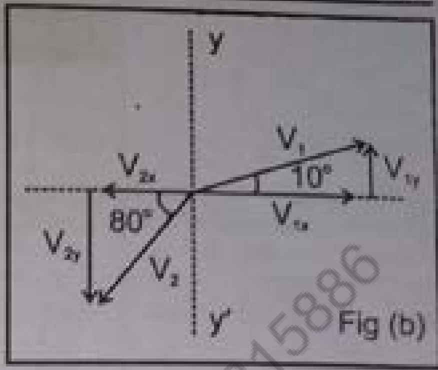
$$v_{1x} = v_1 \cos\theta_1 = 120 \times \cos 10^\circ \text{m/sec}$$

$\Rightarrow \quad v_{1x} = 120 \times 0.985\text{m/sec}$

$\Rightarrow \quad v_{1x} = 118.2\text{m/sec}$

And $\quad v_{1y} = v_1 \sin\theta_1 = 120 \times \sin 10^\circ = 120 \times 0.174\text{m/sec}$

$\Rightarrow \quad v_{1y} = 20.88\text{m/sec}$



Similarly,

$$V_{2x} = v_2 \cos\theta_2 = 30 \times \cos 80^\circ$$
$$= 30 \times 0.174\text{m/sec}$$

$\Rightarrow \quad v_{2x} = 5.22\text{m/sec}$

And $\quad V_{2y} = v_2 \sin\theta_2 = 30 \times \sin 80^\circ$

$$= 30 \times 0.985 \text{ m/sec}$$

$\Rightarrow \quad v_{2y} = 29.55\text{m/sec}$

Point to Ponder

1. When two or more vectors are added or subtracted together, they must all have the same units and they all must be the same type of quantity
2. In addition process, the vectors A, B which result R after addition, are called components of vector R.
3. As we can add any number of vectors, so we can say that a vector may have infinite number of its components.

Now the x-component of resultant velocity is given by,

$$v_x = v_{1x} + (-v_{2x}) = v_{1x} - v_{2x} = (118.2 - 5.22)\text{m/sec}$$

$\Rightarrow \quad v_x = 112.98\text{m/sec}$

Now the y-component of resultant velocity is given by,

$$v_y = v_{1y} + (-v_{2y}) = v_{1y} - v_{2y} = (20.88 - 29.55)\text{m/sec}$$

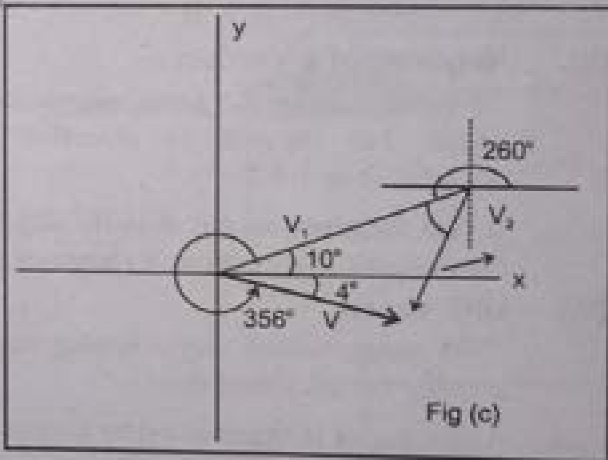
$\Rightarrow \quad v_y = -8.67\text{m/sec}$

Putting the values of "v_x" and "v_y" in equation (1)

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(112.98)^2 + (-8.67)^2}$$

$\Rightarrow \quad v = \sqrt{12764.5 + 75.2} = \sqrt{12839.7}$

$\Rightarrow \quad v = 113.3\text{m/sec}$



The direction of "v" is given by,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.67}{112.98}\right)$$

$\Rightarrow \quad \theta = \tan^{-1}(0.077) \Rightarrow \theta = 4^\circ$

Now the actual angle of "v" with positive x-axis is given by,

$$\theta' = 360^\circ - 4^\circ \Rightarrow \theta' = 356^\circ$$

Q.8 How will you subtract two vectors? Explain in detail.

Ans: **Subtraction of Vectors**

There is no direct method for determining the result of subtraction of two vectors. Even for subtraction we use the method of addition as:

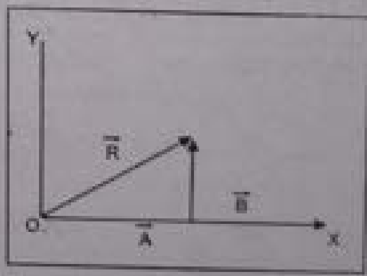


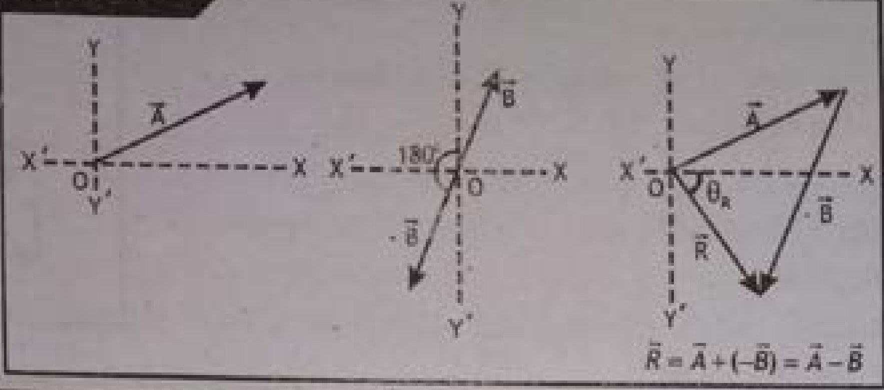
Fig 2.10

i. 1st draw the negative of the vector to be subtracted. If vector $\vec{B}$ is supposed to be subtracted from $\vec{A}$ then first we find the negative of $\vec{B}$, which is $-\vec{B}$

ii. Then join the tail of negative vector with the head of 1st vector according to head to tail rule, i.e. we add this negative of $\vec{B}$ with $\vec{A}$ as shown:

iii. To get their resultant, draw a vector from tail of 1st vector **A** to head of negative vector **–B** as shown by vector **(A - B)**.

FIGURE 2.11

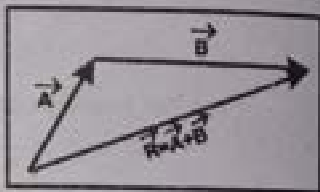


Q.9 Define and explain the flowing terms:

(i) Resultant vector **(ii) Negative of vector** **(iii) Unit vector**
(iv) Null vector **(v) Equal vectors** **(vi) Unequal vectors**

Ans:

(i) Resultant Vector:



"A vector having the same effect as the combined effect of all the vectors to be added is known as resultant vector." Consider two vectors $\overline{A}$ and $\overline{B}$. The sum vector $\overline{A} + \overline{B}$ is known as resultant vector and it can be expressed

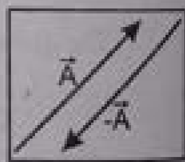
$$\overline{R} = \overline{A} + \overline{B}$$

(ii) Negative of a Vector:

"A vector having the same magnitude as the given vector, but opposite in direction is known as negative of vector."

For Your Information
Resultant of two vectors will be maximum when they are along the same direction and will be minimum when they are opposite in direction.

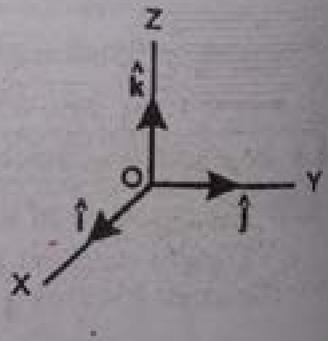
If $\overline{A}$ is the given vector, then the negative of vector $\overline{A}$ has the same magnitude as $\overline{A}$ but opposite in direction as shown in figure.



(iii) Unit vector

"The dimensionless vector having magnitude 1 and used to represent the direction of a vector is called unit vector."

A unit vector is represented by a letter with a cap or hat on it $\hat{A}$.



Explanation:

A unit vector in the direction of $\overline{A}$ is written as $\hat{A}$.

So $\overline{A} = A\hat{A}$

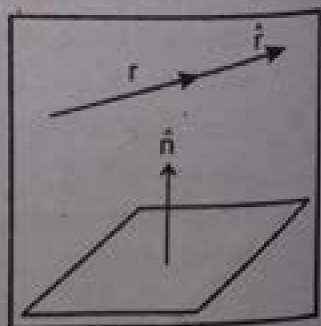
or $\hat{A} = \dfrac{\overline{A}}{A}$

or $\hat{A} = \dfrac{\overline{A}}{|\overline{A}|}$

Special Unit Vectors:

(a) Unit Orthogonal Vectors

These are unit vectors along axes.



Generally, we take unit vectors $\hat{i}, \hat{j}, \hat{k}$ along x-axis, y-axis and z-axis respectively. The magnitude of each unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ is one i.e.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

The three unit vectors $\hat{i}, \hat{j}, \hat{k}$ do not change the magnitude or the dimensions of anything; they only indicate directions.

(b) Normal Unit Vector:

The unit vector $\hat{n}$ is used to represent the direction of normal drawn on a certain surface. e.g. unit vector of area is drawn perpendicular to surface.

(c) Unit Vector of Position Vector:

$\hat{r}$ is the unit vector in the direction of $\vec{r}$.

> **For Your Information**
> - Position vector of origin is null vector
> - The acceleration of a body moving with uniform velocity is null vector

(iv) Null Vector or Zero Vector

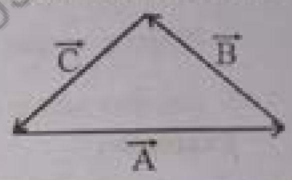
"A vector having zero magnitude and arbitrary direction is called a null vector."

A null vector is denoted by $\vec{O}$.

- If a vector $\vec{A}$ and its negative vector $-\vec{A}$ are to be added then their resultant vector can be expressed as,

$$\vec{A} + (-\vec{A}) = \vec{O}$$

- The resultant of number of vectors arranged by head to tail rule forming a closed polygon is a null vector.



(v) Equal Vectors

"Two vectors are said to be equal vectors if they have the same magnitude and direction, regardless of the position of their initial points."

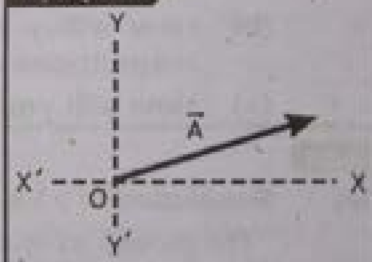
This means that parallel vectors of the same magnitude and direction are equal to each other.

> **For Your Information**
> Two like parallel vectors of equal magnitudes are called equal vectors

(vi) Unequal Vectors

Vectors having either different magnitudes or direction or both are called unequal vectors.

FIGURE 2.12



Q.10 Explain the multiplication of a vector with a scalar.

Ans: **Multiplication of a vector by a scalar**

A vector can be multiplied by

1. a positive number
2. a negative number
3. zero
4. a scalar quantity with dimensions

> **For Your Information**
> If vector $\vec{A}$ is multiplied by a number 'n' either positive or negative, the magnitude of new vector be $|n\vec{A}|$

1. Multiplication with positive number

When a vector $\vec{A}$ is multiplied by a positive number n (i.e. k > 0) Then the product vector will have magnitude equal to *kA* and *same* direction as that of $\vec{A}$.

2. Multiplication with negative number

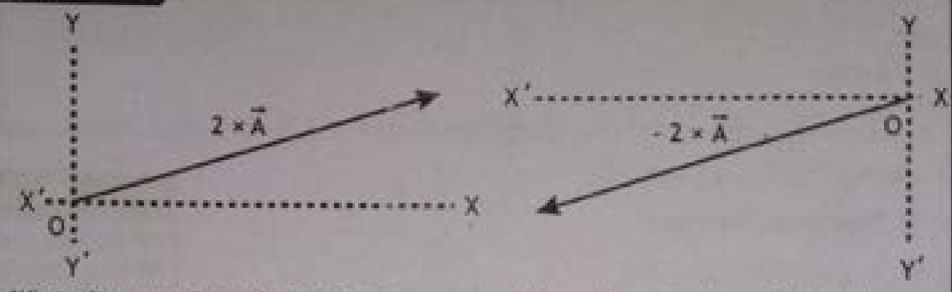
When a vector $\vec{A}$ is multiplied by a negative number n then the product vector will have magnitude equal to *kA* but direction *opposite* to that of $\vec{A}$.

3. Multiplication with Zero:

When a vector is multiplied with zero, we get null vector ($\vec{O}$)

If k = 0 then $k\vec{A} = \vec{O}$

FIGURE 2.13



When the vector is multiplied by a scalar (k) = 2, the length of the vector increases two times and the direction remains the same.

When the vector is multiplied by a scalar (k) = - 2, the length of the vector increases two times and the direction reverses.

4. Multiplication with scalar quantity

When a vector $\vec{A}$ is multiplied by a scalar quantity n, then the product vector will be a *new physical quantity* whose dimensions are equal to the *product of dimensions of k and* $\vec{A}$

Examples

- Product of mass m and velocity $\vec{v}$ is momentum[$\vec{p} = m\vec{v}$]
- Product of mass m and acceleration $\vec{a}$ is force[$\vec{F} = m\vec{a}$]
- Product of force $\vec{F}$ time t impulse [$\vec{J} = \vec{F} \times t$]

Q.11 (a) Define the following vectors: Resolution of a vector, component of a vector and rectangular components of a vector

(b) How will you find rectangular components of a vector? Derive the formulae for magnitudes of rectangular components

(c) How will you find a vector from its rectangular components?

Ans:

(a) Resolution of a Vector

"The process of splitting up a vector into its components is called resolution of a vector."

TIP: The effective value of a vector in a particular direction is called component of a vector.

Components of a vector

The effective value of a vector in a given direction is called component of a vector.

A vector may split up into two or more than two parts these parts are known as components of vector.

Resolution of a vector is the **reverse processes of addition of vectors.**

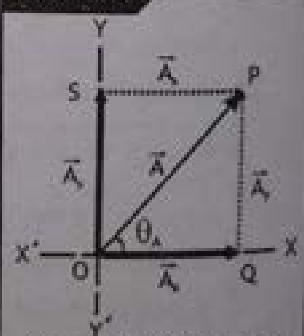
Rectangular Components of Vector

The components of a vector which are perpendicular to each other are called rectangular components.

(b) Resolution of A Vector

- Consider a vector A in the Cartesian coordinate System, represented by the line OP, making an angle θ as shown in the figure.
- Draw perpendiculars from point P on x-axis and y-axis which meets the axis at points Q and S respectively.
- Put arrow head from the direction of O towards Q and S such that they represent vectors as $\overrightarrow{A_x}$ (OQ) or $A_x\hat{i}$ and $\overrightarrow{A_y}$ (OS) or $A_y\hat{j}$, called the x and y rectangular components of vector A.

FIGURE 2.14



Vector represented in term of its components.

Thus by head to tail rule

$$\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$$

OR $\vec{A} = A_x\hat{i} + A_y\hat{j}$ .....(i)

As $A_x\hat{i}$ and $A_y\hat{j}$ are the components of vector $\vec{A}$ and $A_x\hat{i}$ and $A_y\hat{j}$ are mutually perpendicular, so these are called as rectangular components.

Magnitudes of Rectangular Components:

From right angled triangle OPQ

From triangle OPM, we can write

$$Cos\theta = \frac{Base}{Hypotenuse}$$

Or $\cos\theta = \frac{Ax}{A}$

$A_x = A\cos\theta$ .....(ii)

This equation gives the magnitude of the x-component or horizontal of $\vec{A}$.

Again, From right angled triangle OPQ

$$Sin\theta = \frac{Perpendicular}{Hypotenuse}$$

Or $\sin\theta = \frac{Ay}{A}$

Or $A_y = A\sin\theta$ .....(iii)

These equations (ii) and (iii) give magnitudes of rectangular given in term of magnitude and direction of their vector.

Equation (i) can be written as:

$\vec{A} = (A\,Cos\theta)\hat{i} + (A\,Sin\theta)\hat{j}$ .....(iv)

(c) Determination of vector from its rectangular components

A vector can be determined by its rectangular components. As vector needs magnitude and direction (i.e. angle) for its complete description so,

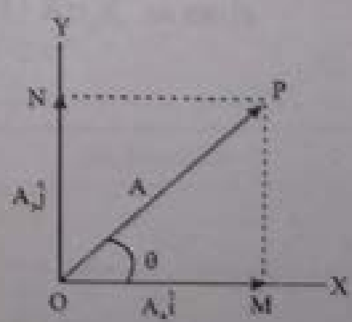
For magnitude:

The magnitude can find out by using Pythagorean theorem. In right triangle ΔPOM,

$(Hyp)^2 = (Base)^2 + (Perp)^2$

$A^2 = A_x^2 + A_y^2$

$A = \sqrt{A_x^2 + A_y^2}$



The magnitude of vector can now be determined if the values of the magnitudes of components are known.

For direction:

$$tan\theta = \frac{Perpendicular}{Hypotenuse}$$

$$tan\theta = \frac{A_y}{A_x}$$

or $\theta = tan^{-1}(\frac{A_y}{A_x})$

This expression gives the direction of vector $\vec{A}$ in terms of angle θ with x-axis.

MCQ's

1. If R_x is negative and R_y is positive then the resultant vector lies in the _____ quadrant:
   (A) 2^{nd} (B) 3^{rd} (C) 4^{th} (D) 5^{th}
2. A force of 10 N is acting along x-axis. Which of the following is magnitude of its y-component?
   (A) 10N (B) $(10)^{1/2}$ (C) 0N (D) 1 N
3. A force of 10 N makes an angle of 30° with y-axis .Which of the following is magnitude of its x-component?
   (A) 5N (B) 8.66 N (C) 10 N (D) Zero
4. If R_x and R_Y both are negative, then in which quadrant the resultant vector lies?
   (A) 1^{st} (B) 2^{nd} (C) 3^{rd} (D) 4^{th}
5. How many rectangular components, a vector have in space?
   (A) 2 (B) 3 (C) 4 (D) 5
6. At which angle with the x-axis, the magnitude of rectangular components of a vector are equal?
   (A) 30° (B) 45° (C) 60° (D) 90°
7. If R_x is positive and R_y is negative, then in which quadrant the resultant vector lies?
   (A) 1^{st} (B) 2^{nd} (C) 3^{rd} (D) 4^{th}
8. If a vector of magnitude 6N is along y-axis, then which of the following is its component along x axis?
   (A) 0 N (B) 5 N (C) 8.66 N (D) 10 N
9. If a vector of magnitude 10 N is along y-axis, then which of the following is its component along x axis?
   (A) 0 N (B) 5 N (C) 8.66 N (D) 10 N
10. If vector A is along x-axis then which of the following is its component along y- axis?
    (A) A sinθ (B) A cosθ (C) A tanθ (D) Zero
11. Which of the following is the resultant of two forces 30 N and 40 N acting parallel to each other?
    (A) 30 N (B) 40 N (C) 70 N (D) 10 N
12. Which of the following is the resultant of two forces 3N and 4N acting at right angle to each other?
    (A) 5N (B) 6N (C) 1N (D) 7N
13. Which of the following is the resultant of two forces 5 N and 12 N making an angle of 90° with each other?
    (A) 17N (B) 7N (C) 13N (D) 15N

Answers Key

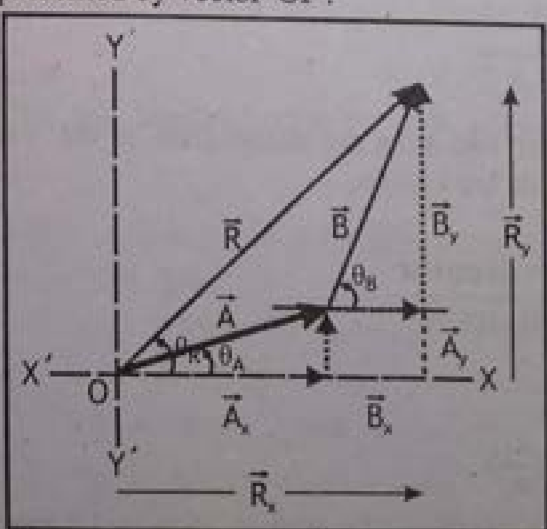
1. A	2. C	3. A	4. C	5. B	6. B	7. D	8. A	9. A	10. D	11. C	12. A
13. C											

Q.12 Describe the method of addition of vectors by rectangular components.

Ans: **Vector Addition by Rectangular Components**

The **analytical method** for addition of vectors is called addition of vectors by rectangular components. This method is more mathematical in nature rather than geometrical; therefore, it is regarded as **more precise and accurate.**

Consider two vector $\vec{A}$ and $\vec{B}$. These vectors are added by head to tail rule. The resultant vector is given as $\vec{R} = \vec{A} + \vec{B}$ which is represented by vector $\overrightarrow{OP}$.



To resolve the vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ into rectangular components, we draw the perpendiculars from points P and M on x-axis shown in figure. OQ,QR and OR are the magnitudes of x-components of vector $\vec{A}$, $\vec{B}$ and $\vec{R}$ respectively. So

$$OR = OQ + QR$$

OR $\quad OR = OQ + MS \qquad \therefore QR = MS$

$$R_x = A_x + B_x \quad \text{----------(1)}$$

OR In vector form

$$\overrightarrow{R_x} = \overrightarrow{A_x} + \overrightarrow{B_x}$$

OR $\quad R_x\hat{\imath} = A_x\hat{\imath} + B_x\hat{\imath} = (A_x + B_x)\hat{\imath}$

- This shows that magnitudes of the sum of x-components of $\vec{A}$ and $\vec{B}$ is equal to the magnitude of x-component of resultant vector $\vec{R}$.

Similarly, we can write

$$RP = RS + SP$$

$$RP = QM + SP \qquad \therefore RS = QM$$

$$R_y = A_y + B_y \quad \text{----------(2)}$$

OR In vector form

$$\overrightarrow{R_y} = \overrightarrow{A_y} + \overrightarrow{B_y}$$

OR $\quad R_y\hat{\jmath} = A_y\hat{\jmath} + B_y\hat{\jmath} = (A_y + B_y)\hat{\jmath}$

- This shows that the magnitudes of the sum of y-components of $\vec{A}$ and $\vec{B}$ is equal to the magnitude of y-component of the resultant vector $\vec{R}$.
- Since $R_x\hat{i}$ and $R_y\hat{j}$ are the rectangular component of resultant vector $\vec{R}$ hence

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

By putting values of magnitudes of R_x and R_y, from equation (1) and (2) we get,

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Magnitude:

The magnitude of resultant vector $\vec{R}$ is.

$$R = \sqrt{R_x^2 + R_y^2}$$

or

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Direction:

The direction of the resultant vector $\vec{R}$ is obtained from

$$\tan\theta = \left(\frac{R_y}{R_x}\right)$$

or

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

Generalization:

For any number of coplanar vectors $\vec{A}, \vec{B}, \vec{C}$.... we can write:

$$\vec{R} = (A_x + B_x + C_x + \cdots)\hat{\imath} + (A_y + B_y + C_y + \cdots)\hat{\jmath}$$

Its magnitude R can be written as:

$$R = \sqrt{R_x^2 + R_y^2}$$

For Your Information

If $\vec{A}$ and $\vec{B}$ be two vectors then the magnitude of their resultant is,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

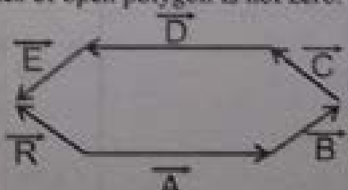
If $\theta = 0°$ $\quad R = \sqrt{A^2 + B^2 + 2AB} = A + B$

If $\theta = 180°$ $\quad R = \sqrt{A^2 + B^2 - 2AB} = A - B$

If $\theta = 90°$ $\quad R = \sqrt{A^2 + B^2}$

Do You Know?

The sum of vector which forms the sides of open polygon is not zero.



$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} \neq 0$$

$$R = \sqrt{(A_x + B_x + C_x + \cdots)^2 + (A_y + B_y + C_y + \cdots)^2}$$

And $$\theta = tan^{-1}\left(\frac{A_y+B_y+C_y+\cdots}{A_x+B_x+C_x+\cdots}\right)$$

Summary

(i) Find the x and y-components of all given vectors.

(ii) Add x-components of all the vectors to find the x-component R_x of the resultant vector.

(iii) Add y-components of all the vectors to find the y-component R_y of the resultant vector.

(iv) Find the magnitude of resultant vector $\vec{R}$ by using $R = \sqrt{R_x^2 + R_y^2}$

(v) Find the direction of resultant vector $\vec{R}$ by using $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

Q.12 How can you determine the angle θ of the vector $\vec{R}$ by its rectangular components.

Ans: Determination of Angle (θ_R) OR Direction of Resultant Vector

It is the angle which resultant vector makes with positive x-axis in anti-clockwise direction.

Table 2.1

	II	Y	I	
X′	R_x − R_y +		R_x + R_y +	X
	R_x − R_y −		R_x + R_y −	
	III	Y′	IV	

1) First find ϕ by the following relation.

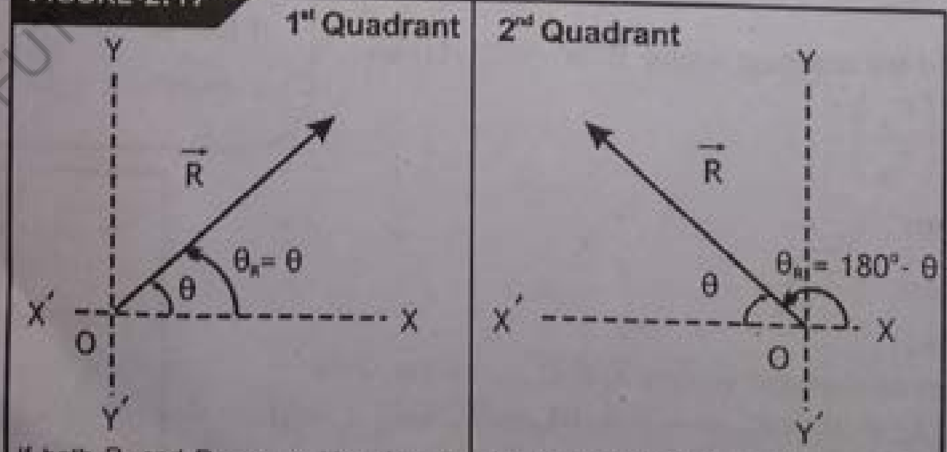
$$\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Where ϕ = the angle which $\vec{R}$ makes with nearest x-axis and while calculating ϕ ignore the signs of R_x and R_y.

2) By the signs of R_x and R_y, find the quadrant in which $\vec{R}$ lies as follows:

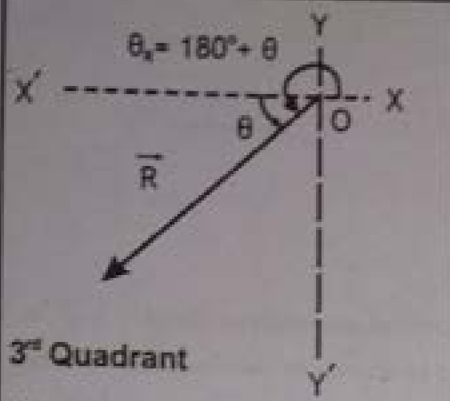
- If both R_x and R_y are positive, the resultant lies in the 1st quadrant and its direction θ_R is $\theta_R = \phi$
- If R_x is – ve and R_y is +ve, the resultant lies in 2nd quadrant and its direction is $\theta_R = 180^\circ - \phi$
- If both R_x and R_y are –ve , the resultant lies in 3rd quadrant and its direction is $\theta_R = 180^\circ + \phi$
- If R_x is + ve and R_y is –ve, the resultant lies in 4th quadrant and its direction is $\theta_R = 360^\circ - \phi$

FIGURE 2.17

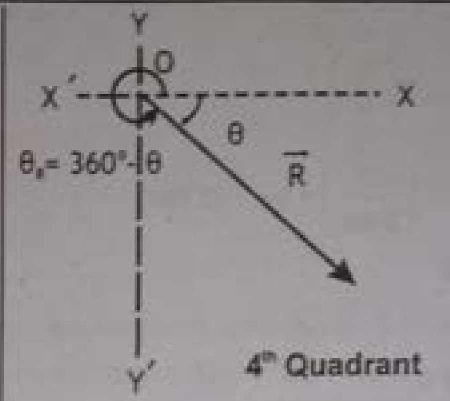


If both R_x and R_y are positive then θ_R lies 1st quadrant and can be determined as $\theta_R = \theta$

If R_x and R_y negative and R is positive then θ_R lies in 2nd quadrant and can be determined as $\theta_R = 180^\circ - \theta$



If both R_x and R_y are negative then θ_x lies 3rd quadrant and can be determined as $\theta_x = 180° + \theta$



If R_x is positive and R_y is negative then θ_x lies 4th quadrant and can be determined as $\theta_x = 360° - \theta$

Assignment 2.2:

A Force $F_1 = 20$ N making an angle $\theta_1 = 30°$ with positive x-axis and Force $F_2 = 30$ N making an angle $\theta_2 = 150°$ with positive x-axis, acts at a point, calculate the resultant force.

Solution:

Given Data:

$F_1 = 20N$ at angle of $\theta_1 = 30°$ with positive x-axis

$F_2 = 30N$ at angle of $\theta_2 = 150°$ with positive x-axis

Required:

Resultant force = F = ?

Calculation:

The magnitude of resultant force is given by,

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

First we resolve $\vec{F}_1$ and $\vec{F}_2$ into its components, as shown in figure (b). For "F_1", we have,

$$F_{1x} = F_1 \cos\theta_1 = 20 \times \cos 30° = 20 \times 0.866\text{ N}$$

$\Rightarrow \quad F_{1x} = 17.32N$

And $F_{1y} = F_1 \sin\theta_1 = 20 \times \sin 30° = 20 \times 0.5N$

$\Rightarrow \quad F_{1y} = 10N$

For F_2, we have,

$$F_{2x} = -F_2 \cos\theta_2 = -30 \times \cos 30° = -30 \times 0.866N$$

$\Rightarrow \quad F_{2x} = -25.98N$

Now the y-component [vertical component] of the resultant force "F" is given by,

$$F_y = F_{1y} + F_{2y} = 10N + 15N$$

$\Rightarrow \quad F_y = 25N$

Putting the values of "F_x" and "F_y" in equation (1), we get,

$$F = \sqrt{(-8.66N)^2 + (25)^2} = \left[\sqrt{74.9956 + 625}\right]N$$

$\Rightarrow \quad F = \left[\sqrt{699.9956}\right]N$

$\Rightarrow \quad F = 26.46N$ or $F = 26.5N$

Direction of $\vec{F}$:

We have, $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{25}{8.66}\right) = \tan^{-1}(2.886)$

$\Rightarrow \quad \theta = 70.8^\circ$

As x-component of $\vec{F}$ is negative, y-component is positive, so resultant force $\vec{F}$ will be acting in second quadrant by making an angle of 70.8° which negative x-axis, as shown in the figure (c). So the actual angle made by $\vec{F}$ with positive x-axis is given by,

$\theta' = 180^\circ - 70.8^\circ = 109.2^\circ \Rightarrow \theta' = 109.2^\circ$

Assignment 2.3:

Two forces of 20 N and 10 N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system.

Solution:

Let $\vec{F}_1 = 20N$ acting along x-axis,

$\vec{F}_2 = 10N$ making an angle of 120° with positive x-axis

(a) (i) Magnitude of resultant of $\vec{F}_1$ and $\vec{F}_2 = R = ?$

(ii) Direction of $\vec{R} = \theta = ?$

(b) Balance force of $\vec{R} = \vec{F} = ?$

(a) First we resolve $\vec{F}_1$ and $\vec{F}_2$ into its horizontal and vertical components. A $\vec{F}_1$ is lying only along x-axis, so it has only horizontal component and its vertical component will be equal to zero. So we have,

$F_{1x} = F_1 \cos\theta = 20 \times \cos 0^\circ$

$\Rightarrow \quad F_{1x} = 20 \times 1N$

$\Rightarrow \quad F_{1x} = 20N$ and $F_{1y} = 0N$

Now we resolve $\vec{F}_2$ into its horizontal and vertical components, as shown in figure (b).

So we have,

$F_{2x} = -F_2\cos\theta = -10 \times \cos 60^\circ$

$\Rightarrow \quad F_{2x} = -10 \times 0.5 = -5N$

$\Rightarrow \quad F_{2x} = -5N$

While,

$F_{2y} = F_2 \sin\theta = -10 \times \cos 60^\circ$

$\Rightarrow \quad F_{2y} = 10 \times 0.866N$

$\Rightarrow \quad F_{2y} = 8.66N$

F_{2x} is in second quadrant so it is taken as negative.

Now the x-component of the resultant is given by,

$R_x = F_{1x} + F_{2x}$

$\Rightarrow \quad R_x = 20 + (-5) = 20 - 5 = 15$

$\Rightarrow \quad R_x = 15\ N$

Now y-component of $\vec{R}$ [resultant] is given by,

$R_y = F_{1y} + F_{2y}$

$\Rightarrow \quad R_y = 0N + 8.66N$

$\Rightarrow \quad R_y = 8.66N$

(i) Now the magnitude of the resultant is given by,

$R = \sqrt{R_x^2 + R_y^2}$

$\Rightarrow \quad R = \sqrt{(15)^2 + (8.66)^2} = \sqrt{225 + 74.9956}$

$\Rightarrow \quad R = \sqrt{299.9956}$

$\Rightarrow \quad R = 17.32\ N$

(ii) The direction of $\vec{R}$ is given by,

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{8.66}{15}\right)$$

$$= \tan^{-1}(0.577)$$

$$= 29.98^\circ = 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

(b) Let $\vec{F}$ be the force which balance the resultant $\vec{R}$ [i.e. resultant of given forces] then $\vec{F}$ must have the same magnitude as $\vec{R}$ but opposite to $\vec{R}$ as shown in the figure. So F = 17.32N and making an angle of 210° with positive x-axis, as shown in the figure (c).

MCQ's

1. Resultant of two perpendicular vectors each of magnitude A is:
(A) A (B) 2A (C) $A\sqrt{2}$ (D) A^2

2. The magnitude of resultant of two vectors of 6 N and 8 N at right angle is:
(A) 6N (B) IN (C) 14 N (D) 10 N

3. The resultant of two forces of magnitude 5 N and 10N cannot be:
(A) 4N (B) 6N (C) 8N (D) 13N

4. A + B = B + A , this shows that vector addition is:
(A) Associative (B) Commutative (C) Additive (D) Additive inverse

5. The vector A has components A_x and A_y , the magnitude of A_x is given by ________;
(A) A – Ay (B) $(A - A_y)^{1/2}$ (C) $(A - A_y)^{1/2}$ (D) $[A^2 - A_y^2]^{1/2}$

6. Two vectors to be combined have magnitudes 60 N and 35 N. The magnitude of resultant vector R may be:
(A) 15N (B) 20 N (C) 70 N (D) 100 N

7. Which of the following pair of forces give the resultant force zero?
(A) 2N & 2N (B) 1N&4N (C) 2N&5N (D) 1N& 2N

8. Dot product of vector with itself is:
(A) Zero (B) 2 A (C) A^2 (D) A

9. If two non-zero vectors $\vec{A}$ and $\vec{B}$ are parallel to each other then:
(A) $\vec{A}.\vec{B} = 0$ (B) $\vec{A}.\vec{B}$ = AB (C) $|\vec{A} \times \vec{B}|$ = AB (D) $|\vec{A} \times \vec{B}| = \vec{A}.\vec{B}$

10. If the magnitude of $\vec{A} \cdot \vec{B} = \frac{1}{2}$ AB then an angle between $\vec{A}$ and $\vec{B}$ is:
(A) 30° (B) 45° (C) 60° (D) 90°

11. If |a + b| = |a – b| then angle between $\vec{a}$ and $\vec{b}$ is:
(A) 0° (B) 45° (C) 90° (D) 180°

12. Two forces are acting together on an object. The magnitude of their resultant is minimum, when the angle between the forces is: (Federal 2017)
(A) 60° (B) 120° (C) 180° (D) 45°

13. The resultant of 120 N and 20 N forces cannot be:
(A) 141 N (B) 100 N (C) 101 N (D) 130 N

14. If two unit vectors perpendicular to each other are added, magnitude of resultant:
(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 4

Answers Key

1. C	2. D	3. A	4. B	5. D	6. C	7. A	8. C	9. B	10. C	11. C	12. C
13. A	14. B										

Q. 13 What are different methods to multiply vectors with vectors?

Ans: When two vectors are multiplied then we get either a scalar quantity or vector quantity.

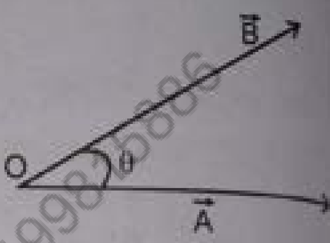
- If two vectors are multiplied and we get a scalar quantity then it is called scalar product.
- If two vectors are multiplied and we get a vector quantity then it is called vector product.

Q.14 Define scalar product of two vectors. Give examples.

Ans: **Scalar Product (Dot Product)**

If the product of two vectors be a scalar quantity then the product is called scalar product.

Vector . Vectors = Scalar



Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\boxed{\vec{A}.\vec{B} = AB\cos\theta}$$

Where A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ and θ is the angle between them.

- Because **dot operation** (.) is used to multiply the two vectors, therefore it is also called **dot product.**

Physical meaning

Dot product of two vectors is equal to product of magnitude of one vector and the component of the second vector in the direction of first vector.

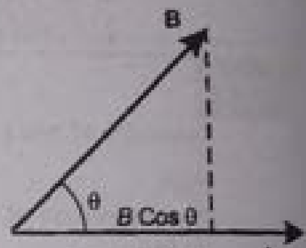


Fig. 2.10 (a)

From figure

$\vec{A}.\vec{B}$ = A (magnitude of component of $\vec{B}$ along $\vec{A}$)

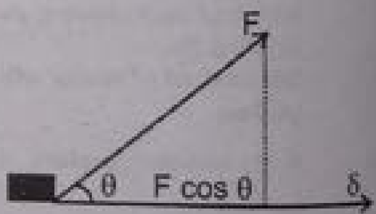
OR $\vec{A}.\vec{B}$ = A (projection of $\vec{B}$ on $\vec{A}$)

$\vec{A}.\vec{B}$ = $A\ (B\cos\theta)$

Examples

1) **Work** is scalar product of force and displacement

[$W = \vec{F}\cdot\vec{d}$]



2) **Power** is scalar product of force and velocity

[$P = \vec{F}\cdot\vec{v}$]

3) **Electric flux** is scalar product of electric intensity and vector area

[$\phi_E = \vec{E}\cdot\vec{A}$]

4) **Magnetic flux** is scalar product of magnetic field strength and vector area

[$\phi_B = \vec{B}\cdot\vec{A}$]

Q.15 Write down the characteristics of scalar product of two vectors.

Ans: **Characteristics of Scalar Product**

(i) **Commutative property**

Scalar product of two vectors is ***commutative.***

If $\vec{A}$ and $\vec{B}$ be two vectors and θ is the angle between them. Then

From figure

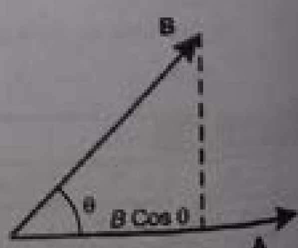


Fig. 2.10 (a)

$\vec{A}.\vec{B}$ = A (magnitude of component of $\vec{B}$ along $\vec{A}$)

OR $\vec{A}.\vec{B}$ = A (projection of $\vec{B}$ on $\vec{A}$)

$\vec{A}.\vec{B}$ = A (B cosθ) = $AB\cos\theta$ ------(1)

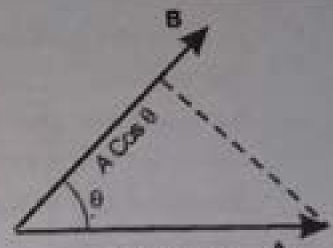


Fig. 2.10 (b)

Similarly

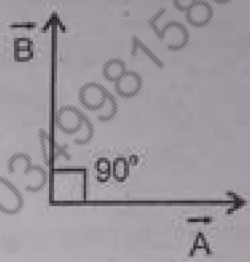
$\vec{B}.\vec{A}$ = B (Projection of $\vec{A}$ on $\vec{B}$)

$\vec{B}.\vec{A}$ = B (magnitude of component of $\vec{A}$ along $\vec{B}$)

$\vec{B}.\vec{A} = B(A\cos\theta) = AB\cos\theta$ ----------(2)

From equations (1) and (2)

$$\boxed{\vec{A}.\vec{B} = \vec{B}.\vec{A}}$$



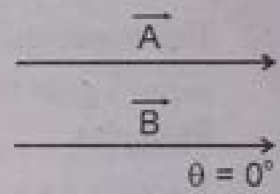
(ii) **Perpendicular vectors**

If two vectors are mutually perpendicular (θ = 90°) to each other then, their scalar product is *zero*. i.e.

$\vec{A}.\vec{B} = AB\cos 90°$

$\vec{A}.\vec{B} = AB\,(0)$

$\vec{A}.\vec{B} = 0$

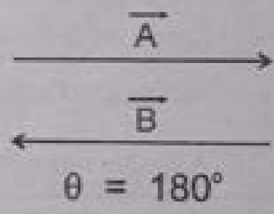


(iii) **Parallel Vectors**

If two vectors are parallel (θ = 0°) to each other, their scalar product is equal to the product of their magnitudes. i.e.

$\vec{A}.\vec{B} = AB\cos 0° = AB\,(1) = AB$

This is the positive maximum value of scalar product.



(iv) **Anti- Parallel Vectors**

If two vectors are anti-parallel (θ = 180°) then their scalar product is negative.

$\vec{A}.\vec{B} = AB\cos 180° = AB\,(-1) = -AB$

This is the negative maximum value of scalar product.

Do You Know?

Magnitude of a vector $\vec{A}$ is $\sqrt{\vec{A}.\vec{A}}$

(iv) **Self Scalar product**

The self-product of a vector is equal to square of its magnitude i.e.,

$\vec{A}.\vec{A} = AA\cos 0° = A^2\,(1) = A^2$

For Your Information

Why K.E is scalar quantity even through velocity is a vector.

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\,(\vec{v}.\vec{v})$

Q.16 Define and explain vector product of two vectors? Give its physical significance. Give examples.

Ans: **Vector Product (Cross Product)**

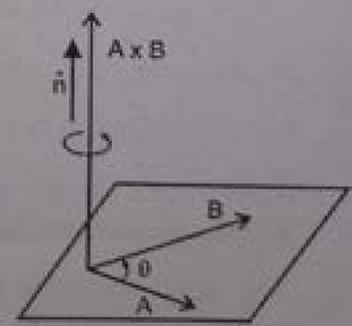
If the product of two vectors be a vector quantity then the product of vectors is called vector product.

Vector × Vector = Vector

The vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\boxed{\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}}$$

Where angle between $\vec{A}$ and $\vec{B}$ is θ and $\hat{n}$ is the unit vector perpendicular



(a)

to the plane containing $\vec{A}$ and $\vec{B}$. (Where θ is smaller angle between the two vectors)

- Because **cross operation** (×) is used to multiply the two vectors, therefore it is also called **Cross product.**

For Your Information

$\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ can be equal in magnitude only when $\vec{A}$ and $\vec{B}$ are parallel or anti – parallel.

Direction of vector product

The direction of vector product $\vec{A}\times\vec{B}$ can be found by right hand rule.

Right hand Rule

Join the tails of the two vectors to define a plane of vectors. "Rotate vector $\vec{A}$ into $\vec{B}$ through smaller of the two possible angles. Curl the fingers of the right hand in the direction of rotation. Erect thumb represents the direction of $\vec{A}\times\vec{B}$."

OR

"Curl the fingers of your right hand from first vector to second vector through smaller angle then erect thumb will point out in the direction of product vector"

Physical Significance:

The magnitude of $\vec{A}\times\vec{B}$ i.e. AB Sinθ gives the area of the plane determined by $\vec{A}$ and $\vec{B}$. The unit vector $\hat{n}$ gives the direction of the area of the plane.

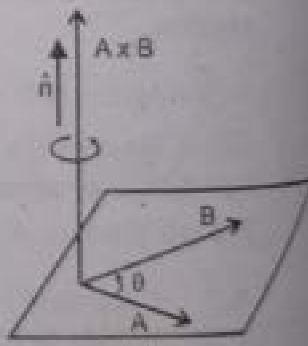
Examples

1) Torque is the vector product of position vector $\vec{r}$ and force $\vec{F}$ i.e. $[\vec{\tau}=\vec{r}\times\vec{F}]$
2) Force on a moving charged particle in magnetic field is vector product of velocity of charged particle and magnetic field $\vec{B}$ .i.e. $[\vec{F}=q(\vec{v}\times\vec{B})]$
3) Angular momentum is vector product of position vector $\vec{r}$ and linear momentum $\vec{p}$ i.e. $[\vec{L}=\vec{r}\times\vec{p}]$

Q.17 Write down the characteristics of vector product of two vectors.

Ans: Characteristics of Vector Product

(i) Violation of Commutative law



(a)

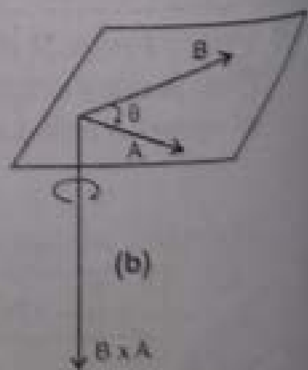
The cross product of two vector $\vec{A}$ and $\vec{B}$ is not commutative.

If $\vec{A}$ and $\vec{B}$ be two vectors and θ is the angle between them. Then

$$\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n} \quad ....(1)$$

and $$\vec{B}\times\vec{A}=BA\sin\theta\left(-\hat{n}\right)$$

$$\vec{B}\times\vec{A}=AB\sin\theta\left(-\hat{n}\right) \quad ....(2) \quad [\text{since } AB=BA]$$



(b)

From equations (1) and (2)

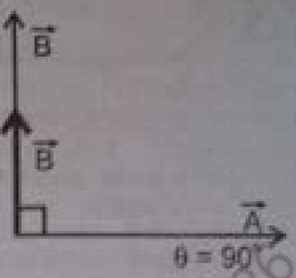
$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

OR $$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$$

(ii) Perpendicular Vectors

The cross product of two perpendicular ($\theta=90^\circ$) vectors $\vec{A}$ and $\vec{B}$ has *maximum* magnitude. i.e.

$$\vec{A}\times\vec{B} = AB\sin 90^\circ\,\hat{n}$$
$$\vec{A}\times\vec{B} = AB\,(1)\,\hat{n}$$
$$\vec{A}\times\vec{B} = AB\,\hat{n}$$

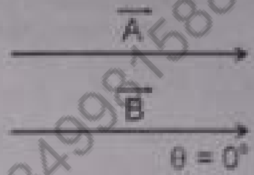


(iii) **Parallel and Anti-parallel Vectors**

The cross product of two parallel ($\theta = 0^\circ$) or two anti-parallel ($\theta = 180^\circ$) vectors is a ***null*** vector. i.e.

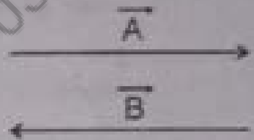
In case of parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 0^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$



In case of anti – parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 180^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$



(iv) **Self Vector product**

The self-product of a vector $\vec{A}$ is ***null*** vector.

$$\vec{A}\times\vec{A} = AA\sin 0^\circ\,\hat{n} = AA\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

In case of unit vectors,

(v) **Area of Parallelogram**

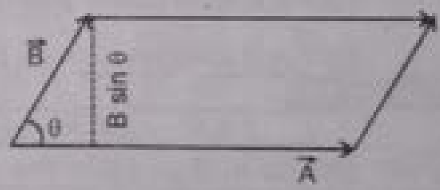
The magnitude of cross product of two vectors represents the ***area of parallelogram*** formed with these vectors taken as its two adjacent sides.

Area of parallelogram = (length) (height)

$= (A)\,(B\sin\theta)$

$= AB\sin\theta$

$=$ magnitude of $(\vec{A}\times\vec{B})$



Area of parallelogram $= \left|\vec{A}\times\vec{B}\right|$

Assignment 2.4:

Show that $|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2$

Solution:

$$\left|\vec{A}\times\vec{B}\right| = AB\,\text{Sin}\,\theta$$

$$|\vec{A}\times\vec{B}|^2 = A^2B^2\,\text{Sin}^2\theta \quad \ldots\ldots(1)$$

$$\vec{A}\cdot\vec{B} = AB\,\text{Cos}\,\theta$$

$$|\vec{A}\times\vec{B}|^2 = A^2B^2\,\text{Cos}^2\theta \quad \ldots\ldots(2)$$

Adding eq. (1) and (2)

$$|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2\,\text{Sin}^2\theta + A^2B^2\,\text{Cos}^2\theta$$
$$= A^2B^2(\text{Sin}^2\theta + \text{Cos}^2\theta)$$

$$|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2(1)$$

$$|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2$$

MCQ's

1. **The dot product of vector A with itself is equal to:**
(A) Zero (B) A (C) 2A (D) A^2

2. **If the angle between the two vectors with magnitude 12 and 4 is 60° then their scalar product is:**
(A) 6 (B) 12 (C) 24 (D) None of these

3. **The scalar product of two vectors is maximum when they are:**
(A) Parallel (B) Perpendicular (C) Null (D) Anti parallel

4. **Projection of $\overline{B}$ on $\overline{A}$ is:**
(A) A Cos θ (B) B Sin θ (C) A Sin θ (D) B Cos θ

5. **Projection of B along A is given as:**
(A) $\hat{A} \cdot \vec{B}$ (B) $\vec{B} \cdot \hat{A}$ (C) $\frac{\vec{B}\vec{A}}{A \cos\theta}$ (D) $\frac{A \cos\theta}{B}$

6. **If the Scalar product of two vectors is $2\sqrt{3}$ and the magnitude of their vector product is 2. The angle between them is:**
(A) 120° (B) 30° (C) 60° (D) 180°

7. **The vector product $(\vec{A} \times \vec{A})$ is:**
(A) $\vec{O}$ (B) 1 (C) A (D) 0

8. **The magnitude of dot and cross product of two vectors are equal when angle between them is:**
(A) Zero (B) 45° (C) 90° (D) 270°

9. **An area of parallelogram formed by vectors A and B as its two adjacent sides is given as:**
(A) AB Sin θ (B) AB cos θ (C) AB tan θ (D) $\vec{A} \cdot \vec{B}$

10. **If $\vec{A} \times \vec{B} = 0$, then angle between the vectors is:**
(A) 90° (B) 45° (C) 0° (D) None of these

11. **Both the dot product and cross product of two vectors $\vec{A}$ and $\vec{B}$ is zero, when:**
(A) $\vec{A}$ and $\vec{B}$ are parallel to each other (B) $\vec{A}$ and $\vec{B}$ are antiparallel
(C) $\vec{A}$ and $\vec{B}$ are perpendicular to each other (D) Either the vector is zero

Answers Key

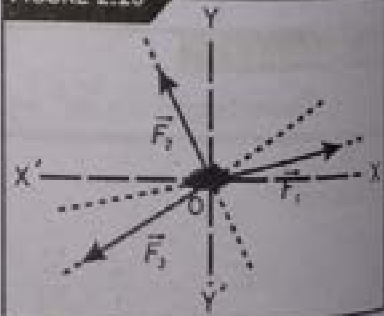
1. D	2. C	3. A	4. D	5. B	6. B	7. A	8. B	9. A	10. C	11. D

Q.18 Define and explain the terms concurrent forces and equilibrant force.

Ans: **Concurrent Forces**

When two or more forces are acting upon a body and the lines of action of these forces pass through a common point, the forces are said to be concurrent forces.

FIGURE 2.26



Example:

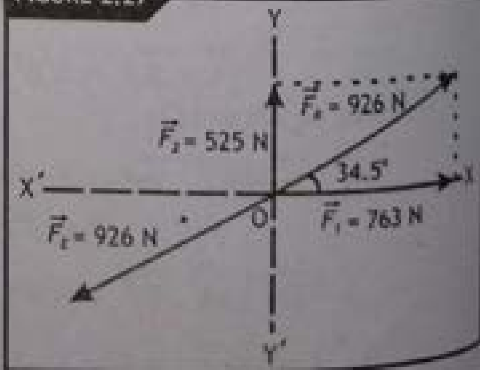
let three dogs are pulling a piece of meat with forces F_1,F_2 and F_3 as shown in the figure, the forces are concurrent as their line of action passes through a common point.

Equilibrant Force:

"Two or more concurrent forces can be balanced by a single force called equilibrant force." OR

"It the single force which is equal in magnitude but opposite to the resultant of different concurrent forces."

FIGURE 2.27



Example:

Consider two concurrent forces F_1 = 763 N and F_2 = 525 N acting at right angle to each other with their resultant F_R = 926 N at 34.5 with x –axis.

The equilibrant force is equal in magnitude to that of the resultant force but it acts in the opposite direction as shown in Figure.

Following diagram may help to understand concurrent forces.

Two or more concurrent forces can be balanced by a single force, as shown.

Q.19 Define and explain the term torque or moment of force.

Ans: **Torque or Moment of Force**

"The turning effect produced in a body about a fix point due to an applied force is called torque or moment of force".

OR

The product of magnitude of force and the perpendicular distance from axis of rotation to line of action of force i.e. moment arm.

$$\tau = \ell F$$

Where

ℓ = perpendicular distance between line of action of force and axis of rotation

F = Magnitude of applied force

OR

Torque is the vector product of position vector $\vec{r}$ and force $\vec{F}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

OR $$\vec{\tau} = rF\sin\theta\,\hat{n}$$

Unit

The SI -unit of torque is newton meter (Nm)

Its dimensions are $[ML^2T^{-2}]$.

Torque is a vector quantity.

Examples of torque

- Tightening of a nut with a spanner.
- A seesaw rotates on and off the ground due to torque imbalance.

Direction:

Its direction is determined by **Right Hand Rule.** "Curl the fingers of your right hand in the direction of rotation then erect thumb will point out in the direction of torque"

Force and Torque Analogy

1. Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration.

2. Torque is the rotational analogue of force.

As F = ma , similarly $\tau = I\alpha$

(where I= Moment of Inertia and α is angular/ rotational acceleration)

3. If the body is at rest or rotating with uniform angular velocity, the angular acceleration will be zero. In this case the torque acting on the body will be zero.

Note:
The force which is perpendicular to moment arm apply torque on the body while the force which passes through axis of rotation cannot apply torque.

The nut is easy to turn with a spanner.

It is easier still if the spanner has a long handle.

Greater the moment arm, easier to rotate the body and vice versa.

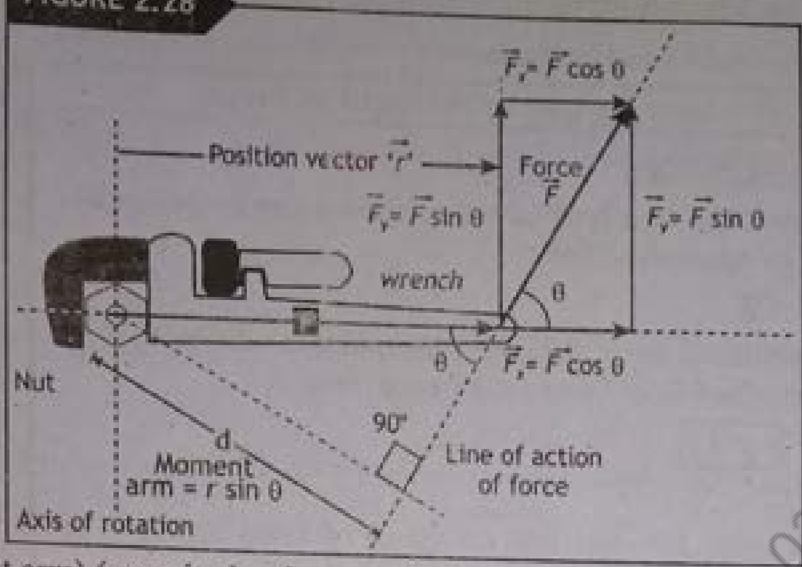
Q.20 Show that torque is the vector product of force and position vector.

Ans: Torque is an example of vector product of position vector $\vec{r}$ and $\vec{F}$.

Proof: Let us consider a spanner on which we apply a force $\vec{F}$ on its end to loose the nut (where its pivot point lies and axis of rotation passes as shown) and $\vec{r}$ is the position vector from pivot point to point of action of force.

- Let us resolve the force $\vec{F}$ into its rectangular components $\vec{F_X}$ and $\vec{F_Y}$ as shown.
- We see that line of action of $F_X = F\cos\theta$ passes through the pivot point i.e. it is parallel to the position vector $\vec{r}$. So, it does not apply torques.
- While the line of action of force $F_Y = F\sin\theta$ is perpendicular to moment arm or position vector $\vec{r}$. So, the effective component of force which produces rotation is F_Y.
- Since magnitude of torque is given by:

FIGURE 2.28



Torque = (moment arm) (magnitude of perpendicular component of force)

$$\tau = (r)\,(F\,sin\theta)$$

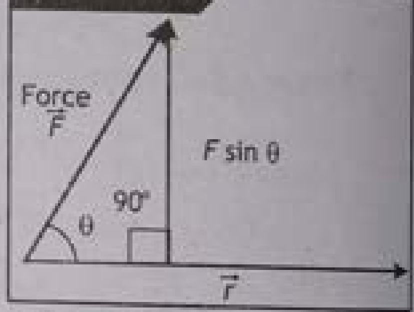
In vector form,

$$\vec{\tau} = (r)\,(F\,sin\theta)\hat{n}$$

OR

$$\vec{\tau} = \vec{r} \times \vec{F}$$

FIGURE 2.29 (a)



Another Proof:

Instead of resolving force, we can prove the same formula by resolving position vector $\vec{r}$ as shown in the figure.

We can write the torque as:

Torque = (moment arm i.e. perpendicular component of position vector $\vec{r}$) (magnitude of force)

$$\tau = (r\,sin\theta)\,(F)$$

OR

$$\tau = (r\,F\,sin\theta)$$

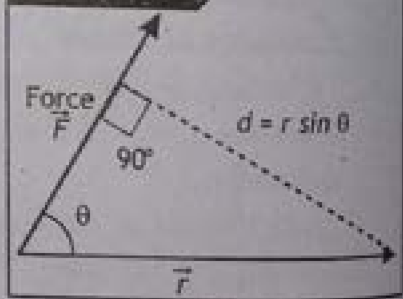
In vector form

$$\vec{\tau} = (r)\,(F\,sin\theta)\hat{n}$$

OR

$$\vec{\tau} = \vec{r} \times \vec{F}$$

FIGURE 2.29 (b)



Dependence of torque

Torque depends on the following factors;

1) Magnitude of ***force***
2) Magnitude of position vector
3) Angle between force vector $\vec{F}$ and position vector $\vec{r}$.

Maximum Torque:

Magnitude of torque applied on the body will be maximum if $\vec{F}$ and $\vec{r}$ are perpendicular ($\theta = 90^o$)

$$\tau_{max} = r\,F\,sin(90^o)$$

$$\Rightarrow \quad \tau_{max} = r\,F\,(1) = r\,F$$

Minimum Torque:

Magnitude of torque applied on the body will be minimum if $\vec{F}$ and $\vec{r}$ are parallel ($\theta = 0^o$) or anti-parallel ($\theta = 180^o$)

If $\theta = 0^o$

$$\tau_{min} = r\,F\,sin(0^o)$$

$$\Rightarrow \quad \tau_{min} = r\,F\,(0) = 0$$

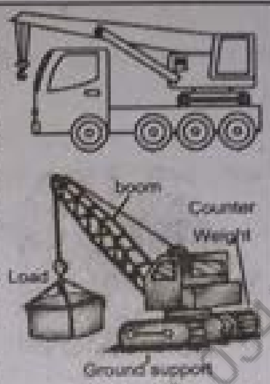
And if $\theta = 180^o$

$$\tau_{min} = r\,F\,sin(180^o)$$

$$\Rightarrow \quad \tau_{min} = r\,F\,(0) = 0$$

DO YOU KNOW

Mobile Crane Counterweights – an Important Safety Factor:
Mobile cranes are vehicles designed to lift, lower and transport heavy loads. A mobile crane moves the heavy loads. Cranes operate on the principle of lever. The shorter end of the beam is applied by a force and the longer end (called boom) can rotate and move the load radially inward or outward, to position the object at the correct location.
Since boom is having large length even a small load lifted at its end will produce large torque and there is a danger for the crane to topple over.
To overcome such a situation, cranes have a counter weight at the other side that moves in an opposite direction from the object that is lifted. The counter weight exerts a torque on the crane in equal and opposite direction to the torque from the load. Mathematically



$$\vec{\tau}_{boom} = \vec{\tau}_{counterweight}$$

Q.21 What is value of torque if the body is at rest or rotating with uniform angular velocity?

Ans: Torque acting on the body will be *zero*,

Reason

In this case, angular acceleration is zero, so torque will be

$$\tau = I\alpha \qquad [2^{nd} \text{ law for rotational motion}]$$

$$\tau = I(0) = 0$$

Q.22 What is couple? Derive its formula

Ans: **Moment of A Couple**

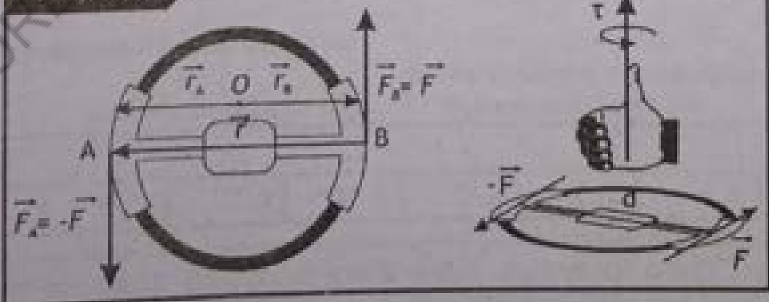
Two parallel forces that have the same magnitude but opposite in direction and are separated by a perpendicular distance, form the **couple**.

The rotational effect produced by couple is called **moment of couple**.

OR

"The moment produced by a couple is called a couple moment."

FIGURE 2:30



Example:

For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate as shown in Figure.

Derivation:

Consider the steering wheel whose pivot point is O. Let two equal and opposite forces $\vec{F_A}$ and $\vec{F_B}$ are acting at points A and B whose position vectors are $\vec{r_A}$ and $\vec{r_B}$ respectively from the pivot point O as shown.

A vector whose point of application is not fixed but its magnitude and direction are fixed, is called **free vector.**

OR

A vector whose point of action is not fixed and can be moved parallel to itself, known as free vector.

- Torque applied due to force $\overrightarrow{F_A}$ is:

$$\overrightarrow{\tau_A} = \overrightarrow{r_A} \times \overrightarrow{F_A}$$

- Torque applied due to force $\overrightarrow{F_B}$ is:

$$\overrightarrow{\tau_A} = \overrightarrow{r_B} \times \overrightarrow{F_B}$$

- Moment of couple i.e. total torque is equal to vector sum of torques due to these forces is:

$$\vec{\tau} = \overrightarrow{\tau_A} + \overrightarrow{\tau_B}$$

$$\Rightarrow \vec{\tau} = (\overrightarrow{r_A} \times \overrightarrow{F_A}) + (\overrightarrow{r_B} \times \overrightarrow{F_B}) \quad \text{-------------} \quad (i)$$

Since both forces are equal in magnitude but opposite in direction i.e.

$$\overrightarrow{F_A} = -\vec{F} \qquad \text{and} \qquad \overrightarrow{F_B} = \vec{F}$$

So, equation (i) becomes:

$$\vec{\tau} = (\overrightarrow{r_A} \times (-\overrightarrow{F})) + (\overrightarrow{r_B} \times \vec{F})$$

$$\Rightarrow \vec{\tau} = (-\overrightarrow{r_A} + \overrightarrow{r_B}) \times \vec{F}$$

$$\Rightarrow \vec{\tau} = (\overrightarrow{r_B} - \overrightarrow{r_A}) \times \vec{F}$$

Since, $\overrightarrow{r_B} - \overrightarrow{r_A} = \vec{r} = position\ vector\ from\ point\ B\ to\ A$

So, $\vec{\tau} = \vec{r} \times \vec{F}$

Direction of couple is measured by **Right Hand Rule.**

MCQ's

1. **The direction of torque can be found by:**
   (A) Head to tail rule (B) Right hand rule (C) Left hand rule (D) Fleming rule
2. **Torque of force is given by $\vec{\tau} = \vec{r} \times \vec{F}$. It has maximum value when $\vec{r}$ and $\vec{F}$ are at an angle of:**
   (A) 90° (B) 0° (C) 45° (D) 30°
3. **If the position vector $\vec{r}$ and $\vec{F}$ are in same direction then torque will be:**
   (A) Maximum (B) Minimum (C) Zero (D) Negative
4. **If the body is at rest or rotating with uniform angular velocity, then torque will be:**
   (A) Maximum (B) Negative (C) Zero (D) Positive
5. **The turning effect of force in a body is called:**
   (A) Work (B) Momentum (C) Power (D) Torque
6. **The direction of torque is:**
   (A) Along the position vector $\vec{r}$ (B) Parallel to the plane containing $\vec{r}$ and $\vec{F}$
   (C) Along the force $\vec{F}$ (D) Perpendicular to the plane $\vec{r}$ and $\vec{F}$
7. **The counterpart of force for rotational motion is called:**
   (A) Linear momentum (B) Angular momentum (C) Angular acceleration (D) Torque

Answers Key

1. B	2. A	3. C	4. C	5. D	6. D	7. D	

Q.23 What is equilibrium? Give its types. Also discuss types of dynamic equilibrium

Ans: **Equilibrium**

The state of a body, under the action of several forces and torques acting together and there is no change in the translational motion as well as its rotational motion, is called equilibrium.

OR

A body is said to be in equilibrium if it is at rest or moving with uniform velocity under the action of a number of forces.

OR

If a body has no acceleration then it is said to be in equilibrium.

Fig: 2.23

Types of Equilibrium:

i. **Static Equilibrium:**

If a body is in rest then it is in state of static equilibrium.

OR When a body is at rest under the action of several forces acting together the body is said to be in static equilibrium.

For example, book placed on the table.

ii. **Dynamic Equilibrium:**

If a body is moving with uniform velocity then it is in dynamic equilibrium.

OR When a body is moving at uniform velocity under the action of several forces acting together the body is said to be in dynamic equilibrium.

- A paratrooper falling down with constant velocity is an example of dynamic translational equilibrium
- A car moving with uniform linear velocity
- A wheel, a disc or a grinding stone rotating about its axis with constant angular speed are examples of dynamic rotational equilibrium.

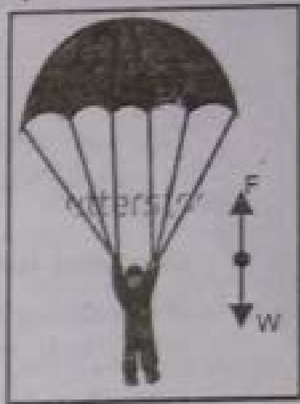
Types of Dynamic Equilibrium

i. **Dynamic Translational Equilibrium:** When a body is moving with uniform linear velocity the body is said to be in dynamic translational equilibrium.

For example, a paratrooper falling down with constant velocity is in dynamic translational equilibrium as shown in Figure

ii. **Dynamic Rotational Equilibrium:** When a body is moving with uniform angular velocity the body is said to be in dynamic rotational equilibrium.

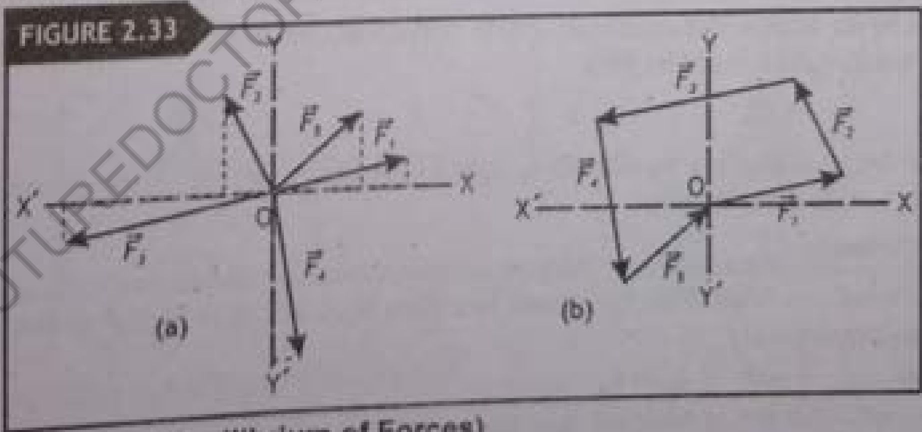
For example a compact disk (CD) rotating in CD Player with constant angular velocity is in dynamic rotational equilibrium as shown in Figure.



Q.24 State the two conditions of equilibrium. What type of equilibrium is guaranteed by each condition of equilibrium?

Ans: For a body to be in complete equilibrium, two conditions must be satisfied.

FIGURE 2.33



First condition of equilibrium (Equilibrium of Forces)

The vector sum of all the forces acting on a body must be null vector.

i.e.

$$\sum \vec{F} = \vec{O} \quad \text{\_\_\_\_\_\_(1)}$$

In case of coplanar force, 1st condition can be expressed as:

$$\sum \vec{F}_x = \vec{O} \quad \text{\_\_\_\_\_\_(2)}$$

And

$$\sum \vec{F}_y = \vec{O} \quad \text{\_\_\_\_\_\_(3)}$$

Where $\sum\vec{F}_x$ = Vector sum of x-directed forces

$\sum\vec{F}_y$ = Vector sum of y-directed forces

Note

- If the rightward forces are taken as positive then leftward forces are taken as negative.
- If upward forces are taken as positive, then downward forces are taken as negative.
- ***Forces which lie in a common plane are said to be coplanar.***

Second Condition of equilibrium (Equilibrium of Torques)

The vector sum of all the torques acting on the body about an axis must be null vector.

i.e. $$\sum\vec{\tau} = \vec{O}$$

OR $$\sum\vec{\tau}_{Clockwise} = \sum\vec{\tau}_{Anti-Clockwise}$$

There are certain situations in which sum of forces is zero but body is still not in equilibrium e.g. motion of steering wheel. Then we required 2nd condition of equilibrium i.e. sum of torques actin on the must be zero.

Condition for Translational Equilibrium

When sum of all the forces acting the body is zero, the linear acceleration of body is zero then the body is said to be in translational equilibrium.

Condition for Rotational Equilibrium

When sum of torques acting on the body is zero, angular acceleration of body is zero and the body is said to be in rotational equilibrium.

Q.25 What is condition for complete equilibrium?

Ans: For a body to be in complete equilibrium, both conditions must be satisfied. i.e. both ***linear*** acceleration and ***angular*** acceleration must be zero.

OR

For a body to be in complete equilibrium, sum of all forces acting on the body is zero and sum of all torques acting on the body is zero.

Complete Equilibrium:

- When the first condition is satisfied this means that there is no net force acting on the body, so it will represent translational equilibrium only.

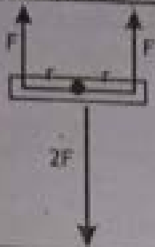
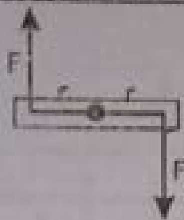
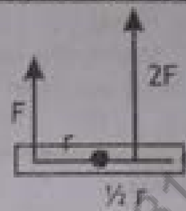
  $F_{net} = 0$ and $a_{net} = 0$
- When the second condition is satisfied this means that there is no net torque acting on the body, so it will represent rotational equilibrium only.

  $\tau_{net} = 0$ and $\alpha_{net} = 0$ (α_{net} = net angular acceleration)
- For complete equilibrium, both the first and second conditions of equilibrium must be satisfied.
- There are certain situations in which one of the two conditions are satisfied, so body will not be in complete equilibrium.

  For example, in case of couple we see that the first condition is satisfied, but still the object can rotate with angular acceleration, therefore the object is not in equilibrium with respect to rotation, hence we cannot say that object is in complete equilibrium.

Q.26 Describe different situations which could be helpful in understanding the translational, rotational and complete equilibrium.

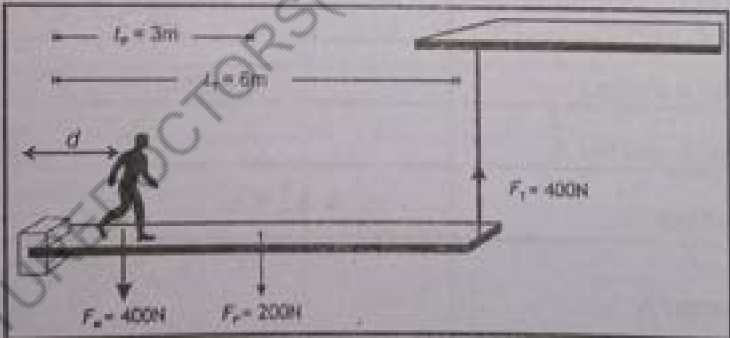
Ans: In these diagrams the axis of rotation is taken perpendicular to figure and is represented by a red dot.

		
(a) This body is in static equilibrium.	(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.	(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.
First Condition Satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole. **Second Condition Satisfied:** Not torque about the axis = 0, so body at rest no tendency to start rotating.	**First Condition Satisfied:** Net force = 0, so body at rest has no tendency to start moving as a whole. **Second Condition Satisfied:** There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.	**First Condition NOT Satisfied:** There is a net upward force, so body at will start moving upward. **Second Condition Satisfied:** Not torque about the axis = 0, so body at rest no tendency to start rotating.

To be in static equilibrium, a body at rest must satisfy both conditions for equilibrium: It can have no tendency to accelerate as a whole or to start rotating.

Assignment 2.5:

A uniform plank of weight 200 N and length 6 m is supported by a rope as shown in the figure. If the breaking tension in the rope is 400 N. How far can a boy of weight 400 N walk towards the support?



Solution: Apply 2[nd] condition of equilibrium

$$\Sigma\tau = 0$$

Σ Clockwise torque = Anti Clockwise torque

$\tau_P + \tau_M = \tau_T$ ---------------- (i)

Where τ_P = torque due to weight of the plank = $F_P \times l_P = 200 \times 3$ (Nm)

τ_M = torque due to weight of the man = $F_M \times d = 400 \times d$ (Nm)

τ_T = torque due to tension of the rope = $F_T \times l_T = 400 \times 6$ (Nm)

Put values in equation (i)

⇨ $200 \times 3 + 400 \times d = 400 \times 6$

⇨ $600 + 400 \times d = 2400$

⇨ $400 \times d = 2400 - 600$

⇨ $400 \times d = 1800$

⇨ $d = 1800/400$

⇨ $d = 4.5$ m

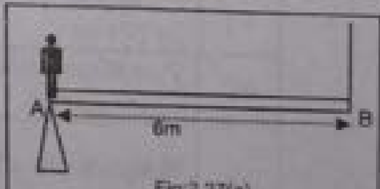


Fig 2.27(a)

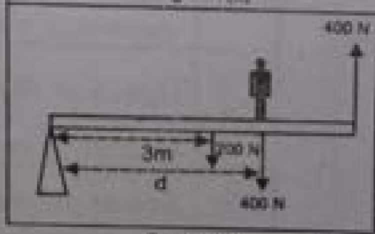


Fig 2.27(b)

MCQ's

1. **The first condition of equilibrium implies that:**
(A) $\Sigma F = 0$ (B) $\Sigma F_x = 0$ only (C) $\Sigma F_y = 0$ only (D) $\Sigma F_x = \Sigma F_y$
2. **A body is in static equilibrium only when it is:**
(A) At rest (B) Moving with variable velocity
(C) Moving with uniform acceleration (D) Moving with uniform velocity
3. **A body will be in translational equilibrium if:**
(A) $\Sigma \tau = 0$ (B) $\Sigma F = 0$ (C) $\Sigma p = 0$ (D) $\Sigma F_x = 0$

Answers Key

1. A	2. A	3. B

FORMULAE

1	Commutative law for vector addition	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$	
2	Subtraction of vectors	$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$	
3	Vector representation	$\vec{A}=A\hat{A}$	
4	Unit vector	$\hat{A}=\frac{\vec{A}}{A}$	
5	Null vector	$\vec{A}+(-\vec{A})=\vec{O}$	
6	Vector in terms of rectangular components	$\vec{A}=A_x\hat{i}+A_y\hat{j}$	
7	x-component of a vector $\vec{A}$	$A_x = A\cos\theta$	
8	y-component of a vector $\vec{A}$	$A_y = A\sin\theta$	
9	Magnitude of vector $\vec{A}$	$A^2 = A_x^2 + A_y^2$	$A=\sqrt{A_x^2+A_y^2}$
10	Direction of vector $\vec{A}$	$\theta = \tan^{-1}(\frac{A_y}{A_x})$	
11	Position vector of a point P(a, b) in plane	$\vec{r}=a\hat{i}+b\hat{j}$	
12	Position vector of a point P(a, b, c) in space	$\vec{r}=a\hat{i}+b\hat{j}+c\hat{k}$	
13	Magnitude of resultant ($\vec{R}$) of vectors $\vec{A}$ and $\vec{B}$	$R=\sqrt{(A_x+B_x)^2+(A_y+B_y)^2}$	
14	Direction of resultant ($\vec{R}$) of vectors $\vec{A}$ and $\vec{B}$	$\theta=\tan^{-1}(\frac{A_y+B_y}{A_x+B_x})$	
15	Scalar product of two vectors	$\vec{A}\cdot\vec{B}=AB\cos\theta$	
16	Scalar product of two perpendicular vectors	$\vec{A}\cdot\vec{B}=0$	

17	Scalar product of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$	$\hat{i}\cdot\hat{j}=0$	$\hat{j}\cdot\hat{k}=0$	$\hat{k}\cdot\hat{i}=0$
18	Scalar product of two parallel vectors	$\vec{A}\cdot\vec{B}=AB$		
19	Scalar product of two anti-parallel vectors	$\vec{A}\cdot\vec{B}=-AB$		
20	Self dot product of vector $\vec{A}$	$\vec{A}\cdot\vec{A}=A^2$	$A=\sqrt{\vec{A}\cdot\vec{A}}$	
21	Self scalar product of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$	$\hat{i}\cdot\hat{i}=1$	$\hat{j}\cdot\hat{j}=1$	$\hat{k}\cdot\hat{k}=1$
22	Scalar product in terms of rectangular components	$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$		
23	Angle between two vectors $\vec{A}$ and $\vec{B}$	$\theta=\cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$	$\theta=\cos^{-1}\left(\frac{A_xB_x+A_yB_y+A_zB_z}{AB}\right)$	
24	Projection of vector $\vec{A}$ on $\vec{B}$	$A\cos\theta=\frac{\vec{A}\cdot\vec{B}}{B}$	$A\cos\theta=\vec{A}\cdot\hat{B}$	
25	Projection of vector $\vec{B}$ on $\vec{A}$	$B\cos\theta=\frac{\vec{A}\cdot\vec{B}}{A}$	$B\cos\theta=\hat{A}\cdot\vec{B}$	
26	Vector product of two vectors	$\vec{A}\times\vec{B}=AB\sin\theta\cdot\hat{n}$		
27	Vector product of two perpendicular vectors	$\vec{A}\times\vec{B}=AB\hat{n}$		
28	Vector product of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$	$\hat{i}\times\hat{j}=\hat{k}$	$\hat{j}\times\hat{k}=\hat{i}$	$\hat{k}\times\hat{i}=\hat{j}$
29	Vector product of two parallel or anti-parallel vectors	$\vec{A}\times\vec{B}=\vec{O}$		
30	Self cross product of vector $\vec{A}$	$\vec{A}\times\vec{A}=\vec{O}$		
31	Self Vector product of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$	$\hat{i}\times\hat{i}=\vec{O}$	$\hat{j}\times\hat{j}=\vec{O}$	$\hat{k}\times\hat{k}=\vec{O}$
32	Vector product in terms of rectangular components	$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB$		
33	Angle between two vectors $\vec{A}$ and $\vec{B}$	$\theta=\sin^{-1}\left(\frac{\lvert\vec{A}\times\vec{B}\rvert}{AB}\right)$		
34	Area of a parallelogram and vector product	Area of a parallelogram = $\lvert\vec{A}\times\vec{B}\rvert=AB\sin\theta$		
35	Torque	$\tau=\ell F$	$\vec{\tau}=\vec{r}\times\vec{F}$	$\vec{\tau}=rF\sin\theta\,\hat{n}$

36	1st condition of equilibrium	$\sum \vec{F} = 0$	$\sum \vec{F}_x = 0$, $\sum \vec{F}_y = 0$
37	2nd condition of equilibrium	$\sum \vec{\tau} = 0$	

Key Points

- Cartesian coordinates system provides a framework to sketch vector quantities.
- Head to tail rule is a geometrical approach to add vectors.
- Addition of vectors by rectangular components method is somehow mathematical method.
- Scalar product and vector product helps to understand the physical difference between physical quantities i.e

  e.g. work = Force. Displacement = $\vec{F} . \vec{S} = \vec{F} . \vec{d}$

  and torque = Moment arm × Force = $\vec{r} \times \vec{F}$
- The state in which an object has zero acceleration is said to be in equilibrium.
- If the sum of all the forces acting on an object is equal to zero then the object is said to be in equilibrium i.e. $\Sigma \vec{F} = 0$.
- If the sum of all the torques acting on a body is equal to zero then the body is said to be in equilibrium i.e. $\Sigma \vec{\tau} = 0$.
- For perfect equilibrium a body must satisfy both the following conditions.

  i.e. $\Sigma \vec{F} = 0$. and $\Sigma \vec{\tau} = 0$.

Solved Examples

Example 2.1:

A ship leaves port and travels 200 km at 30° north of east. Then it changes its o direction and travels 350 km in a direction 140° north of east to reach destination. Calculate straight line distance covered by ship?

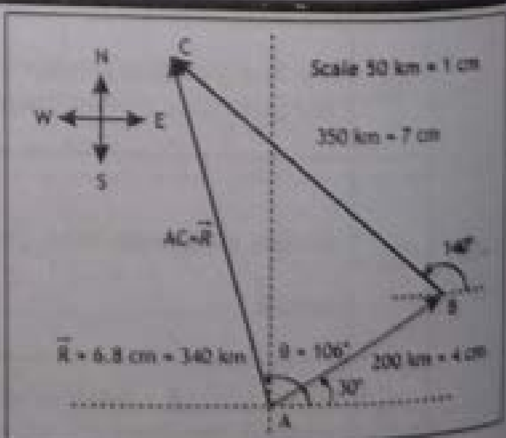
Given: Procedure

1. Selecting scale:

Let 50km = 1 cm

200 km = 4cm, $\theta = 30^\circ$ with the north of east.

350 km = 7 cm, $\theta = 140^\circ$ with the north of east.



To Find: Resultant $\vec{R}$ = ?

Solution: 2. Finding the resultant: Using head to tail rule to get the resultant $\vec{R}$. We measure the length of vector $\vec{R}$ (with scale) which was about 6.8 cm (6.8 x 50 = 340 km), and with the protector we also calculate the value of angle, which is $\theta = 106^\circ$ with east.

Answer: 34 km, 106° with north of east

36	1[st] condition of equilibrium	$\sum \vec{F} = 0$	$\sum \vec{F}_x = 0$, $\sum \vec{F}_y = 0$
37	2[nd] condition of equilibrium	$\sum \vec{\tau} = 0$	

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- If the sum of all the torques acting on a body is equal to zero then the body is said to be in equilibrium

  i.e. $\sum \vec{\tau} = 0$.
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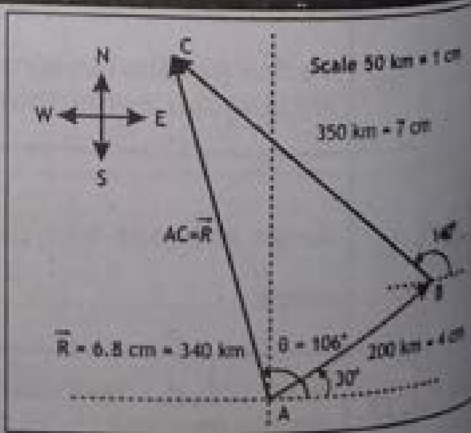


To Find: Resultant $\vec{R}$ = ?

Solution: **2.** Finding the resultant: Using head to tail rule to get the resultant $\vec{R}$. We measure the length of vector $\vec{R}$ (with scale) which was about 6.8 cm (6.8 x 50 = 340 km), and with the protector we also calculate the value of angle, which is $\theta = 106^\circ$ with east.

Answer: 34 km, 106° with north of east

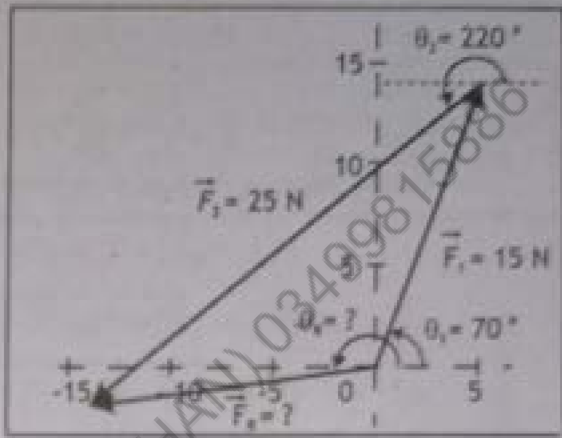
Example 2.2:

Two forces $\vec{F}_1$ = 15 N making an angle θ_1 = 70° with positive x-axis and Force $\vec{F}_2$ = 25 N making an angle θ_2 = 220° with positive x-axis, act at a point, calculate the resultant force $\vec{F}_R$.

Given: Force F1 = 15 N, angle $\theta_1 = 70°$

Force $F_2 = 25$ N, angle $\theta_2 = 220°$

To Find: Resultant Force $\vec{F}_R = ?$



Solution:

By rectangular components

$\vec{F}_R = \vec{F}_{Rx} \quad F_{Ry}$

or $\quad F_R = F_{Rx}\hat{i} + F_{Ry}\hat{j}$

or $\quad F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$ ______(1)

By addition of vectors by rectangular components

$F_{Rx} = F_{1x} + F_{2x}$ ______(2) and $F_{Ry} = F_{1y} + F_{2y}$ ______(3)

Now from the given data we can easily determine the rectangular components of each vector as shown in Figure

For vector F_1

$F_{1x} = F_1 \cos\theta_1$ $\qquad$ $F_{1y} = F_1 \sin\theta_1$

or $\quad F_{1x} = 15 \cos 70°$ $\qquad$ or $\quad F_{1y} = 15 \sin 70°$

Hence $\quad F_{1x} = 5.13$ N ________(4) $\qquad$ Hence $\quad F_{1y} = 14.09$ N ________(5)

For vector F_2

$F_{2x} = F_2 \cos\theta_2$ $\qquad$ $F_{2y} = F_2 \sin\theta_2$

or $\quad F_{2x} = 25 \cos 220°$ $\qquad$ or $\quad F_{2y} = 25 \sin 220°$

Hence $\quad F_{2x} = -19.15$ N ______(6) $\qquad$ Hence $\quad F_{2y} = -16.07$ N ________(7)

putting values of F_{1x} and F_{2x} from equation 4 and 6 in equation 2, we get F_{Rx} as

$F_{Rx} = 5.13\text{N} - 19.15\text{N}$ $\quad$ Therefore $F_{Rx} = -14.02$N ________(8)

putting values of F_{1y} and F_{2y} from equation 5 and 7 in equation 3, we get F_{Ry} as

$F_{Ry} = 14.09\text{N} - 16.07\text{ N}$ $\quad$ Therefore $F_{Ry} = -1.98$N ______(9)

putting values of F_{Rx} and F_{Ry} from equation 8 and 9 in equation 1, we get F_R as

$F_R = \sqrt{(-14.02\text{N})^2 + (-1.98\text{N})^2}$ $\qquad$ or $\qquad F_R = 14.16$ N **Answer:**

Now to determine the angle, we use $\qquad \theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}}$

$\theta = \tan^{-1}\dfrac{1.98\text{ N}}{14.02\text{ N}}$ $\qquad$ Therefore $\theta = 7.97°$

As F_{Rx} and F_{Ry} are both negative therefore the resultant lies in the third quadrant where

$\theta_R = 180° + \theta$, therefore

$\theta_R = 180° + \theta = 180° + 7.97°$

Answer: Hence $\quad \theta_R = 187.97°$

Example 2.3:

Find the angle between two forces of equal magnitude such that the magnitude of their resultant is also equal to either of them.

Solution:

Let θ be the required angle between $\vec{F_1}$ and $\vec{F_2}$. Let $\vec{F_1}$ is taken along +x-direction and $\vec{F_2}$ makes angle θ with $\vec{F_1}$. To find their resultant first we find their rectangular components as:

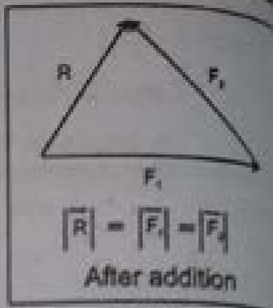


Fig: (a)

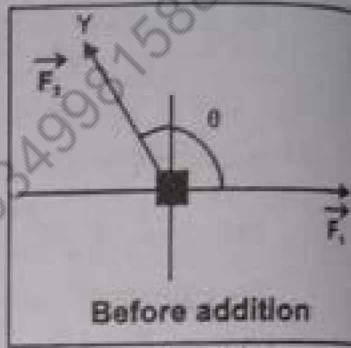


Fig: (b)

	$|\vec{F}_{1x}| = F_1 \cos 0^\circ = F_1$
and	$|\vec{F}_{1y}| = F_1 \sin 0^\circ = 0\ N$
Similarly,	$|\vec{F}_{2x}| = F_2 \cos\theta$
And	$|\vec{F}_{2y}| = F_2 \sin\theta$
Thus,	$R_x = F_{1x} + F_{2x} = (F_1 + F_2 \cos\theta)$
And	$R_y = F_{1y} + F_{2y} = 0 + F_2 \sin\theta = F_2 \sin\theta$

Hence, the magnitude of the resultant is

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

$$\Rightarrow \quad |\vec{R}| = \sqrt{F_1^2 + F_2^2\cos^2\theta + 2F_1F_2\cos\theta + F_2^2\sin^2\theta} \qquad [\cos^2\theta + \sin^2\theta = 1]$$

$$\Rightarrow \quad |\vec{R}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \qquad \text{\_\_\_\_\_\_\_\_ (1)}$$

As stated in the question $|\vec{R}| = |\vec{F_1}| = |\vec{F_2}| = F$

Therefore, Eq. (i) becomes as:

$$F = \sqrt{F^2 + F^2 + 2F^2\cos\theta} = \sqrt{2F^2 + 2F^2\cos\theta}$$

$$\Rightarrow \quad F = F\sqrt{2 + 2\cos\theta}$$

Dividing both sides by F

$$1 = \sqrt{2 + 2\cos\theta}$$

Taking square of both sides:

$$1 = 2 + 2\cos\theta \Rightarrow 2\cos\theta = -1 \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$$

Answer: $\theta = 120^\circ$

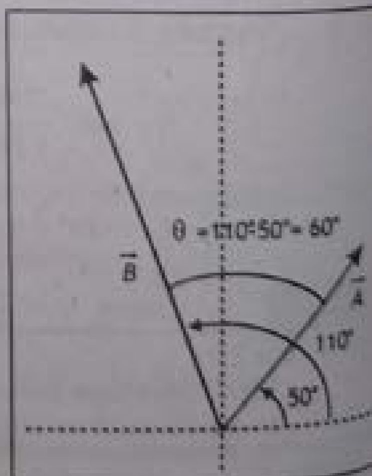
Thus the angle between the forces must be 120°.

Example 2.4:

Vector $\vec{A}$ having magnitude 3.2 makes 50° with x-axis and vector $\vec{B}$ with magnitude 5.2 make 110° with x-axis. What is the magnitude of their dot and cross products?

Given: Vector $|\vec{A}| = 3.2$, angle $\theta_1 = 50^\circ$ with x-axis

Vector $|\vec{B}| = 5.1$, angle $\theta_2 = 110^\circ$ with x-axis



Solution:

The angle θ is the smaller of the angle between two vectors as shown in Figure, we have

$$\theta = \theta_2 - \theta_1 = 110^\circ - 50^\circ = 60^\circ$$

The magnitude of dot product is

$$|\vec{A} \cdot B| = AB\cos\theta$$

Putting values

$$|\vec{A} \cdot B| = (3.2)(5.1)(\cos 60^\circ)$$

Hence $|\vec{A} \cdot B| = 8.2$ **Answer:**

The magnitude of cross product is

$$\left|\vec{A}\times B\right| = AB\sin\theta$$

Putting values $\left|\vec{A}\times B\right| = (3.2)(5.1)(\sin 60^\circ)$

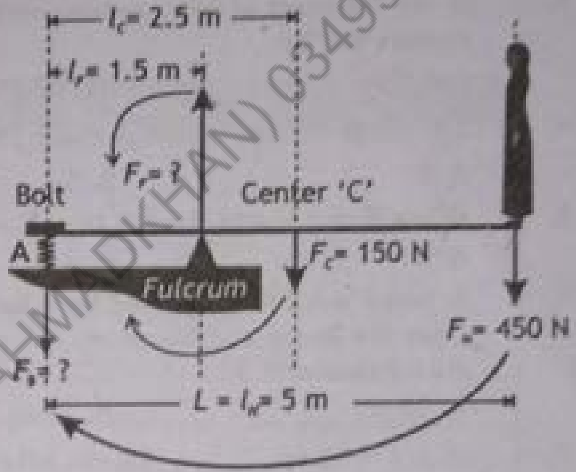
Therefore $\left|\vec{A}\times B\right| = 14.1$ **Answer:**

Example 2.5:

Hina weighing 450 N is standing at the edge of the uniform diving board 5 m in length. Weight of the board is 150 N, and is bolted down at the left end, while being supported 1.50 m away by a fulcrum, as in Figure. Find the forces that the bolt and the fulcrum, exert on the board.

Given:
Weight of Hina $F_H = 450$ N
Weight of Board $F_C = 150$ N
Center of gravity of board $l_C = 2.5$ m
Distance of fulcrum $l_F = 2.5$ m
Length of board $L = l_H = 5$ m

To Find:
(a) Resultant $\vec{R} = ?\ \Rightarrow\ F_F = ?$
(b) $F_B = ?$



Solution:

For the axis of rotation at point A, let the torque produced by support fulcrum is τ_F, the torque produced by weight of board is τ_C and the torque produced by weight of girl Hina is τ_H.

By second condition of equilibrium $\sum\tau = 0$

Therefore $\tau_F - \tau_C - \tau_H = 0$

The sign convention is adopted,

Hence $(F_F)(\ell_F) - (F_C)(\ell_C) - (F_H)(\ell_H) = 0$

or $(F_F)(\ell_F) = (F_C)(\ell_C) + (F_H)(\ell_H)$

or $F_F = \dfrac{(F_C)(l_C) + (F_H)(l_H)}{l_F}$

Putting values $F_F = \dfrac{(150)(2.5) + (450)(5)}{1.5}$

or $F_F = \dfrac{375 + 2250}{1.5}$

or $F_F = \dfrac{2625}{1.5}$

Hence $F_F = 1750$ N **Answer:**

Now the force due to bolt 'F_B' can be easily found out by solving first condition of equilibrium along y-axis. Such that $\sum F_y = 0$ or $-F_B + F_F - F_C - F_H = 0$

Or $F_B = F_F - F_C - F_H$

Putting values $F_B = 1750\text{ N} + 150\text{ N} + 450\text{ N}$

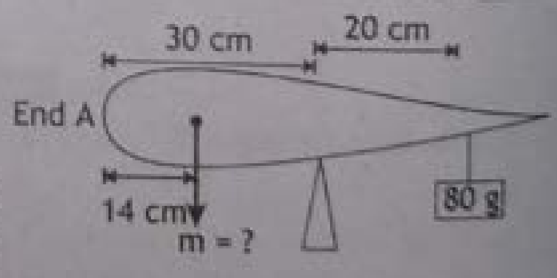
Hence $F_B = 1150$ N **Answer:**

Text Book Exercises

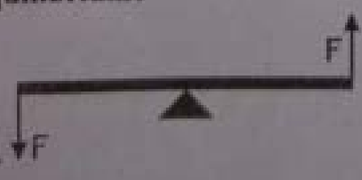
Q.1 Select the correct answer of the following questions.

Choose the best possible answer

1. **Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:**
   A. 0 B. 60° C. 70° D. 80°
2. **The minimum number of vectors of unequal magnitude required to produce a zero resultant is**
   A. 2 B. 3 C. 4 D. 5
3. **If the resultant of two vectors, each of magnitude A is also a magnitude of A. the angle between the two vectors will be:**
   A. 30° B. 45° C. 60° D. 120°
4. **The magnitude of vector $\overline{A} = 2\hat{i} + \hat{j} + 2\hat{k}$ is**
   A. 9 B. 5 C. 3 D. 1
5. **When $F_x = 3$ N and $F = 5$ N then $F_y =$**
   A. 6 N B. 4 N C. 2 N D. 0 N
6. **A meter stick is supported by a knife-edge at the 50-cm mark. Arif hangs masses of 0.40 kg and 0.60 kg from the 20-cm and 80-cm marks, respectively. Where should Arif hang a third mass of 0.30 kg to keep the stick balanced?**
   A. 20 cm B. 70 cm C. 30 cm D. 25 cm
7. **If $\overline{A}x = 1.5$ cm, $\overline{A}y = -1.0$ cm, into which quadrant do the vector $\overline{A}$ point?**
   A. I B. II C. III D. IV
8. **$\overline{A} \bullet (\overline{A} \times \overline{B}) = ?$**
   A. 0 B. 1 C. AB D. A^2B
9. **Two forces of magnitude 20 N and 50 N act simultaneously on a body. Which one of the following forces cannot be a resultant of the two forces?**
   A. 20 N B. 30 N C. 40 N D. 70 N
10. **If the dot product of two nonzero vectors A and B is zero then the magnitude of their cross product is __**
    A. 0 B. 1 C. AB D. –AB
11. **The sum of magnitudes of two forces is 16N. If the resultant force is 8N and its direction is perpendicular to minimum force then the forces are**
    A. 6N and 10N B. 8N and 8N C. 4N and 12N D. 2N and 14N
12. **Find the mass of the uneven rod shown in the figure. If its center of gravity is 14 cm from end A is ___**
    A. 100 g
    B. 150 g
    C. 80 g
    D. 5 g

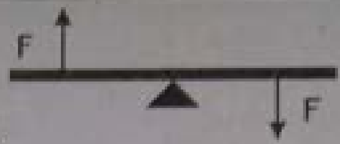


13. **The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces F are applied on the rod, as shown in the Figure. Which diagram shows the rod in equilibrium?**

A. 

B. 

C. 

D. 

14. For which angle the equation $|\vec{A}\cdot\vec{B}|=|\vec{A}\times\vec{B}|$ is correct.

A. 30° B. 45° C. 60° D. 90°

A. B. C. D.

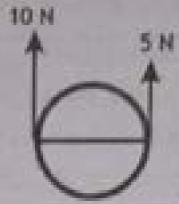
15. What is the net torque on wheel radius 2 m as shown?

A. 10 N anticlockwise

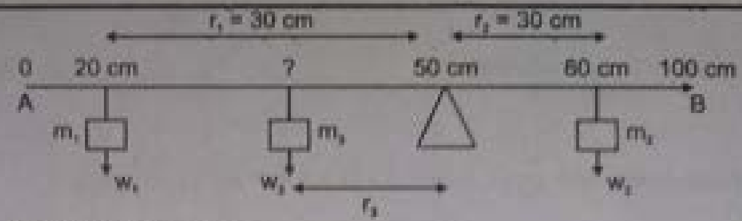
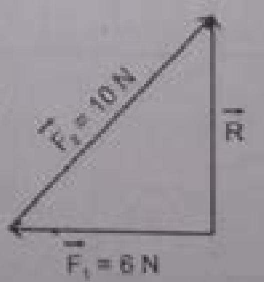
B. 10 Nm anticlockwise

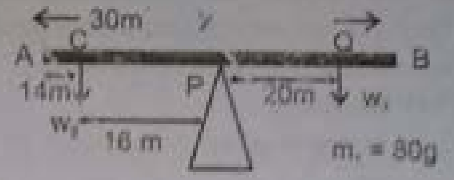
C. 10 Nm clockwise

D. 5 Nm clockwise



No.	Option	ANSWER	EXPLANATION
1.	D	80°	$\vec{A}.\vec{B} = AB\,Cos\theta$ Put $\theta = 80^o$ $\vec{A}.\vec{B} = AB\,Cos80^o = AB\,(0.174) = 0.174\,AB$ Now Put $\theta = 100^o$, we get $\vec{A}.\vec{B} = AB\,Cos100^o = AB\,(-0.174) = -0.174\,AB$ $\theta = 100^o = 80^o + 20^o$, so angle is increased by 20^o
2.	B	3	If they form a closed triangle
3.	D	120°	$R = \sqrt{A^2 + B^2 + 2AB\,Cos\theta}$ Put $R = B = A$ $R = \sqrt{A^2 + B^2 + 2AB\,Cos\theta}$ $A = \sqrt{A^2 + A^2 + 2AA\,Cos\theta}$ $A^2 = 2A^2 + 2A^2\,Cos\theta$ $A^2 = 2A^2(1 + Cos\theta)$ $1 = 2(1 + Cos\theta)$ $\frac{1}{2} = (1 + Cos\theta)$ $\frac{1}{2} - 1 = Cos\theta$ $-\frac{1}{2} = Cos\theta$ $\theta = Cos^{-1}(-\frac{1}{2})$ $\theta = 120^o$
4.	C	3	$\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ $A = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{4+1+4}$ $A = \sqrt{9} = 3$
5.	B	4N	$F_x = 3\,N$ and $F = 5\,N$ From Pathaygoras Theorem $F^2 = F_x^2 + F_y^2$ $F_y^2 = F^2 - F_x^2$ $F_y^2 = 5^2 - 3^2 = 25 - 9 = 16$ $F_y = \sqrt{16} = 4\,N$

6.	C	30cm	 Under balancing Condition: $\sum \tau = 0$ $\sum a \cdot c \cdot w \cdot \tau = \sum c \cdot w \cdot \tau$ $\Rightarrow (w_1 \times r_1) + (w_3 \times r_3) = w_2 \times r_2$ $\Rightarrow (m_1 g \times r_1) + (m_3 g \times r_3) = (m_2 g \times r_2)$ $\Rightarrow g(m_1 r_1 + m_3 r_3) = g(m_2 \times r_2)$ $\Rightarrow m_3 r_3 = m_2 r_2 - m_1 r_1$ $\Rightarrow r_3 = \frac{m_2 r_2 - m_1 r_1}{m_3}$ $\Rightarrow r_3 = \frac{(0.6)(30) - (0.4)(30)}{0.30}$ $r_3 = \frac{18 - 12}{0.3} = \frac{6}{0.3} = 20$ cm ⇒ mass m_3 is hung at distance of 20 cm from central mark of 50cm. So, its location from 1st end of stick (from o mark) is 30cm.
7.	D	IV	When x component is positive and y component is negative then vector lies in 4th Quadrant.
8.	A	0	**A** x **B** will give a product vector which is perpendicular to both vectors **A** and **B**. If **A** x **B** = **C** then Angle between **A** and **C** is 90°. So, **A**.(**A** x **B**) = **A**.**C** = AC cos 90° = AC (0) = 0
9.	A	20N	$R_{min} = 50 - 20 = 30$ N $R_{max} = 50 + 20 = 70$ N So, their resultant may have magnitude in the range from 30 N to 70 N. So, 20 N resultant magnitude is incorrect.
10.	C	AB	**A**.**B** = 0 for two non-zero vectors then these vectors are perpendicular. (θ = 90°). **A** x **B** = AB Sin90 = AB (1) = AB
11.	A	6N and 10N	$F_1 + F_2 = 16$ N $F_2 = 16 - F_1$ $R = 8$ N $F_2^2 = F_1^2 + R^2$ Putting values $(16 - F_1)^2 = F_1^2 + (8)^2$ $256 + F_1^2 - 32 F_1 = F_1^2 + 64$ $32 F_1 = 256 - 64$ $32 F_1 = 192$ $F_1 = \frac{192}{32}$ $F_1 = 6$ N $F_2 = 16 - F_1$ $F_2 = 16 - 6$ $F_2 = 10$ N 

12.	A	100g	a.c.w.τ = c.w.τ $(pc)(w_2) = (Qp)(w_1)$ $(16)(m_2g) = Qp \times m_1g$ $(16)(m_2) = Qp \times (m_1)$ $16 \times m_2 = 20 \times 80$ $m_2 = \frac{20 \times 80}{16} = 100g$ 
13.	C		Anticlockwise torque = clockwise torque $\sum \tau = 0$
14.	B	45°	$\lvert\vec{A}\cdot\vec{B}\rvert = \lvert\vec{A}\times\vec{B}\rvert$ $AB\cos\theta = AB\sin\theta$ $\frac{\sin\theta}{\cos\theta} = 1$ $\tan\theta = 1$ $\theta = \tan^{-1}(1)$ $\theta = 45^\circ$
15.	C	10Nm clockwise	c.w. torque = − r.F = −2 × 10 = −20 Nm a.w. torque = r.F = 2 × 5 = 10 Nm $\sum\tau = -20 + 10 = -10$ Nm −ve sign for clockwise torque

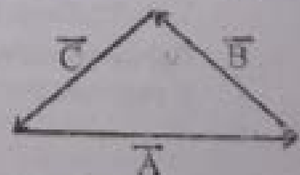
Short Answers of the Exercise

Q.2 Write short answers of the following questions.

1. Is it possible to add three vectors of equal magnitude but different directions to get a null vector? Illustrate with a diagram.

Ans: Yes, it is possible to add three vectors of equal magnitude but different directions to get a null vector if they are added to form a closed triangle.

Explanation:



Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as shown in figure. It is clear that sum of the vectors is zero because tail of the first vector *coincides* with the head of the last vector. There is no space to draw the resultant vector, so resultant will be a null vector.

2. The magnitudes of three vectors are 2 m, 3 m, and 5 m, respectively. The directions are at your disposal. Can these three vectors be added to yield zero? Illustrate with a diagram..

Fig (a)

Ans: Yes, it is possible.

Explanation:

If the three vectors are represented by the *sides of triangle* joined by head to tail rule i.e. they form a closed triangle, their vector sum will be zero vector.

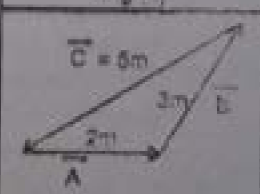


Fig (b)

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as shown in figure. It is clear that sum of the vectors is zero because tail of the first vector *coincides* with the head of the last vector. There is no space to draw the resultant vector, so resultant will be a null vector.

3. What units are associated with the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$?

Ans: Unit vectors are unitless and dimensionless vectors. So, no units are associated with unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, they are used only to give direction.

Explanation:

Unit vector is given by formula:

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

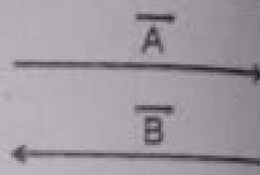
We see that unit vector is obtained by dividing a vector with its magnitude. As both vector and its magnitude have same units, so their ratio gives no unit of unit vector.

That is why, $\hat{i}$, $\hat{j}$, and $\hat{k}$ **have no units. They give direction of components of vector along axes.**

4. Can a scalar product of two vectors be negative? Provide a proof and give an example.

Ans: Yes, scalar product of two vectors can be negative.

Proof:



If two vectors are anti-parallel ($\theta = 180°$) then their scalar product is negative.

$$\vec{A}.\vec{B} = AB\cos 180°$$
$$= AB\,(-1) = -AB$$

Example:

If a body is pushed against the along the ground, then force of friction ($\vec{f}$) acts opposite to displacement ($\vec{d}$) covered by the body, then work done by the friction is:

$$W = \vec{f}.\vec{d}$$
$$= f\,d\cos\theta$$

Since, friction and displacement are opposite to each other ($\theta = 180°$)

So, $W = f\,d\cos 180 = f\,d\,(-1)$

⇨ $W = -f\,d$

5. A and B are two non zero vectors. How can their scalar product be zero? And how can their vector product be zero?

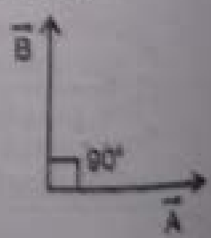
Ans: **(a) Scalar Product Case**

Yes, the scalar product of two non-zero vectors can be zero if both vectors are perpendicular to each other.

Perpendicular vectors

If two vectors are mutually perpendicular ($\theta = 90°$) to each other then, their scalar product is *zero*, i.e.

$$\vec{A}.\vec{B} = AB\cos 90°$$
$$\vec{A}.\vec{B} = AB\,(0)$$
$$\vec{A}.\vec{B} = 0$$



(b) Vector Product Case

Vector product of two vectors can not be zero because zero is a number i.e. scalar and vector product always gives vector. It can give null vector.

But magnitude of vector product can be zero if both vectors are parallel or anti-parallel.

Parallel and Anti-parallel Vectors

The cross product of two parallel ($\theta = 0°$) or two anti-parallel ($\theta = 180°$) vectors is a ***null*** vector. i.e.

- **In case of parallel vectors**

$$\vec{A}\times\vec{B} = AB\sin 0°\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero

- In case of anti – parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 180^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero

6. Suppose you are given a known non zero vector A. The scalar product of A with an unknown vector B is zero. Likewise, the vector product of A with B is zero. What can you conclude about B ?

Ans: Vector $\vec{B}$ is a null vector.

Reason:

Case 1:

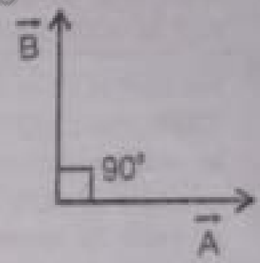
- If $\vec{B}$ is non- zero vector and it is perpendicular to $\vec{A}$ then their scalar product will be zero.

If two vectors are mutually perpendicular ($\theta = 90^\circ$) to each other then, their scalar product is *zero*. i.e.

$$\vec{A}.\vec{B} = AB\cos 90^\circ$$

$$\vec{A}.\vec{B} = AB\,(0)$$

$$\vec{A}.\vec{B} = 0$$



Case 2:

- If $\vec{B}$ is non-zero vector and it is parallel or anti-parallel to $\vec{A}$ then the magnitude of their vector product will be zero.

  ⇨ In case of parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 0^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero

  ⇨ In case of anti – parallel vectors

$$\vec{A}\times\vec{B} = AB\sin 180^\circ\,\hat{n} = AB\,(0)\,\hat{n} = 0\,\hat{n} = \vec{O}$$

So, in this case, magnitude of $\vec{A} \times \vec{B}$ is equal to zero.

⇨ From above situations, we conclude that these vector can never be perpendicular and parallel (or antiparallel) simultaneously, therefore, **vector $\vec{B}$ is a null vector**. Only then magnitudes of its scalar product and vector product with non-zero vector $\vec{A}$ can be zero.

7. Why a particle experiencing only one force cannot be in equilibrium?

Ans: Yes, A particle experiencing only one force cannot be in equilibrium.

Explanation:

A body will be in equilibrium, if net force is zero, as according to first condition of equilibrium:

$$\sum F = 0$$

But under action of single force, net force cannot be zero.

Also, body will have acceleration. Therefore, body cannot be in equilibrium under action of single force.

8. To open a door that has the handle on the right and the hinges on the left a torque must be applied. Is the torque clockwise or counterclockwise when viewed from above? Does your answer depend on whether the door opens toward or away from you?

Ans: Consider the door that has the handle on the right and the hinges on the left as shown. When we pull or push the door, it rotates due to applied torque.

When Door Is Pulled Towards US:

When we pull the handle of the door towards us to close it, the door will rotate clockwise (Moment of force will be inward by Right Hand Rule).

When Door Is Pulled Towards US:

When we push the door away from us to open it, the door will rotate anti-clockwise (Moment of force will be outward by Right Hand Rule).

Does your answer depend on whether the door opens toward or away from you?

Yes, the direction of torque will be reversed if door opens or closes opposite to the above-mentioned situation.

9. Explain the warning 'Never use a large wrench to tighten a small bolt'.

Ans: We should not use large wrench to tighten a small bolt because it may damage the nut due to application of large torque.

Explanation:

We know that torque acting on the body is:

$$\tau = r F$$

$\Rightarrow$ Torque = (Moment arm) (magnitude of force)

It is obvious from above equation that large wrench will provide greater moment arm and hence can apply greater torque on nut even with the small force. This greater torque acting on the small nut may damage it.

10. A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

Ans: No, central force e.g. Centripetal force can not apply torque on the body.

Example: Centripetal force is example of central force. Since Gravitational force between earth and satellite acts as centripetal force. Moment arm (Displacement between earth and satellite) i.e. radius of orbit makes angle of 180° with the gravitational force.

Formula for torque is:

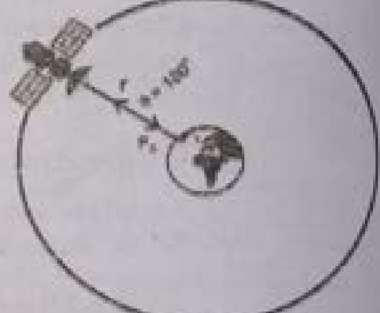
$$\tau = r F \sin \theta$$

since $\theta = 180^\circ$

So, $\tau = r F \sin 180$

$$\tau = r F (0)$$

$$\tau = 0$$



So, we conclude that central force can not apply torque on any body.

Comprehensive Questions

Q3. Give a short response to the following questions.

1. Define what is meant by vectors. Give five example of each. Also with the help of an example explain the graphical representation of a vector.

Ans: See Q # 1, 2

2. Explain in detail whether for two vectors of equal magnitude is it possible to give a resultant of magnitude equal to their individual magnitude. Justify your answer mathematically.

Ans: Yes, it is possible if angle between these vectors of equal magnitudes is 120°.

Reason: The magnitude of the resultant vector is given by:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

If $F_1 = F_2 = F$ then:

$$F = \sqrt{F^2 + F^2 + 2FF \cos\theta}$$

$$F = \sqrt{2F^2 + 2F^2 \cos\theta}$$

$$F = \sqrt{2F^2 (1 + \cos\theta)}$$

$$F^2 = 2F^2(1 + \cos\theta)$$

$$1 = 2(1 \cos\theta)$$

$$\frac{1}{2} = 1 + \cos\theta$$

$$\frac{1}{2} - 1 = \cos\theta$$

$$-\frac{1}{2} = \cos\theta$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120^\circ$$

So, two vectors of equal magnitudes will give resultant of the magnituds equal to individual added vectors.

3. What does rectangular components of a vector mean? Explain addition of vectors by rectangular components.

Ans: See Q # 9,10

4. Explain scalar product of two vectors.

Ans: See Q # 12

5. Explain vector product of two vectors.

Ans: See Q # 14

6. Define torque. Explain why it is equal to the vector product of force and moment arm.

Ans: See Q # 16

7. Write the solution steps of a numerical problem of equilibrium.

Ans:
(i) First draw vector diagram for all forces involved in problem.
(ii) Draw their rectangular components.
(iii) Apply 1[st] condition of equilibrium i.e. $\Sigma F = 0$
(iv) Then apply 2[nd] condition of equilibrium i.e. $\Sigma \tau = 0$ (if required).
(iv) then solve the obtained equation to get results.

NUMERICAL QUESTIONS

1. A person throws a ball straight up with a speed of 12 m/s. If the bus is moving at 25 m/s, what is the velocity of the ball to an observer on o ground?

Solution:

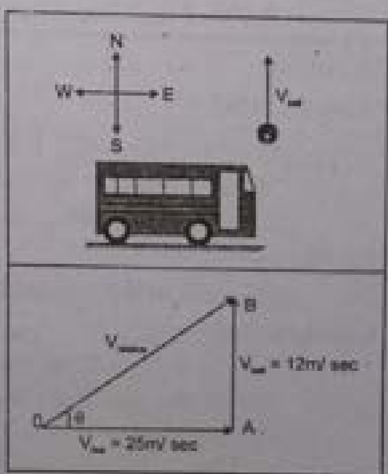
Given Data:

Velocity of ball = V_{ball} = 12m/sec
Velocity of bus = V_{bus} = 25m/sec

To Find:

Velocity of ball to an observer on ground = $V_{relative}$ = ?

Calculation:



Let the ball is thrown straight North and the bus is moving East. Here we add the velocities of ball and bus by head to tail rule as shown in figure.

Now for finding the resultant velocity, we combine the tail of first vector $\overline{OA}$ with the head of last vector $\overline{AB}$. Thus $\overline{OB}$ gives us the resultant velocity or relative vector of the ball to an observer on ground. Now

$$(OB)^2 = (OA)^2 + (AB)^2$$

$$\Rightarrow \quad V_{rel}^2 = V_{bus}^2 + V_{ball}^2 (25m/se)^2 + (12m/sec)^2$$

$$\Rightarrow \quad V_{rel}^2 = (625m^2/sec^2) + (144m^2/sec^2)$$

$\Rightarrow \quad V_{rel}^2 = 769 m^2/sec^2$

$\Rightarrow \quad V_{rel} = \sqrt{769 m^2/sec^2}$

$\Rightarrow \quad \boxed{V_{rel} = 27.7 m/sec}$

OR $\quad \boxed{V_{rel} = 28 m/sec}$

$\boxed{\text{Perpendicular} = V_{ball} \; \text{Base} = V_{bus}}$

As $\quad \theta = \tan^{-1}\left(\frac{V_{ball}}{V_{bus}}\right)$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{12}{25}\right) = 25.6°$

$\Rightarrow \quad \boxed{\theta = 25.6°}$ or $\boxed{\theta = 26°}$

Thus the relative velocity of ball will make an angle of 25.6° with horizontal [i.e. with direction of bus].

2. A football leaves the foot of a punter at an angle of 54° positive x- direction) at a speed of 21 m/s. Determine the horizontal and vertical components of the velocity.

Solution:

Given Data:

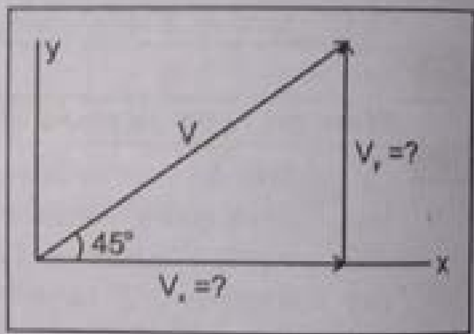
Velocity of foot-ball = V = 21m/sec

Angle of football with x-axis = θ = 54°

To Find:

(a) Horizontal component of velocity = V_x = ?

(b) Vertical component of velocity = V_y = ?



Calculation:

(a) $V_x = V\cos\theta$ ________ (1)

Putting the values in equation (1), we get,

$V_x = [21 \times \cos 54°] \; [21 \times 0.588]$

$\Rightarrow \quad \boxed{V_x = 12.4 m/sec}$ or $\boxed{V_x = 12 m/sec}$

(b) We also know that,

$V_y = V\sin\theta$ ________ (2)

Putting the values in equation (2), we get,

$V_y = (21 \times \sin 54°) = (21 \times 0.809)$

$\boxed{V_y = 16.99 m/sec}$ or $\boxed{V_y = 17 m/sec}$

3. A 1.84-kg school bag hangs in the middle of a clothesline, causing it to sag by an angle q = 3.50°. Find the tension T in the clothesline. **(148 N)**

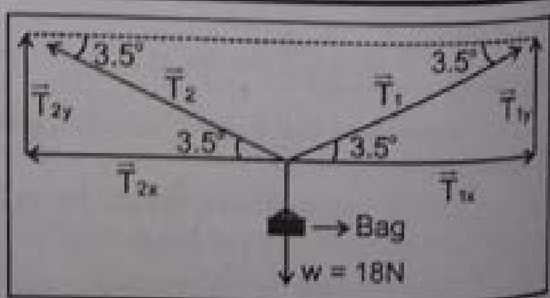
Solution:

Given Data:

Mass of bas = m = 1.84kg

Weight of bag = w = mg = 1.84 × 9.8N = 18N

Angle = θ = 3.50°



To Find:

Tension in the clothesline

Calculation:

Let $T_1 = T_2 = T$ be the tension in each part of the cloth line. Here we resolve the tension in each part into their horizontal and vertical components, as shown in the figure. As the system is in equilibrium state, so we apply the first condition of equilibrium i.e.

$\sum F_x = 0 \;\Rightarrow T_{1x} + (-T_{2x}) = 0 \;\Rightarrow T_{1x} = T_{2x}$ ________ (1)

And $\sum F_y = 0 \Rightarrow \sum T_y = 0 \Rightarrow T_{1y} + T_{2y} + (-w) = 0$

$\Rightarrow T_{1y} + T_{2y} - w = 0 \Rightarrow T_1 \sin\theta_1 + T_2 \sin\theta_2 = w$

$\Rightarrow T\sin\theta + T\sin\theta = w \Rightarrow 2T\sin\theta = w$

$$\begin{bmatrix}\theta_1 = \theta_2 = \theta \\ T_1 = T_2 = T\end{bmatrix}$$

$$\Rightarrow T = \frac{w}{2\sin\theta} = \left(\frac{18}{2\times\sin 3.5^\circ}\right)N$$

$$\Rightarrow T = \left(\frac{18}{2\times 0.061}\right)N = \left(\frac{18}{0.122}\right)N = 147.5N$$

$\boxed{T = 147.5N}$ or $\boxed{T = 148N}$

4. Find the magnitude and direction of vector represented by the following pair of components

(a) $\vec{A}_x = -2.3$ cm, $\vec{A}_y = +4.1$ cm (b) $\vec{A}_x = +3.9$ m, $\vec{A}_y = -1.8$ m

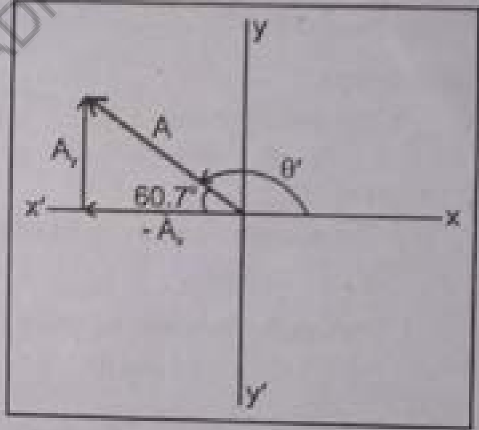
(a) $\vec{A} = 4.7$ and $\theta_A = 119.3^\circ$ (b) $\vec{A} = 4.3$ and $\theta_A = 335.2^\circ$)

Solution:

Given Data:

(a) $A_x = -2.3cm, A_y = +4.1cm$

(b) $A_x = +3.9m, A_y = -1.8m$



To Find:

Magnitude of vector in each case = A = ?

Direction of $\vec{A}$ in each case = θ' =?

Calculation:

(a) We know that the magnitude of a vector is given by,

$$A = \sqrt{A_x^2 + A_y^2} \text{ ______ (1)}$$

Putting $A_x = -2.3cm$ and $A_y = +4.1cm$ in equation (1), we get,

$$A = \sqrt{(-2.3cm)^2 + (4.1cm)^2} = \sqrt{5.29cm^2 + 16.81cm^2}$$

$$\Rightarrow A = \sqrt{(5.29 + 16.81)cm^2} = \sqrt{22.1cm^2} = 4.7cm$$

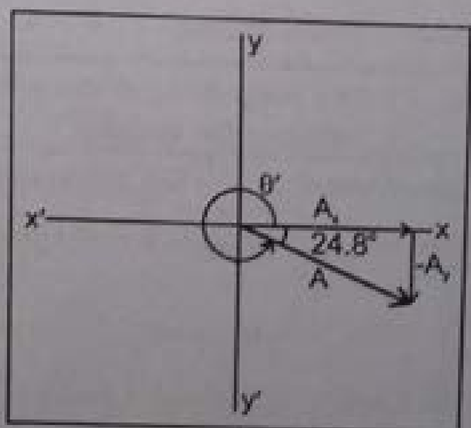
$\Rightarrow \boxed{A = 4.7 \text{ cm}}$

We also know that,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{4.1}{2.3}\right) = \tan^{-1}(1.78) \Rightarrow \theta = 60.7^\circ$$

As the x-component of $\vec{A}$ is negative and y-component is positive, so vector $\vec{A}$ lies in second quadrant. Thus the actual angle made by the vector $\vec{A}$ with positive x-axis is given by,

$\theta' = 180^\circ - \theta \Rightarrow \theta' = 180^\circ - 60.7^\circ \Rightarrow \theta' = 119.3^\circ$



(b) Now putting $A_x = +3.9m$ and $A_y = -1.8m$ in equation (1), we get,

$$A = \sqrt{(3.9m)^2 + (-1.8m)^2} = \sqrt{15.21m^2 + 3.24m^2}$$

$$\Rightarrow A = \sqrt{18.45m^2}$$

$$\Rightarrow A = 4.3m$$

Now $1 \Rightarrow \theta = 24.8^\circ$

As the x-component of $\vec{A}$ is positive and y-component is negative, so the vector $\vec{A}$ lies in fourth quadrant. Thus the actual angle made by the vector $\vec{A}$ with positive x-axis is given by,

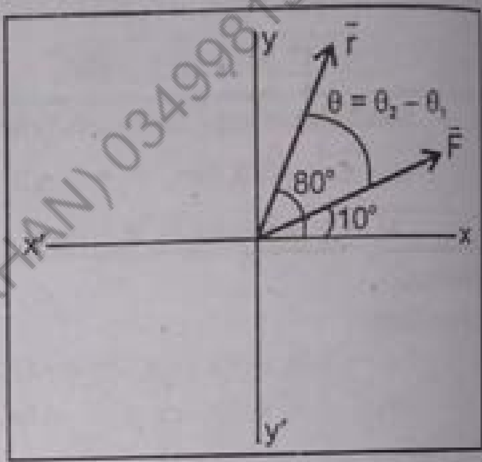
$\theta' = 360^\circ - \theta$

$\Rightarrow \theta' = 360^\circ - 24.8^\circ$

$\Rightarrow \theta' = 335.2^\circ$

5. Vector $\vec{F}$ having magnitude 5.5 N makes 10° with x-axis and vector $\vec{r}$ with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products?

Solution:



Given Data:

Magnitude of vector $\vec{F} = F = 5.5N, \theta_1 = 10^\circ$

Magnitude of vector $\vec{r} = r = 4.3m, \theta_2 = 80^\circ$

To Find:

(a) $|\vec{F}.\vec{r}| = ?$

(b) $|\vec{F} \times \vec{r}| = ?$

Calculation:

The angle between $\vec{F}$ and $\vec{r}$ is given by,

$\theta = \theta_2 - \theta_1 = 80^\circ - 10^\circ$

$\Rightarrow \theta = 70^\circ$

(a) The magnitude of dot product of $\vec{F}$ and $\vec{r}$ is given by,

$|\vec{F} \cdot \vec{r}| = Fr\cos\theta$ ________ (1)

Putting the values in equation (1), we get,

$|\vec{F} \cdot \vec{r}| = 5.5 \times 4.3 \times \cos 70^\circ$

$= 5.5N \times 4.3m \times 0.342$

$\Rightarrow$ $\boxed{|\vec{F} \cdot \vec{r}| = 8.09Nm}$ or $\boxed{|\vec{F} \cdot \vec{r}| = 8.1Nm}$

(b) The magnitude of cross product of $\vec{F}$ and $\vec{r}$ is given by,

$|\vec{F} \times \vec{r}| = Fr\sin\theta$ __________ (2)

Putting the values in equation (2), we get,

$|\vec{F} \times \vec{r}| = 5.5 \times 4.3 \times \sin 70^\circ = 5.5 \times 4.3 \times 0.939$

$\Rightarrow |\vec{F} \times \vec{r}| = 22.2Nm$

6. The magnitude of dot and cross product of two vectors $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

Given data: Let $\vec{A}$ and $\vec{B}$ be two given vectors.

Magnitude of dot product of two vectors $= |\vec{A}.\vec{B}| = 6\sqrt{3}$

Magnitude of cross product of two vectors $= |\vec{A} \times \vec{B}| = 6$

To Find: The angle between two vectors $= \theta = ?$

Calculation:

$$|\vec{A}.\vec{B}| = AB\cos\theta = 6\sqrt{3} \quad \text{----------(1)}$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta = 6 \quad \text{------------(2)}$$

Dividing eq. (2) by equation (1) we get.

$$\frac{AB\sin\theta}{AB\cos\theta} = \frac{6}{6\sqrt{3}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\boxed{\theta = 30^\circ}$$

7. A uniform rod 1 m long with weight 6 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance?

Solution:

Given Date: Length of rod = r = 1m

Weight of rod = w = 6N

Weight suspended from one end of rod = w_1 = 10N

Weight suspended from other end of rod = w_2 = 15N

Let the midpoint of the rod represents the pivot point, then r_1 = 0.5m, r_2 = 0.5m

Let the center of gravity is situated at a distance "r_3" from pivot point.

To Find:

According to the 2nd condition of Equilibrium

$\Sigma\tau = 0$

Anti clock wise torque = clock wise torque

$$AC \times W_1 + BC \times W_a = CD \times W_2$$

$$0.5 \times 10 + r_3 \times 6 = 0.5 \times 15$$

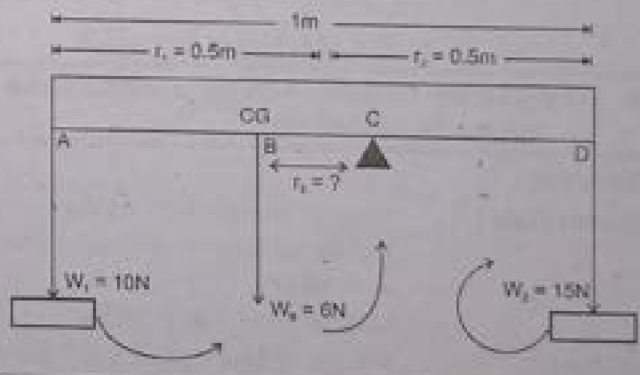
$$5 + 6r_3 = 7.5$$

$$6r_3 = 7.5 - 5$$

$$6r_3 = 2.5$$

$$r_3 = \frac{2.5}{6}$$

$$\boxed{r_3 = 0.41\ m}$$



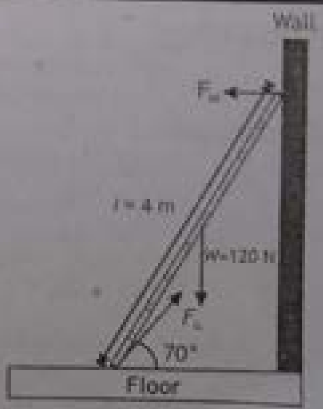
8. A 4.0-m-long uniform ladder with weight of o 120 N leans against a wall making 70° above a cement floor as shown in Figure. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

Solution:

Given Data: Length of ladder = AB = L = 4m

Weight of ladder = w = 120N

Centre of gravity from ground = $AG = \frac{L}{2}$



Required:

(a) Force of reaction on the ladder due to wall = F_{wall} = ?

(b) Force of reaction on the ladder due to ground = F =?

Calculation:

(a) According to first condition of equilibrium, we have,

$$\sum F_x = 0 \Rightarrow F_x + (-F_{wall}) = 0$$

$\Rightarrow \quad F_x - F_{wall} = 0 \qquad \Rightarrow F_x = F_{wall}$ ________ (1)

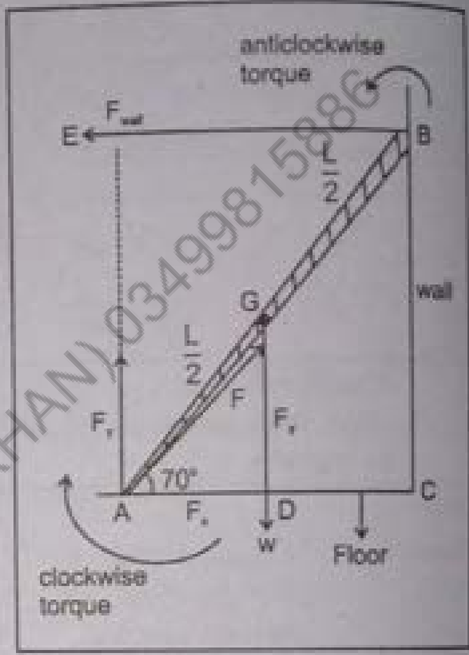
And $\quad \sum F_y = 0 \;\Rightarrow\; F_y + (-w) = 0 \;\Rightarrow F_y - w = 0$

$\Rightarrow \quad F_y - 120N = 0 \qquad \Rightarrow F_y = 150N$ ________ (2)

Now applying second condition of equilibrium, we get, Sum of anticlockwise moments = Sum of clockwise moments.

$\Rightarrow \quad AE \times F_{wall} = AD \times w$

$\Rightarrow \quad Fwall = \dfrac{AD \times w}{AE}$ ________ (3)



Now from the figure, we have, $AD = AG \cos\theta$

$\Rightarrow \quad AD = \dfrac{L}{2}\cos 70^\circ = \dfrac{4}{2}\cos 70^\circ = 2\cos 70^\circ = 2 \times 0.342 = 0.684$

$\Rightarrow \quad AD = 0.684m$

And $AE = CB = AB\sin\theta = L\sin 70^\circ = 4 \times 0.939m \Rightarrow AE = 3.756m$

Putting the values of "AD", "AE" and "w" in equation (3), we get,

$F_{wall} = \left(\dfrac{0.684 \times 120}{3.756}\right)N \qquad \Rightarrow \boxed{F_{wall} = 21.8N}$

Putting the value of "F_{wall}" in equation (1), we get,

$F_x = 21.8N$ ________ (4)

(b) Now $F = \sqrt{F_x^2 + F_y^2}$ ________ (5)

Putting equation (2) and equation (4) in equation (5), we get,

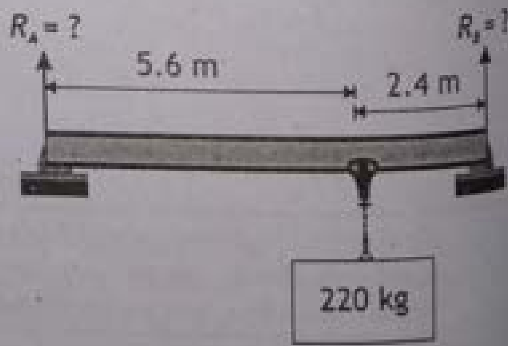
$F = \left[\sqrt{(21.8)^2 + (120)^2}\right]N$

$\Rightarrow \quad F = \left[\sqrt{475.24 + 14400}\right]N = \left[\sqrt{14875.24}\right]N$

$\Rightarrow \quad \boxed{F = 121.96N}$ or $\boxed{F = 122N}$

9. The 450-kg uniform I-beam supports the load of 220 kg as shown. Determine the reactions at the supports. (2850 N)

Solution:



Given Data:

Mass of beam = M = 450kg

Weight of beam = w = mg = 450 × 9.8N

$\Rightarrow$ w = 4410N

Mass of load = m = 220kg

Weight of load = w_1 = mg = 220 × 9.8N

$\Rightarrow$ w_1 = 2156N

Moment arm of "w" = r = 4m

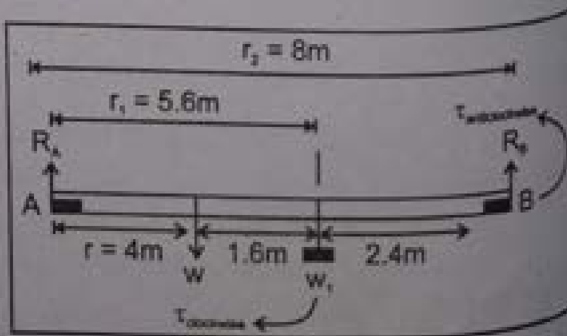
Moment arm of "w_1" = r_1 = 5.6m

Moment arm of "R_B" = r_2 = 8m

Required:

(a) Reaction at "B" = R_B =?

(b) Reaction of "A" = R_A =?



Calculation:

(a) Applying second condition of equilibrium, we get,

$\tau_{anticlock} = \tau_{clockwise}$

$\Rightarrow \quad R_B \times \ldots (w_1 \times r_1)$

$\Rightarrow \quad R_B = \dfrac{(wr) + (w_1 r_1)}{r_2}$ ________ (1)

Putting the values in equation (1), we get,

$\Rightarrow \quad R_B = \left(\frac{29713.6}{8}\right)N \Rightarrow \boxed{R_B = 3714.2N}$ or $\boxed{R_B = 3714N}$

Now applying first condition of equilibrium, we get,

$$\sum F = 0 \Rightarrow R_A + R_B + (-w) + (-w_1) = 0$$

$\Rightarrow \quad R_A + R_B - w - w_1 = 0 \qquad \Rightarrow R_A = w + w_1 - R_B$

$\Rightarrow \quad R_A = (4410 + 2156 - 3714)N = (6566 - 3714)N$

$\Rightarrow \quad R_A = 2852N$

Additional Conceptual Short Questions With Answers

1. A plane is moving at 100 km hr⁻¹ at an angle of 60° with the ground. Then what is speed of its projection (shadow) on the ground if the sun is just above the plane at noon?

Ans: It means that projection of plane is moving on ground along x-axis. So, we are to find x-component of the velocity of the plane i.e., V_x is the velocity of the projection.

Velocity of plane = V = 100 km h^{-1}

So, velocity of projection is

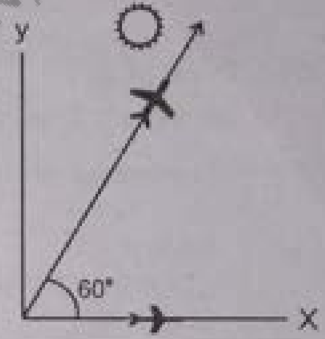
$V_x = V \cos\theta$

$\Rightarrow \quad V_x = (100)(\cos 60^\circ)$

$= (100)\left(\frac{1}{2}\right)$

$V_x = 50 \text{ km h}^{-1}$

So, velocity of projection on the ground is 50 km h^{-1}.



2. Show that $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2$

Ans:

$|\vec{A} \times \vec{B}| = AB \sin\theta$

$|\vec{A} \times \vec{B}|^2 = A^2B^2 \sin^2\theta \text{ ......(1)}$

$\vec{A} \cdot \vec{B} = AB \cos\theta$

$|\vec{A} \times \vec{B}|^2 = A^2B^2 \cos^2\theta \text{ ......(2)}$

Adding eq. (1) and (2)

$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2 \sin^2\theta + A^2B^2 \cos^2\theta$

$= A^2B^2(\sin^2\theta + \cos^2\theta)$

$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2(1)$

$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2$

3. Under what circumstances would a vector has components that are equal in magnitude?

Ans: It is possible only when the vector makes an angle of <u>45° with x-axis</u>.

Proof

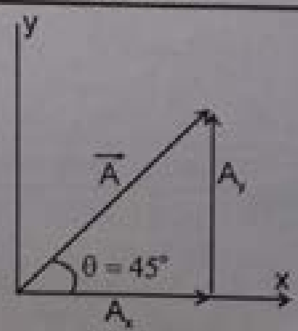
Let A_x and A_y be the magnitude of rectangular components of vector $\vec{A}$.

If $\quad A_y = A_x$

OR $\quad A \sin\theta = A \cos\theta$

OR $\quad \sin\theta = \cos\theta$

OR $\quad \frac{\sin\theta}{\cos\theta} = 1$



OR $\tan\theta = 1$

OR $\theta = \tan^{-1}(1)$

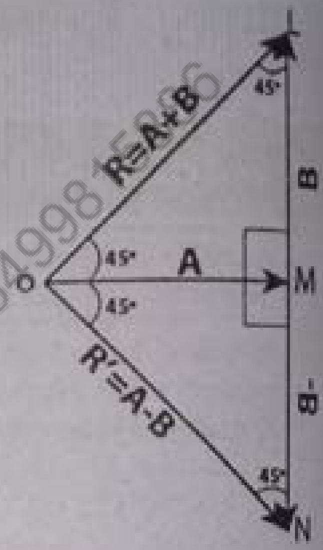
$\Rightarrow$ $\boxed{\theta = 45^\circ}$

4. Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length?

Ans: Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in figure.

By using head to tail rule,

$\vec{R} = \vec{A} + \vec{B}$ and $\vec{R}' = \vec{A} - \vec{B}$



Given Data: $A = B$

Angle between two vectors $\vec{A}$ and $\vec{B}$ is 90°.

Proof

Magnitude of $\vec{R}$

$$R = \sqrt{(A)^2 + (B)^2} = \sqrt{A^2 + B^2} \quad \text{———} (1)$$

Magnitude of $\vec{R}'$

$$R' = \sqrt{(A)^2 + (-B)^2} = \sqrt{A^2 + B^2} \quad \text{———} (2)$$

From equations (1) & (2), it is clear

$R = R'$ ——— (3)

Since $A = B$

$\angle LOM = \angle NOM = 45^\circ$

Therefore,

the angle between $\vec{R}$ and $\vec{R}' = \angle LON$

$= \angle LOM + \angle NOM = 45^\circ + 45^\circ$

$\Rightarrow \angle LON = 90^\circ$

So $\vec{R}$ and $\vec{R}'$ are perpendicular to each other.

Hence proved.

5. Name the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = \vec{O}$?

Ans: If $\vec{A}_1$ and $\vec{A}_2$ are two vectors then

$$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta \hat{n}$$

Conditions

$\vec{A}_1 \times \vec{A}_2$ is zero if

1) $\vec{A}_1$ or $\vec{A}_2$ is a ***null*** vector.	
2) $\vec{A}_1$ and $\vec{A}_2$ are ***parallel***. [i.e., $\theta = 0^\circ$]	$A_1 \times A_2 = A_1A_2 \sin 0^\circ = A_1A_2(0) = 0$
3) $\vec{A}_1$ and $\vec{A}_2$ are ***anti-parallel***. [i.e., $\theta = 180^\circ$]	$A_1 \times A_2 = A_1A_2 \sin 180^\circ = A_1A_2(0) = 0$
4) $\vec{A}_1$ and $\vec{A}_2$ are ***equal*** vectors.	

6. Can a body rotate about its center of gravity under the action of its weight?

Ans: ***No***, it is not possible.

Reason

In this case, the line of action of force (weight) passes through ***pivot point*** (center of gravity), so the moment arm becomes ***zero***.

As $\tau = \ell F$

So $\tau = (0)(F)$ (as $\ell = 0$)

OR $\tau = 0$

Hence the torque will also be zero.

7. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ then what is the angle between $\vec{A}$ and $\vec{B}$?

Ans: As $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

$$(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = (\vec{A} - \vec{B}).(\vec{A} - \vec{B})$$

$$A^2 + B^2 + 2\vec{A}.\vec{B} = A^2 + B^2 - 2\vec{A}.\vec{B}$$

$$4\vec{A}.\vec{B} = 0$$

$$\vec{A}.\vec{B} = 0$$

Since the scalar product of $\vec{A}$ and $\vec{B}$ be equal to zero so $\vec{A}$ and $\vec{B}$ are mutually perpendicular to each other.

8. If $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are perpendicular to each other, show that $\vec{A}$ and $\vec{B}$ are of same magnitude?

Ans: As $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are mutually perpendicular to each other then

$$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$$

$$\vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$$

Or $\vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{A}.\vec{B} - \vec{B}.\vec{B} = 0$

or $A^2 - B^2 = 0$

or $A - B = 0$

or $A = B$

So $\vec{A}$ and $\vec{B}$ are of equal magnitude.

9. Explain dot product of unit vectors.

Ans: $\hat{i}.\hat{j} = |\hat{i}||\hat{j}|\cos 90^\circ$

$\hat{i}.\hat{j} = (1)(1)\cos 90^\circ = (1)(1)(0) = 0$

Similarly $\hat{j}.\hat{k} = 0$ and $\hat{k}.\hat{i} = 0$

Thus $\boxed{\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0}$

$\hat{i}.\hat{i} = |\hat{i}||\hat{i}|\cos 0^\circ$

$\hat{i}.\hat{i} = (1)(1)\cos 0^\circ = (1)(1)(1) = 1$

Similarly

$\hat{j}.\hat{j} = 1$ and $\hat{k}.\hat{k} = 1$

Thus $\boxed{\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1}$

10. Explain cross product of unit vectors.

Ans: $\hat{i} \times \hat{j} = |\hat{i}||\hat{j}|\sin 90^\circ$

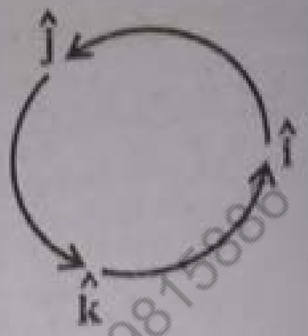
$\hat{i} \times \hat{j} = (1)(1)\sin 90^\circ\,\hat{k} = (1)(1)(1)\,\hat{k} = \hat{k}$

Similarly,

$$\hat{j}\times\hat{k}=\hat{i} \text{ and } \hat{k}\times\hat{i}=\hat{j}$$

Hence

$$\hat{i}\times\hat{j}=\hat{k}$$
$$\hat{j}\times\hat{k}=\hat{i}$$
$$\hat{k}\times\hat{i}=\hat{j}$$
$$\hat{j}\times\hat{i}=-\hat{k}$$
$$\hat{k}\times\hat{j}=-\hat{i}$$
$$\hat{i}\times\hat{k}=-\hat{j}$$
$$\hat{i}\times\hat{i}=|\hat{i}|\,|\hat{i}|\sin 0^\circ$$
$$\hat{i}\times\hat{i}=(1)(1)\sin 0^\circ\ \hat{n}=(1)(1)(0)\ \hat{n}=\vec{O}$$



Similarly

$$\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{O}$$

So,

$$\hat{i}\times\hat{i}=\vec{O}$$
$$\hat{j}\times\hat{j}=\vec{O}$$
$$\hat{k}\times\hat{k}=\vec{O}$$

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **Minimum number of forces of different magnitudes which can give zero resultant are ______**
   A. 2 B. 3 C. 4 D. 1
2. **If V = 4i − 4j then the angle of vector with positive x − axis is:**
   A. 315^0 B. 180^0 C. 90^0 D. 45^0
3. **Angle at which magnitude of dot product and cross product, are equal is:**
   A. 90^0 B. 60^0 C. 45^0 D. 30^0
4. **A force of 5N acts parallel to moment arm of 5m , then torque is**
   A. 25Nm B. 10 Nm C. 5 Nm D. zero
5. **i.(k x j) + j.(k x i) = ?**
   A. -1 B. 2 C. -2 D. 0
6. **If A = 5 i + 7 j − 3 k and B = 2 i + 2 j −a k are perpendicular vectors then which of the following is value of 'a' ?**
   A. −2 B. −7 C. 8 D. −8
7. **If $|A+B|=|A-B|$ then angle between A and B is**
   A. 30 B. 60 C. 90 D. 180
8. **Moment of force is called**
   A. Torque B. Impulse C. Force D. Angular Momentum
9. **x-component of a force of 10N acting along horizontal direction is**
   A. 5N B. 10N C. 15N D. Zero
10. **If $A_x=-1, A_y=-1$ then resultant vectors make angle**
    A. 45° B. 180° C. 90° D. 225°

11. At which angle two vectors of equal magnitude are oriented such that, their resultant is also equal to magnitude of either of the vector.

A. $60°$ B. $72°$ C. $120°$ D. $360°$

12. Two vectors will have maximum resultant when angle between them is______.

A. $0°$ B. $90°$ C. $45°$ D. $180°$

13. When angle between vectors is 45°. The magnitude between of their resultant is

A. $\sqrt{A^2+B^2}$ B. $\sqrt{A^2+B^2+\frac{1}{\sqrt{2}}AB}$ C. $\sqrt{A^2+B^2+\sqrt{2}AB}$ D. Zero

14. Two forces are acting together on an object. The magnitude of their resultant is minimum, when the angle between the forces is: (ANNUAL 2017)

A. $120°$ B. $180°$ C. $45°$ D. $60°$

15. If the Scalar product of two vectors is $2\sqrt{3}$ and the magnitude of their vector product is 2. The angle between them is: (ANNUAL 2017)

A. $30°$ B. $60°$ C. $180°$ D. $120°$

16. When two reference lines are drawn at right angles to each other, their point of intersection, is called: (ANNUAL 2017)

A. Coordinate system B. Origin C. Coordinate Axis D. Rectangular components

17. Maximum number of components of a vector may be: (ANNUAL 2017)

A. Infinite B. One C. Two D. Three

18. Counter clockwise Torque is: (ANNUAL 2018)

A. Zero B. Infinite C. Negative D. Positive

19. If $\vec{A}=a\hat{i}$ and $\vec{B}=a\hat{j}$ then: (ANNUAL 2018)

A. $\vec{A}.\vec{B}=-a$ B. $\vec{A}.\vec{B}=0$ C. $\vec{A}.\vec{B}=a$ D. $\vec{A}.\vec{B}=a^2$

20. SI unit of Torque is: (ANNUAL 2018)

A. Ns B. JC C. Js D. Nm

21. In case of unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$. Which of the following is valid? (ANNUAL 2018)

A. $\hat{j}\times\hat{i}=0$ B. $\hat{j}\times\hat{i}=1$ C. $\hat{j}\times\hat{i}=-\hat{k}$ D. $\hat{j}\times\hat{i}=\hat{k}$

22. A person walks first 10 km north and 20 km east. The magnitude e of the resultant vector is: (ANNUAL 2019)

A. 22.36 km B. 22.46 km C. 25.23 km D. 20.36 km

23. For which angle the equation, $|\vec{A}.\vec{B}|=|\vec{A}\times\vec{B}|$ is correct: (ANNUAL 2019)

A. $45°$ B. $60°$ C. $90°$ D. $0°$

24. If the scalar product $\vec{A}.\vec{B}=0$, then which of the following is NOT correct? (ANNUAL 2019)

A. $|\vec{A}|=0$, $|\vec{B}|\neq 0$, $\theta\neq 0°$, $\cos\theta\neq 0°$

B. $|\vec{A}|\neq 0$, $|\vec{B}|=0$, $\theta\neq 0°$, $\cos\theta\neq 0°$

C. $|\vec{A}|\neq 0$, $|\vec{B}|\neq 0$, $\theta=0°$, $\cos\theta\neq 0°$

D. $|\vec{A}|\neq 0$, $|\vec{B}|\neq 0$, $\theta\neq 0°$, $\cos\theta=0°$

Answers Key

1.	B	2.	A	3.	C	4.	D	5.	B
6.	D	7.	C	8.	A	9.	B	10.	D
11.	C	12.	A	13.	C	14.	B	15.	A
16.	B	17.	A	18.	D	19.	B	20.	D
21.	C	22.	A	23.	A	24.	C		

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 Choose the correct answer from the given options. (1 x 6 = 6)

SECTION – A

1. **If a vector of magnitude 10 N is along y-axis, then which of the following is its component along x–axis?**
   (A) 0 N (B) 5 N (C) 8.66 N (D) 10 N
2. **The magnitude of resultant of two vectors of 6 N and 8 N at right angle is:**
   (A) 6N (B) IN (C) 14 N (D) 10 N
3. **If the magnitude of $\vec{A} \cdot \vec{B} = \frac{1}{2} AB$ then an angle between $\vec{A}$ and $\vec{B}$ is:**
   (A) 30° (B) 45° (C) 60° (D) 90°
4. **If the body is at rest or rotating with uniform angular velocity, then torque will be:**
   (A) Maximum (B) Negative (C) Zero (D) Positive
5. **A meter stick is supported by a knife-edge at the 50-cm mark. Arif hangs masses of 0.40 kg and 0.60 kg from the 20 cm and 80 cm marks, respectively. Where should Arif hang a third mass of 0.30 kg to keep the stick balanced?**
   (A) 20 cm (B) 70 cm (C) 30 cm (D) 25 cm
6. **For which angle the equation $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ is correct:**
   (A) 30° (B) 45° (C) 60° (D) 90°

Question.No.2 Give short answers of followings: (3 x 7 = 21)

(i) Is it possible to add three vectors of equal magnitude but different directions to get a null vector? Illustrate with a diagram.

(ii) A and B are two nonzero vectors. How can their scalar product be zero? And how can their vector product be zero?

(iii) A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

(iv) A uniform rod 1 m long with weight 6 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance?

(v) Show that $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = A^2B^2$

(vi) State the two conditions of equilibrium. Name the type of equilibrium that is guaranteed by each condition of equilibrium?

(vii) Write down steps to add vectors by rectangular components vectors.

Question.No.3 Extensive Questions. (13)

SECTION – C

(a) Define Scalar product. Give two examples. Write down its four properties. (07)

(b) Define moment of force. On what factors it depends. Show that moment of force acting on the rigid body is: $\vec{\tau} = \vec{r} \times \vec{F}$ (06)

The End

CHAPTER 3

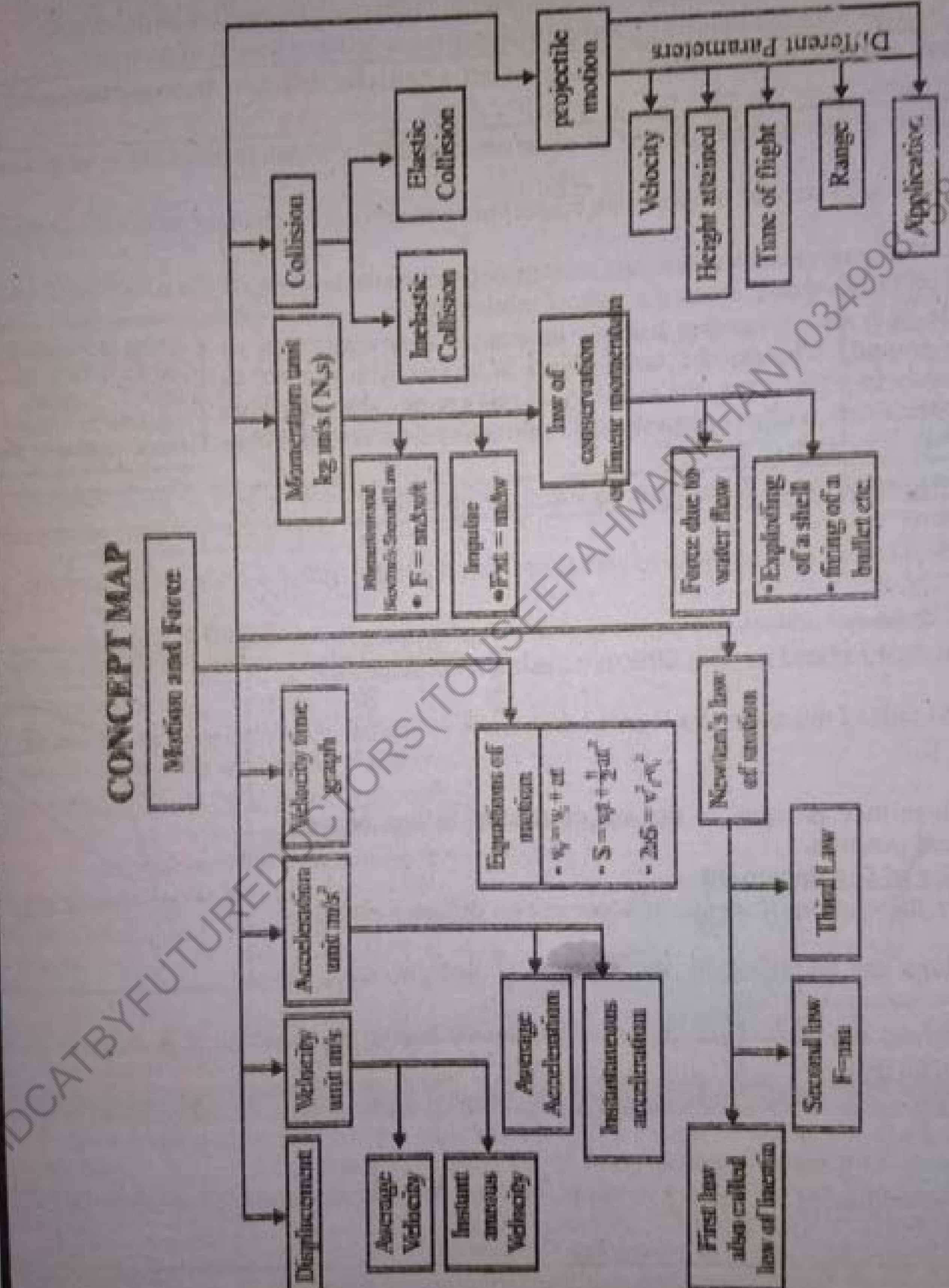
MOTION & FORCE

Learning Objectives

- Describe vector nature of displacement.
- Describe average and instantaneous velocities of objects.
- Compare average and instantaneous speeds with average and instantaneous velocities.
- Interpret displacement-time and velocity-time graphs of objects moving along the same straight line.
- Determine the instantaneous velocity of an object moving along the same straight line by measuring the slope of displacement-time graph.
- Define average acceleration (as rate of change of velocity $a_{av} = \Delta v/\Delta t$) and instantaneous acceleration (as the limiting value of average acceleration when time interval Δt approaches zero).
- Distinguish between positive and negative acceleration, uniform and variable acceleration.
- Determine the instantaneous acceleration of an object measuring the slope of velocity-time graph.
- Manipulate equation of uniformly accelerated motion to solve problems.
- Explain that projectile motion is two dimensional motion in a vertical plane.
- Communicate the ideas of a projectile in the absence of air resistance that:
  - (i) Horizontal component of velocity is constant.
  - (ii) Acceleration is in the vertical direction and is the same as that of a vertically freely falling object.
  - (iii) The horizontal motion and vertical motion are independent of each other.
- Evaluate using equations of uniformly accelerated motion that for a given initial velocity of frictionless projectile:
  - (i) How higher does it go?
  - (ii) How far would it go along the level land?
  - (iii) Where would it be after a given time?
  - (iv) How long will it remain in air?
- Determine for a projectile launched from ground height.
  - (i) Launch angle that results in the maximum range.
  - (ii) Relation between the launch angles that result in the same range.

- Describe how air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile.
- Apply Newton's Laws to explain the motion of objects in a variety of context.
- Define mass (as the property of a body which resists change in motion).
- Describe the Newton's second Law of motion as rate of change of momentum.
- Co-relate Newton's third Law of motion and conservation of momentum.
- Show awareness that Newton's Laws are not exact but provide a good approximation, unless an object is moving close to the speed of light or is small enough that quantum effects become significant.
- Define impulse (as a product of impulsive force and time).
- Describe the effect of an impulsive force on the momentum of an object, and the effect of lengthening the time, stopping, or rebounding from the collision.
- Describe that while momentum of a system is always conserved in interaction between bodies some change in K.E. usually takes place.
- Solve different problems of elastic and inelastic collisions between to bodies in one dimension by using law of conservation of momentum.
- Describe that momentum is conserved in all situations.
- Identify that for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation.
- Differentiate between explosion and collision.

Chapter No. 3


CONCEPT MAP
Motion and Force
Displacement
Velocity unit m/s
Acceleration unit m/s²
Velocity time graph
Momentum unit kg m/s (N.s)
Collision
projectile motion
Average Velocity
Instantaneous Velocity
Average Acceleration
Instantaneous acceleration
Equations of motion
Newton's law of motion
Inelastic Collision
Elastic Collision
Impulse
law of conservation of linear momentum
Force due to water flow
Exploding of a shell
firing of a bullet etc.
Different Parameters
Velocity
Height attained
Time of flight
Range
Application
First law also called law of Inertia
Second law F=ma
Third Law

Q.1 What is mechanics? What are its branches?

Ans: **Mechanics:**

It is the branch of physics in which we study about motion of bodies, causes and effects of motion.

Types of Mechanics

i. **Kinematics:** It deals with the motion of bodies without reference to force (cause of motion)

ii. **Dynamics:** It deals with the motion of bodies with reference to force (cause of motion).

Q.2 Define Rest and Motion. With the help of an example, explain that motion and rest are relative?

Ans: **Rest:** A body is at rest with respect to an observer if it does not change its position with respect to an observer.

Motion: A body is in state of motion with respect to an observer if it changes its position with respect to that observer.

Rest and motion are relative. Rest and motion depend upon the state of the observer. Two observers can disagree on observations about the state of motion or rest.

For example, a body in moving train is in motion with respect to an observer on ground. Whereas the same object is at rest with respect to another observer in train. Thus, the motion and rest are not absolute. This means that specification of the observer is important while inferring about the state of rest or motion of the body

> **Point to Ponder**
> When sitting on a chair, your speed is zero relative to Earth but 30 km/s relative to the Sun.

Q.3 What is displacement? Describe its vector nature?

Ans: **Displacement:**

The change in position of body from its initial position to final position is called displacement.

OR

The shortest distance between two points is called displacement.

It tells us that *how far* and in what *direction* the body has displaced.

Unit

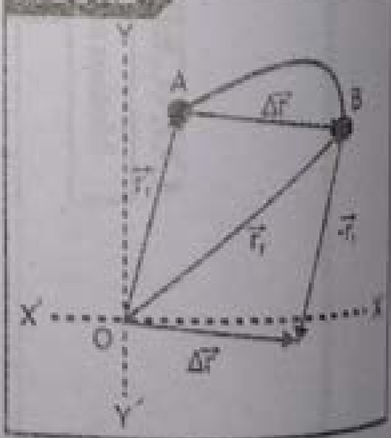
The SI unit of displacement is *meter* (m).

Dimension: [L]

Magnitude

Its magnitude is equal to the length of straight line between initial and final position.

FIGURE



Vector Nature of Displacement

Figure shows the motion of an object between two different positions 'A' and 'B'.

- These positions are identified by the vectors 'r_i' and 'r_f' w.r.t. an arbitrary coordinate origin 'O'.
- The displacement $\vec{d}$ or $\vec{\Delta r}$ of the object is the vector drawn from the initial position A to the final position B. Such that

$$\vec{d} = \vec{\Delta r} = \vec{r_f} - \vec{r_i}$$

Note

- Distance is the actual length covered by the body. It is scalar.
- When a body moves along a *straight line in a specific direction*, the displacement is equal to distance.
- Otherwise magnitude of displacement < distance

Q.4 What is velocity? Define average and instantaneous velocity.

Ans: **Velocity**

The time rate of change of displacement of a body is called velocity.

Its direction is along the direction of change in displacement of body.

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

Unit and Direction

(i) SI unit of velocity is m/s (i.e. ms^{-1}).

(ii) Its dimensions are $[L\,T^{-1}]$.

(iii) It is a vector quantity and its direction is along the direction of change of displacement $\Delta\vec{d}$.

Average Velocity

The ratio of the total displacement covered by body to the total time taken to cover this displacement is called as average velocity.

If $\Delta \vec{d}$ is the total displacement covered by body in time Δt, then average velocity is

$$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$$

Instantaneous Velocity

The velocity of the body at any instant is called instantaneous velocity.

OR

The limiting value of $\frac{\Delta \vec{d}}{\Delta t}$, as time interval Δt following the time t, approaches to zero is called instantaneous velocity.

Mathematically

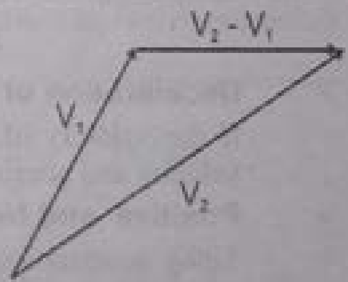
$$\vec{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

> **Remember:**
> 1. Average speed is always positive.
> 2. The average velocity of an object in one dimension can be either positive or negative (For example, if change in displacement towards right is positive then velocity is also taken positive and vice versa).

Q.5 Define uniform and variable velocity.

Ans: **Uniform velocity**

If the body covers equal displacements in equal intervals of time, the body is said to be moving with uniform velocity.



Non-uniform velocity (Variable Velocity)

If the body covers unequal displacements in equal intervals of time, the body is said to be moving with non-uniform velocity.

Condition for uniform velocity

If the average and instantaneous velocities of a body are ***equal,*** the body is moving with uniform velocity.

OR

If the instantaneous velocity ***does not change***, the body is said to be moving with uniform velocity.

Q.6 Differentiate between speed and velocity.

Speed	Velocity
(1) The distance covered by an object in unit time, is called speed.	(1) Rate of change in displacement of a body is called velocity.
(2) Speed = $\frac{\text{distance covered}}{\text{time taken}}$	(2) Velocity = $\frac{\text{displacement Covered}}{\text{time taken}}$
(3) Its symbol is V.	(3) Its symbol is $\vec{V}$.
(4) $V = \frac{S}{t}$	(4) $\vec{V} = \frac{\vec{d}}{t}$

(5) Its SI unit is metre per second (ms^{-1})	(5) Its SI units is metre per second (ms^{-1})
(6) It is scalar.	(6) It is vector.
(7) Average Speed is always positive.	(7) Average Velocity can be positive or negative.

.7 What is acceleration? Explain its different terms

Acceleration

The time rate of change in velocity of a body is called acceleration.

Unit

SI unit of acceleration is m/s^2. The dimensions of acceleration are $[LT^{-2}]$

Direction

Acceleration is also a ***vector*** quantity and the direction of acceleration is along the direction of ***change in velocity.***

Acceleration is a measure of how rapidly the velocity is changing.

➢ **Average Acceleration**

The ratio of the total change in velocity to the total time taken, is called average acceleration.

$$\overrightarrow{a_{av}} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{\Delta t}$$

➢ **Instantaneous Acceleration**

Acceleration of the body at any instant of time is called instantaneous acceleration.

OR

The limiting value of $\frac{\Delta \vec{v}}{\Delta t}$ as the time interval Δt, following the time t, approaches to zero is called instantaneous acceleration.

Mathematically

$$\overrightarrow{a_{ins}} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t}$$

➢ **Deceleration or Retardation:**

If the velocity of body is decreasing then it has retardation or deceleration. Under this condition, velocity and acceleration are in opposite direction i.e. ($\vec{v}$ ***anti*** // $\vec{a}$).

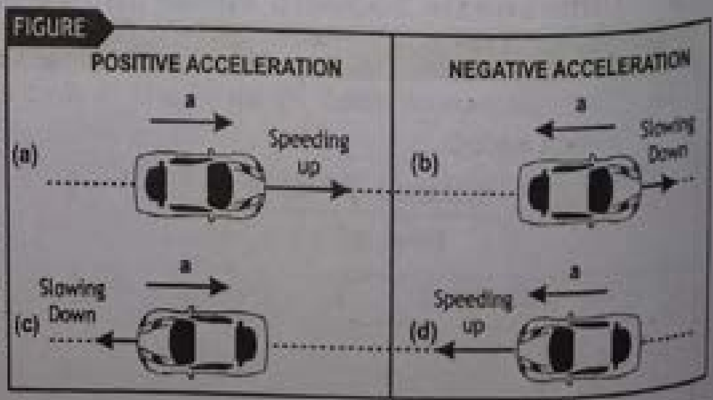
➢ **Positive and Negative Accelerations**

Since acceleration is a vector quantity, it has a direction associated with it. The direction of the acceleration vector depends on two things:

- whether the object is speeding up (velocity increasing) or slowing down (velocity decreasing)
- whether the object is moving in the + or – direction as defined for displacement.
(e.g towards right or along positive x-axis, velocity is taken positive and towards left or along negative x-axis, velocity is taken negative)

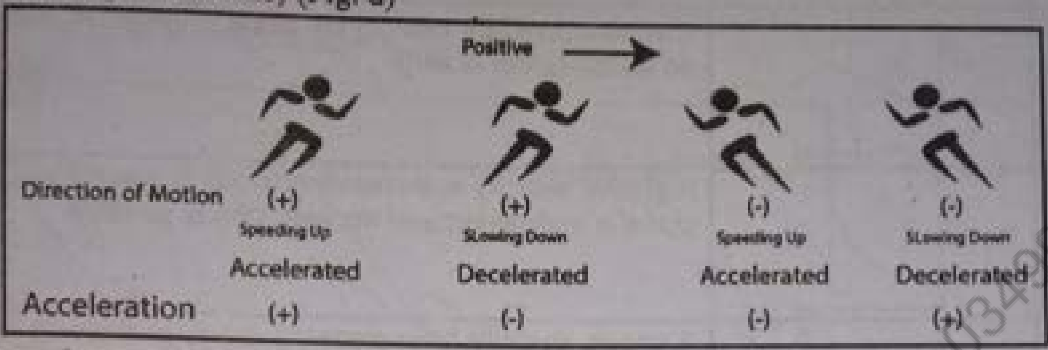
Case 1: If the car is moving towards right (along positive x-axis) with increasing velocity, it has accelerated motion, then it has positive acceleration towards right (towards positive direction). (Fig. a)

Case II: If the car is moving towards right (along positive x– axis) with decreasing velocity (slowing down), it has retarded motion, then it has negative acceleration towards left (along negative x-axis). (Fig. b)



Case III: If the car is moving towards left (along negative x – axis) with decreasing velocity (slowing down), it has retarded motion, then it has positive acceleration opposite to motion towards right (along positive x-axis) (Fig. c)

Case IV: If the car is moving towards left (along negative x – axis) with increasing velocity (speeding up), it has accelerated motion, then it has negative acceleration in the direction of motion towards left (along negative x-axis) (Fig. d)



Uniform Acceleration:

If velocity of the body changes equally in equal intervals of time then it has uniform acceleration.

Variable Acceleration:

If velocity of the body does not change equally in equal intervals of time then it has variable acceleration.

Condition for uniform acceleration

For a body moving with uniform acceleration, its average and instantaneous accelerations are ***equal.***

Q.8 (a) Discuss the displacement – time graph.

(b) Consider a car moving back and forth along the straight line as shown in figure and also consider data of its position of the car every 10 s, as shown in table, Draw its displacement-time. Discuss its motion on different parts of the graph.

(c) What should you do to calculate the instantaneous velocity of the car (as mentioned above)

Ans: **Displacement - Time Graph**

It is the graph plotted between displacement of the body and time taken by it during its journey.

Displacement of the body is taken along y-axis and time is taken along x-axis.

Slope of a graph:

Slope at a point is the "ratio of change in value on y-axis to the change in value on x-axis between two points on the tangent line at that point."

$$\text{Slope} = \frac{\text{change in value on Y}-\text{axis}}{\text{change in value on X}-\text{axis}}$$

Slope or gradient of Displacement – time graph

$$\text{Slope of displacement} - \text{time graph} = \frac{\Delta d}{\Delta t} = \text{Velocity}$$

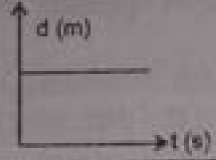
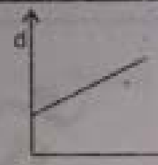
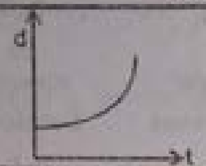
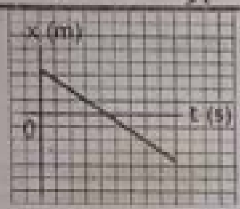
Slope or gradient of Displacement – time graph

$$\text{Slope of distance} - \text{time graph} = \frac{\Delta S}{\Delta t} = \text{Speed}$$

Difference between Displacement – Time and Distance – Time Graphs

- As displacement can be positive and negative, so, slope of displacement time graph may be negative which indicates reverse journey,
- But distance is always positive, so slope of distance - time graph can never be negative, which mean distance is always positive and increases during motion, no matter whichever is the direction of motion.

Graph Between Displacement and Time

(1)		The graph is a straight line parallel to the time axis i.e., displacement (d) does not change with time. Hence the body is at rest. Velocity is zero. Also a = 0.
(2)		It shows constant velocity as slope of graph is constant, so acceleration is zero.
(3)		It shows velocity is increasing (body is speeding up) as slope is increasing and acceleration is positive.
(4)		It shows velocity is decreasing (body is slowing down) as slope decreases and acceleration is negative.
(5)		Displacement is decreasing linearly with time. Slope is constant i.e. uniform velocity. Graph is extending in negative direction, object has not only approached the reference or starting point but also crossed it.

(b) Motion of Car Moving Forward and Backward on A Straight Road

Let us draw the graph using above data.

FIGURE 3.4

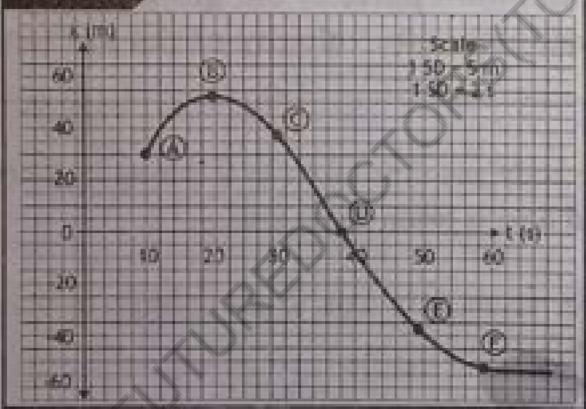
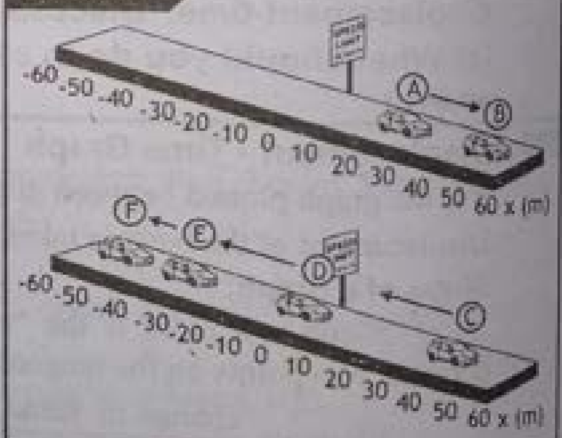


FIGURE 3.3



Let us motion of the car on different parts of the graph

i. **From A to B,** it has positive velocity as displacement is positive. Its average velocity is calculated as:

$$v = \frac{x_B - x_A}{t_B - t_A} = \frac{52 - 30}{20 - 10} = \frac{22}{10} = 2.2\ m/s$$

ii. **At point B,** car comes to rest for a moment. If we draw tangent at point B, it is parallel to time axis, so its velocity is zero at point B.

iii. After B it has reversed its direction and is heading back towards the reference 'O'.

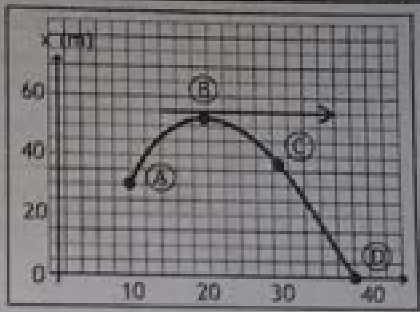
iv. **Between B and C,** the average velocity is:

$$v = \frac{x_C - x_B}{t_C - t_B} = \frac{38 - 52}{30 - 20} = -\frac{14}{10} = -1.4\ m/s$$

Negative sign indicates the reverse direction.

Table 3.2: POSITION OF CAR VARIOUS TIMES

Position	Time (s)	Displacement (m)
A	10	30
B	20	52
C	30	38
D	40	0
E	50	-37
F	60	-53

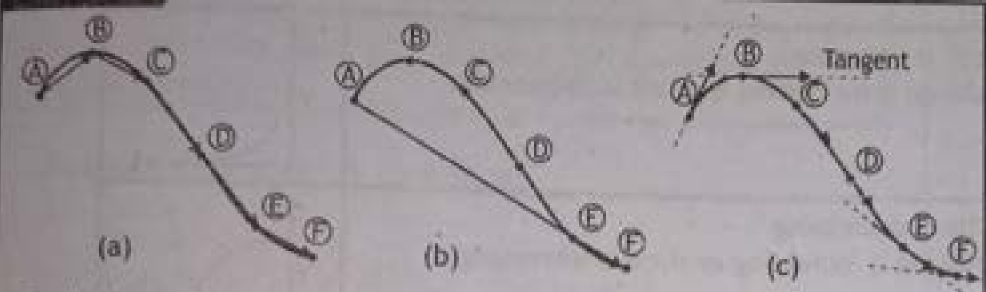


(c) **Calculation of instantaneous velocity**

Why do we need instantaneous velocity?

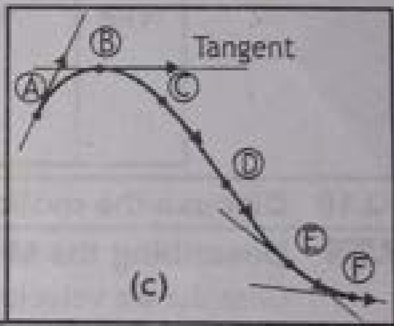
Calculating the average velocity of the car over relatively long-time intervals will not give us the complete description of motion as shown in Figure 3.5 (b), since the car was not moving all the way through with this speed. To describe the motion exactly, we need to know the car velocity at every instant of time.

FIGURE 3.5



The displacement time graph car through points A, B, C, D, E and F is shown in the figure. (a) The average velocity in shorter interval of time is dissimilar both in magnitude and direction at different points. (b) The average velocity over longer interval of time remain same at all points. (c) The instantaneous velocity is tangent to the curved path.

How to calculate instantaneous velocity?



The instantaneous velocity is obtained by making the time intervals shorter (mathematically we say that the limit in which time approach to zero) in displacement-time graph.

This gives us a series of shorter straight-line segments which have the same direction as the tangent to the curve, as shown in Figure.

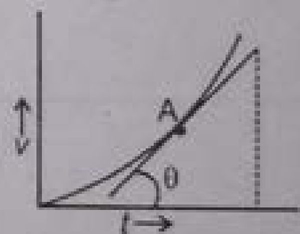
Q.9 Discuss the Velocity – time graph to discuss motion of the body.

Ans: Velocity - Time Graph

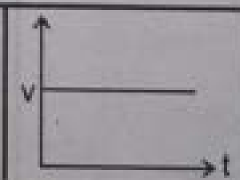
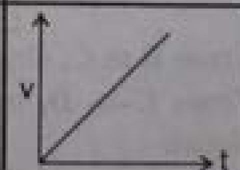
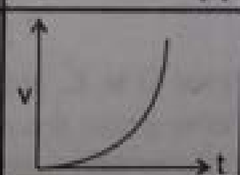
"The graph which represents the variation of velocity with time is called velocity-time graph."

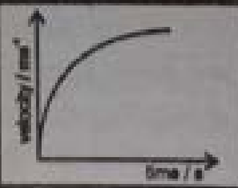
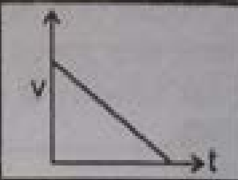
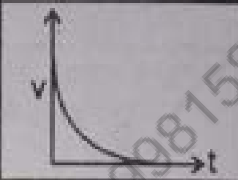
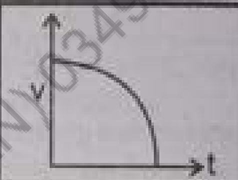
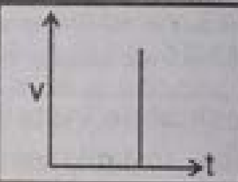
Slope or gradient of Velocity – time graph

$$\text{Slope of velocity-time graph} = \frac{\Delta v}{\Delta t} = \text{Acceleration}$$



- Slope of Velocity – Time graph is equal to acceleration.
- Instantaneous acceleration can be calculated by finding the slope of a tangent to the v-t graph at that required point on the curve, as shown in Fig.
- Area under velocity time graph gives distance covered by the body.

(i)	**A.** Uniform velocity **B.** acceleration is zero	
(ii)	**A.** Velocity is increasing **B.** acceleration is positive **C.** Acceleration is also uniform as slope of velocity time graph is constant.	
(iii)	**A.** Velocity is increasing **B.** Acceleration is positive **C.** Also it is variable acceleration. **D.** Its acceleration is increasing as slope is increasing.	

(iv)	A. Velocity is increasing B. Acceleration is positive C. Also it has variable acceleration. D. Acceleration is decreasing as slope is decreasing.	
(v)	A. Velocity is decreasing B. Uniform deceleration	
(vi)	A. Velocity is decreasing B. Retardation is decreasing as slope is decreasing.	
(vii)	A. Velocity is decreasing B. Retardation is increasing as slope is increasing.	
(viii)	Infinite acceleration	

Q.10 Discuss the motion of a body as described by the velocity time graph

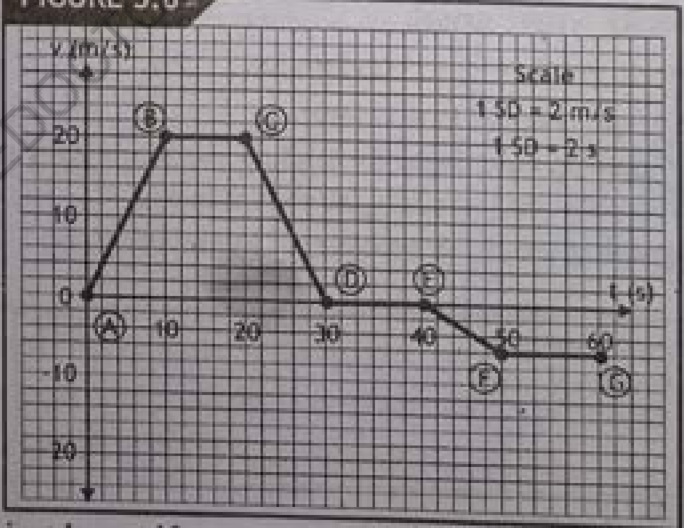
Ans: **Describing the Motion Using Velocity – Time Graph**

Consider the velocity time graph of an object as shown.

i. **From A to B,** its velocity is increasing and it has uniform acceleration (as its slope is constant)

$$a = \frac{v_B - v_A}{t_B - t_A} = \frac{20-0}{10-0} = \frac{20}{10} = 2\ m/s^2$$

FIGURE 3.6



ii. **From B to C,** object has unifrom velocity i.e. zero acceleration.

iii. **From C to D,** its velocity is decreasing, it has uniform retardation (it has constant negative slope).

$$a = \frac{v_C - v_B}{t_C - t_B} = \frac{0-20}{10-0} = -\frac{20}{10} = -2\ m/s^2$$

iv. **From D to E,** object is in rest as velocity of the object is zero here.

v. **From point E to F,** the object accelerates in the opposite direction (opposite to initial direction of its motion from A to C), its acceleration in this part is:

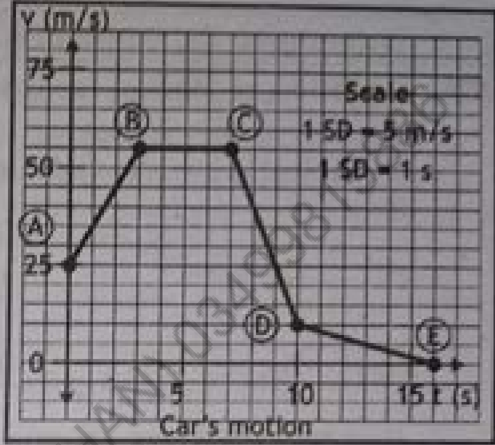
$$a = \frac{v_F - v_E}{t_F - t_E} = \frac{-6-0}{10-0} = -\frac{6}{10} = -0.6\ m/s^2$$

Object is accelerating still but its acceleration is negative. This negative sign with acceleation shows that object has reversed its direction of its motion.

vi. **From F to G,** object has constant speed but in opposite direction (as velocity is negative).

Assignment 3.1:

The Velocity time graph shows the motion of car in a straight line. By reading the scale carefully, calculate (a) the acceleration of the car between segment A and B, B and C, C and D and D and E, from the slope of the graph. Also (b) Calculate the car's average acceleration for the complete journey.



Solution:

i. **Acceleration of car from A to B**

$$a = \frac{v_B - v_A}{t_B - t_A}$$

⇨ $a = \text{slope of line AB} = \frac{55-25}{3-0} = \frac{30}{3} = 10\ ms^{-2}$

ii. **Acceleration of car from B to C is zero,** because it has uniform velocity of 55 m/s.

$$a = \text{Slope of line BC} = \frac{55-55}{7-4} = \frac{0}{3} = 0$$

iii. **Acceleration of car from C to D**

$$a = \text{Slope of line CD} = \frac{v_D - v_C}{t_D - t_C}$$

⇨ $a = \frac{10-55}{10-7} = -\frac{45}{3} = -15\ ms^{-2}$

Car has retardation in this part as it has negative value of acceleration, it is slowing down.

iv. **Acceleration from D to E**

$$a = \frac{v_E - v_D}{t_E - t_D}$$

⇨ $a = \frac{0-10}{16-10} = -\frac{10}{6} = -1.67\ ms^{-2}$

v. **Average acceleration of whole motion is equal to slope of line from A to E**

$$a = \frac{v_E - v_A}{t_E - t_A} = \frac{0-25}{16-0} = -\frac{25}{16} = -1.56\ ms^{-2}$$

MCQ's

1. **The slope of velocity time graph shows the:**
   (A) Total distance covered by the body (B) Average acceleration of body
   (C) Average force by acting on body (D) Total work done on the body
2. **When a body moves with constant acceleration, the velocity time graph is:**
   (A) Parabola (B) Hyperbola (C) Inclined straight line (D) Curve
3. **The area under the velocity time graph is:**
   (A) Force (B) Acceleration (C) Distance (D) Torque
4. **The average acceleration of the car during the interval of time is given by the slope of its:**
   (A) Displacement time graph (B) Force time graph (C) Velocity time graph (D) Acceleration time graph
5. **The slope of velocity time graph at any instant represents:**
   (A) Instantaneous velocity (B) Instantaneous acceleration (C) Power (D) Force
6. **An ant walk with speed:**
   (A) 0.1 ms^{-1} (B) 1.0 ms^{-1} (C) 0.01 ms^{-1} (D) 100 ms^{-1}
7. **When a body is moving with uniform velocity, its?**
   (A) Speed changes (B) Direction of motion change
   (C) Displacement from origin changes (D) Acceleration changes
8. **If the slope of the Velocity Time Graph remains constant then body is moving with:**
   (A) Uniform Velocity (B) Negative Variable Acceleration (C) Variable Acceleration (D) Uniform Acceleration

9. **When average velocity becomes equal to instantaneous velocity then the body is called moving with ______.**
(A) Instantaneous acceleration (B) Constant acceleration (C) Constant velocity (D) Variable velocity

Answers Key

1. B	2. C	3. C	4. C	5. B	6. C	7. C	8. D	9. C

Q.11 Write down the equations of motion for uniformly accelerated bodies?

Ans: **Equation of Motion for Uniformly Accelerated Bodies**

Consider, a body is moving with uniform acceleration along a straight line. If its initial velocity is v_i and after time interval t its final velocity become v_f then

(i) $v_f = v_i + at$ (ii) $S = v_i t + \frac{1}{2}at^2$

(iii) $2aS = v_f^2 - v_i^2$ (iv) $S = \frac{(v_i + v_f)}{2} \times t$

How to apply these equations in problems

- These equations are useful only for ***linear*** motion with ***uniform*** acceleration.
- When the object moves along a ***straight line***, the direction of motion does not change. In such cases, all vectors can be treated like scalars.
- In problems where vectors can be treated like scalars, the direction of initial velocity is taken *positive*.
- A ***negative*** sign is applied to all those quantities whose direction is opposite to that of initial velocity.
- The above equations can also be applied to ***free fall*** motion of bodies replacing 'a' by 'g'.

Acceleration due to Gravity (g)

In the absence of air, all bodies fall freely near the surface of earth under the action of gravity with uniform acceleration called acceleration due to gravity. It is denoted by g.

➢ Its average value near the surface of earth is taken as ***9.8m/sec²*** or **32 ft s^{-2}**.

Do You Know?

At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.

EXPLANATION:
According to Newton, the Earth attracts the bodies toward its centre, with force of gravity. The body falls with constant acceleration 'g' due to gravity. The force is called weight of the body, given by,
W=mg, where g = W/m = 9.8 ms^{-2}, independent of the weight W i.e. (Ratio between weight and mass remains same)

MCQ's

1. **A bullet shot straight up, returns to its starting point in 10 sec. Find its initial speed?**
(A) 9.8 m/s (B) 24.5 m/s (C) 40 m/s (D) 98 m/s
2. **A cricket ball is hit so that it travels straight up in air and it requires 3 seconds to reach the maximum height. Its initial velocity is:**
(A) 10 m/s (B) 15 m/s (C) 29.4 m/s (D) 12.2 m/s
3. **The equation of motion are not useful for objects moving with:**
(A) Uniform velocity (B) Uniform acceleration (C) Variable acceleration (D) Variable velocity
4. **A ball is allowed to fall freely from certain height. How much distance will it cover in first second?**
(A) 2g (B) g (C) g/2 (D) None
5. **The distance covered by free falling body in two seconds is:**
(A) 9.8m (B) 19.6m (C) 44.4m (D) 49m
6. **If the force acting on a body is doubled, then the acceleration becomes:**
(A) double (B) half (C) one-fourth (D) constant
7. **The velocity of a free falling body just before hitting the ground is 9.8 ms^{-1}, what will be the height through which it fall?**
(A) 98m (B) 19.6 (C) 4.9 (D) 9.8m
8. **A body having uniform acceleration of 10 ms^{-2} has a velocity of 100 ms^{-1}. In what time its velocity will be doubled?**
(A) 8 sec (B) 10 sec (C) 12 sec (D) 14 sec

9. What is the value of 'g' at the centre of the earth?
(A) Infinite (B) 2 g (C) 3 g (D) Zero

10. Acceleration of bodies of different masses allowed to fall freely is, (air friction is negligible)
(A) Same in magnitude and direction (B) Same in magnitude only
(C) Same in direction only (D) Different for different bodies

11. A ball is thrown up vertically, it takes 5 sec to reach maximum height. What was its initial velocity?
(A) 10 ms^{-1} (B) 12.2 ms^{-1} (C) 15 ms^{-1} (D) 49 ms^{-1}

12. If a body is moving with constant velocity of 10m s^{-1} towards west, then what is its acceleration?
(A) 1 m s^{-2} (B) 10 m s^{-2} (C) 30 m s^{-2} (D) Zero

13. A car starts from rest and covers a distance of 100m in one second with uniform acceleration. Its acceleration is:
(A) 50 m/s^2 (B) 100 m/s^2 (C) 200 m/s^2 (D) 300 m/s^2

14. When a body is thrown straight up at highest point, its velocity becomes zero and its acceleration will be:
(A) Zero (B) + 9.8 ms^{-2} (C) − 9.8 ms^{-2} (D) Cannot be determined

Answers Key

1. C	2. C	3. C	4. C	5. B	6. A	7. C	8. B	9. D	10. A	11. D	12. D
13. C	14. B										

Assignment 3.2:

A proton moving with a speed of $1.0 \times 10^7 ms^{-1}$ passes through a 0.020 cm thick sheet of paper and emerges with a speed of $2.0 \times 10^6 ms^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Given Data:

Initial speed of proton $= v_i = 1.0\times10^7 ms^{-1}$

Distance covered $= S = 0.02cm = 0.02\times10^{-2} m$

Final speed of electron $= v_f = 2.0\times10^6 ms^{-1}$

To Find:

Retardation $= a = ?$

Time taken by proton $= t = ?$

Calculations:

Using the third equation of motion

$2as = v_f^2 - v_i^2$ putting values, we get

$2\times a\times 0.02\times10^{-2} = (2\times10^6)^2 - (1.0\times10^7)^2$

$a(0.04\times10^{-2}) = 4\times10^{12} - 1.0\times10^{14}$

$a(0.04\times10^{-2}) = (4 - 1.0\times10^2)10^{12}$

$a(0.04\times10^{-2}) = (4 - 100)10^{12}$

$a(0.04\times10^{-2}) = -96\times10^{12}$

$$a = \frac{-96\times10^{12}}{0.04\times10^{-2}}$$

$a = -2400\times10^{14}\ ms^{-2}$

Or $\boxed{a = -2.4\times10^{17} ms^{-2}}$

Now using the equation, for calculating time 't'

$v_f = v_i + at$

Putting value, we get

$2.0\times10^6 = 1.0\times10^7 - 2.4\times10^{17}\times t$

$(2.4\times10^{17})\ t = 1.0\times10^7 - 2.0\times10^6$

$(2.4\times10^{17})\ t = (10-2)10^6$

$$2.4 \times 10^{17}\, t = 8 \times 10^{6}$$

$$t = \frac{8 \times 10^{6}}{2.4 \times 10^{17}} = 3.33 \times 10^{6-17}$$

$$\boxed{t = 3.33 \times 10^{-11} \text{sec.}}$$

Q.12 Discuss Newton's laws of motion briefly.
***(Comprehension Q.3)* Explain three Newton's laws of motion. Give example in each case for further elaboration.**

For Your Information
First law of motion gives the definition of force while second law gives the measurement of force and give definition of acceleration.

Ans. There are three laws of motion given by Newton:

Newton's First Law of Motion / Law of Inertia

Statement

"In the absence of external force, A body at rest will remain at rest and a body moving with uniform velocity will continue to do so forever."

This is also known as law of inertia.

Mathematically $F_{ext} = 0$ then $a = 0$.

Inertia

The property of a body due to which it tends to maintain its state of rest or uniform motion is called inertia.

OR

It is the opposition offered by the body to any change in its state either of rest or of uniform motion.

Explanation

It is a natural resistance to ***acceleration*** that all objects have. The greater the object's mass, the greater this resistance. So,

*The **mass** of the object is a **quantitative** measure of its inertia.*

The larger the mass, the greater is the inertia. As greater net force is required to change the velocity of objects with large mass.

FIGURE 3.7

Isaac Newton (1642-1727) was born in England, he proposed a theory of the causes of motion in a book written in Latin with title "*Philosophiae Naturalis Principia Mathematica*"

Examples

- When you make a turn while driving a car, you move the opposite way in which the car turns the corner.
- Passenger get pressed back in your seat when an airplane takes off. It is due to inertia, body wants to remain in rest and feels get back pressed with the seat.
- Driver's face is smashed against the windshield if your car suddenly stops against a brick wall. Due to inertia, driver continues its motion in forward direction and hits the windshield.

Newton's Second Law of Motion

Statement

"When a force is applied on a body, it produces the acceleration in its own direction. The magnitude of acceleration is directly proportional to applied force and inversely proportional to its mass."

For Your Information
Action and reaction never balance each other.

Mathematical form $a = \frac{F}{m}$

OR $\boxed{\vec{F} = m\vec{a}}$

S.I unit of force is newton.

$$1\text{N} = 1\text{ kg }(1\text{ m/s}^2)$$

Newton: The force which produces acceleration of $1 m/s^2$ in a body of mass 1 kg is called 1 N force.

In C.G.S system unit of force is dyne.

1 dyne = 1 g (1 cm/s^2)

$$\boxed{1 N = 10^5 dy}$$

Newton's Third Law of Motion

Statement

"Action and reaction are equal in magnitude but opposite in direction."

Explanation

- When two bodies interact with each other, action and reaction forces act for the same length of time.
- They never act on the same body but always act on different bodies. That's why cannot balance each other.

If a body A exerts force F_{AB} on body B then body B exerts the reaction force F_{BA} on body A which is equal in magnitude but opposite in direction, so mathematically

$$F_{AB} = -F_{BA}$$

FOR YOUR INFORMATION

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light—that is, less than 3,000 km/s . Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's laws. However they were refined further at the beginning of the 20th century when Einstein developed his theories of relativity. His theories of relativity extended the concept of Newtonian mechanics to be applied to all objects, even objects traveling close to the speed of light.

Point to Ponder

A car accelerates along a road. Which force actually moves the car?

EXPLANATION:

Reactional force of the road is actually responsible to move the car. The engine of the car simply rotates the wheels but does not move it.

FIGURE 3.8

For your Information

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light—that is, less than 3,000 km/s . Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's laws. However they were refined further at the beginning of the 20th century when Einstein developed his theories of relativity. His theories of relativity extended the concept of Newtonian mechanics to be applied to all objects, even objects traveling close to the speed of light.

Assignment 3.3

Suppose that the mass of the spacecraft 'm_s' is 11 000 kg and that the mass of the astronaut 'm_A' is 92 kg. In addition, assume that the astronaut pushes with a force of F = + 36 N (along x-axis) on the spacecraft. Find the accelerations of the spacecraft and the astronaut.

Solution:

Given Data: $m_s = 11000$ kg

$m_A = 92$ kg

$F = 36$ N

To Find:

Acceleration of spacecraft = a_s = ?

Acceleration of astronaut = a_A = ?

Solution:

From newton's 3rd law, magnitude of force on both is same but opposite in direction i.e. $F_S = -F_A$

$F_s = 36$ N and $F_A = -36$ N

Acceleration of spacecraft is:

From newtons, 2nd law, $a_S = \frac{F_S}{m_S} = \frac{36}{11000} = 0.0033\ ms^{-2}$

Acceleration of astronaut ois:

From newtons, 2nd law, $a_A = \frac{F_A}{m_A} = \frac{-36}{92} = -0.39\ ms^{-2}$

Negative sign is due to the forces which are acting in opposite direction.

MCQ's

1. **The mass of an object is the quantitative measure of its ______**
   (A) Momentum (B) Acceleration (C) Inertia (D) Energy

2. **If the force acting on a body is doubled then the acceleration becomes________.**
(A) Double (B) Half (C) One fourth (D) Constant

3. **When a ball is thrown straight up, what is direction of acceleration at its highest point?**
(A) Upward (B) Downward (C) Zero (D) Horizontal

4. **The law of inertia was first formulated by:**
(A) Aristotle (B) Galileo (C) Newton (D) Einstein

5. **An object of mass 1 kg moving with acceleration of 1 ms^{-2} will experience a force of ______.**
(A) 10^{-2} N (B) 10^{-3}N (C) 1 N (D) 1 dyne

6. **The weight of the body at the centre of the earth is:**
(A) Slightly less (B) Slightly greater (C) Zero (D) Infinite

7. **If the mass of a body is doubled and F constant then acceleration becomes:**
(A) Double (B) Half (C) One fourth (D) Constant

8. **A mass of 1Kg is freely falling. The force of gravity is:**
(A) 1N (B) 9.8N (C) 0.5N (D) Zero

9. **Relativistic Mechanics was developed by:**
(A) Newton (B) Faraday (C) Kepler (D) Einstein

Answers Key

1. C	2. A	3. B	4. C	5. C	6. C	7. B	8. B	9. D

Q. 13 Define and explain linear momentum.

Ans: **Momentum**

The quality of moving body which describes the quantity of motion is called momentum.

OR

The product of mass and velocity of moving body is called linear momentum.

Mathematically

$$\vec{P} = m\vec{v}$$

Where $\vec{P}$ = Momentum of the body
m = Mass of the body
$\vec{v}$ = Velocity of the body

Direction

Since $\vec{v}$ is a vector quantity, so momentum is also a vector quantity having same direction is same as that of velocity.

Unit

SI – unit of momentum is **kg m/s or Ns.** Its dimensions are $[MLT^{-1}]$

When a moving car stops quickly, the passengers move forward toward the windscreen. Seat belts 'change the forces of motion and prevent the passengers from moving Thus the chance of injury is greatly reduced.
EXPLANATION:
When you fall towards forward direction, you apply force on the belt. As a reaction the belt applies force on you in back direction and thus you are saved from injury.

Q.14 Show that N s is equivalent to kg ms^{-1}.

Ans: **Proof**

Ns

$= kg\frac{m}{s^2} \times s \qquad \left[1N = kg\frac{m}{s^2}\right]$

$= kg\frac{m}{s} = kg\ ms^{-1}$

$kg\frac{m}{s}$

Multiplying and dividing by S

$= kg\frac{m}{s}\frac{s}{s}$

$= \left(kg\frac{m}{s^2}\right)s$

$=$ Ns

Q. 15 How force and momentum are related to each other? State Newton's second law in terms of momentum.

Ans: **Momentum and Newton's Second Law of Motion**

Consider a body of mass m moving with velocity $\vec{v_i}$. A force $\vec{F}$ is applied on the body for time t and its velocity changes to $\vec{v_f}$

Acceleration produced by the force is

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

According to Newton's second law of motion

$$\vec{F} = m\vec{a} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Using equation (1) in (2), we have

$$\vec{F} = m\frac{\vec{v}_f - \vec{v}_i}{t}$$

OR

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t} \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

Where

$m\vec{v}_i$ = initial momentum of body = $\vec{P}_i$

$m\vec{v}_f$ = final momentum of body = $\vec{P}_f$

$\Rightarrow \quad F = \frac{P_f - P_i}{t}$

$\Rightarrow \quad F = \frac{\Delta P}{t}$

For Your Information

$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$ is more general form of force than $\vec{F} = m\vec{a}$

2nd Law in terms of momentum

"Time rate of change of momentum of body is equal to the applied force."

This is more general form of Newton's second law of motion. Because it can easily be applicable for the cases when mass is changing.

Q. 16 Define impulse how it is related to momentum?

Ans. **Impulse**

When large force acts on a body for a very short interval of time, then the product of force and time for which the force acts, is called impulse of force.

Mathematically

$$\boxed{\vec{J} = \vec{F} \times \Delta t} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

Unit

- SI unit of impulse is **kg-m/s or Ns.** It is same as that of linear momentum.
- The dimensions of impulse are $[MLT^{-1}]$.
- It is a vector quantity.

Relation between impulse and momentum

According to Newton's second law of motion

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Using equation (2) in (1), we have

$$\vec{J} = \frac{m\vec{v}_f - m\vec{v}_i}{t} \times t$$

OR

$$\boxed{\vec{J} = m\vec{v}_f - m\vec{v}_i}$$

$$J = \Delta P \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

Thus, impulse = change in momentum of the body

Instantaneous change in momentum of body due to impulsive force is called impulse.

Concept of Impulse

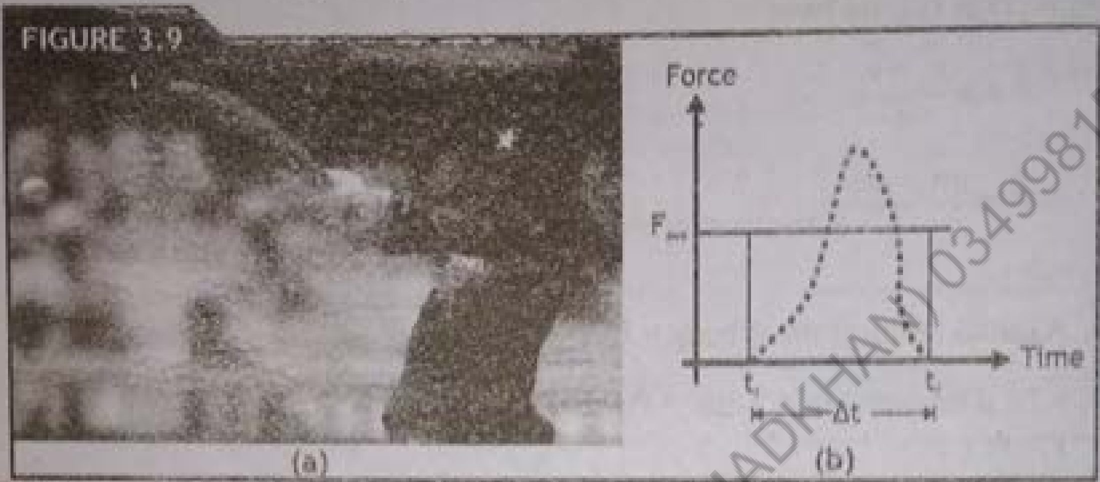
Sometimes the applied force is not constant and it acts for short time. e.g. when a bat hits a cricket ball, the ball force varies from instant to instant during collision. In such cases

$$\vec{J} = \overrightarrow{F_{av}} \times \Delta t$$

Impulsive Force

Force acting on a body for very short interval of time is called impulsive force

FIGURE 3.9



(a) (b)

Assignment 3.4:

A girl of mass 48.0 kg is rescued from a building fire by leaping into a firefighters' net. The window from which she leapt was 12.0 m above the net. She lands in the net so that she is brought to a compete stop in 0.45 s. During this interval (a) what is his change in momentum? (b) What is the impulse on the net due to the girl? (c) What is the average force on the net due to the girl?

Given Data: $m = 48$ kg

$v_i = 0$

$h = 12$ m

$t = 0.45$ s

To Find:

(a) $\Delta P = ?$

(b) $J = ?$

(c) $F_{av} = ?$

Solution:

(a) The final velocity, when the girl just reaches the level of net is:

$v_f^2 - v_i^2 = 2gh$

$\Rightarrow \quad v_f^2 - 0 = 2gh$

$\Rightarrow \quad v_f = \sqrt{2gh}$

This final velocity is equal to initial velocity (vi) when she hits the net and finally comes to rest ($v_f' = 0$)

i.e. $v_i' = v_f = \sqrt{2gh}$

Therefore, change in her momentum when she strikes the net is:

$\Delta P = m v_f' - m v_i'$

$\Delta P = m(v_f' - v_i') = m(0 - \sqrt{2gh})$

$\Delta P = -m\sqrt{2gh} = -48(\sqrt{2 \times 9.8 \times 12})$

$\Delta P = -734.4$ NS

−ve sign shows that ΔP is upward (opposite to motion).

(b) Total impulse applied by the net on the girl is:

$J_{total} = F \times \Delta t + \Delta P$

$= -mg \times \Delta t + \Delta P$

$= (-48 \times 9.8 \times 0.45) + (-734.4)$

$J_{total} = -211.7 - 734.4 = -946.1 \text{ NS}$

This is the impulse applied by net in upward direction by net.

Therefore, impulse applied by girl on net in downward direction is:

$J_{girl} = J_{net} = -(946.1 \text{ NS})\ 946.1 \text{ NS (Down)}$

(c) $\therefore \quad J = F_{av} \times t$

$$\Rightarrow \quad F_{av} = \frac{J}{t} = \frac{946.1}{0.45} = 2102.4 \text{ N (Down)}$$

MCQ's

1. The rate of change in momentum of a body is equal to ___________.
   (A) Displacement (B) Velocity (C) Acceleration (D) Applied force
2. Impulse can be defined as _________ :
   (A) J = F × d (B) J = F× t (C) J = F × v (D) J = F/p
3. If a force of 10 N acts on a body of mass 5 kg for one second, what is its rate of change of momentum?
   (A) 10 $kgms^{-2}$ (B) 50 kg ms^{-2} (C) 5 kg ms^{-2} (D) 2 kg ms^{-2}
4. Total change in momentum of moving body is equal to its:
   (A) K.E (B) Impulse (C) Force (D) Inertia
5. An alternate unit to kgm/s is ___________:
   (A) J s (B) N s (C) N m (D) N
6. SI unit of impulse is:
   (A) Kg ms^{-2} (B) N.m (C) N.s (D) Nm^2
7. At what speed the momentum and kinetic energy of a body having the same value?
   (A) 1 ms^{-1} (B) 2 ms^{-1} (C) 4 ms^{-1} (D) 8 ms^{-1}
8. What are the dimensional unit of impulse?
   (A) (MLT) (B) (MLT^{-1}) (C) $(ML^{-1}T^{-1})$ (D) $(M^{-1}L^{-1}T^{-1})$

Answers Key

1. D	2. B	3. A	4. B	5. B	6. C	7. B	8. B

Q.17 What is isolated system? Give its examples.

Ans: **Isolated system**

It is a system in which no external agency exerts any force.

Example

The molecules of a gas enclosed in a glass vessel at a constant temperature.

Explanation

An isolated system is a collection of particles that can interact with each other but whose interactions with the environment outside the collection have a negligible effect on their motions. It is such system in which external forces like friction, air resistance or any other forces don't have any affect on particles of the system.

(Note: If external forces like friction etc are negligibly smaller than the forces between the particles then we can take such system as isolated system)

Q.18 State Law of conservation of Momentum.

Ans: **Law of conservation of Momentum**

Statement

The total linear momentum of an isolated system remains constant.

OR

It states that if there is no external force applied to a system, the total linear momentum of that system remains constant in time.

According to Newton's 2^{nd} law of motion in term of momentum,

$$\frac{\Delta P}{\Delta t} = F_{ext}$$

For Isolated system, $F_{ext} = 0$, so

$$\frac{\Delta P}{\Delta t} = 0$$

or $\Delta P = 0$

$\Rightarrow$ $P_f - P_i = 0$

$\Rightarrow$ $P_i = P_f$

Where P_i is initial momentum and P_f is the final momentum.

DO YOU KNOW?

An isolated system is a collection of particles that can interact with each other but whose interactions with the environment outside the collection have a negligible effect on their motions.

Q.19 Define elastic and inelastic collision?

Ans: **Collision:**

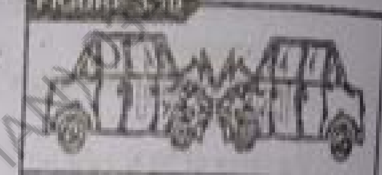
An event during which particles come close to each other and interact by means of forces is called collision.
The forces due to the collision are assumed to be much larger than any external forces present.

Example:

i. Collision of snooker balls
ii. Collision of balls
iii. Interaction between protons and nuclei

For collision to occur the colliding object must not necessarily touch.
For example, consider the collision of a proton with the nucleus of the helium atom, illustrated in Figure. Because the two particles are positively charged, they repel each other in their approach. A collision has occurred, but the colliding particles were never in 'contact'.

FIGURE 3.10



Colliding Vehicles

Collision between players and football

Elastic and Inelastic Collision

Elastic Collision

The collision, in which momentum and kinetic energy of the system is conserved, is called elastic collision.

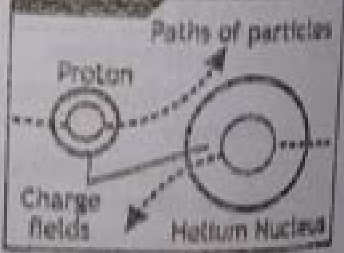
Example

Bouncing back of a hard ball from a marble floor is approximately an elastic collision.

Inelastic Collision

The collision in which only momentum of system is conserved but the kinetic energy of the system is not conserved, is called inelastic collision.

FIGURE 3.11



Example

- Bouncing back of a hard ball from sandy floor
- Collision of two tennis balls.

Note

Momentum and ***total energy*** are conserved in all types of collision.
Perfect elastic collision cannot be possible.

Solid reasons for loss of kinetic energy:

In case of inelastic collision, the loss of kinetic energy is due to

- friction of ball with floor
- friction of ball and air
- sound and heat produces during collision

Head-On Collision

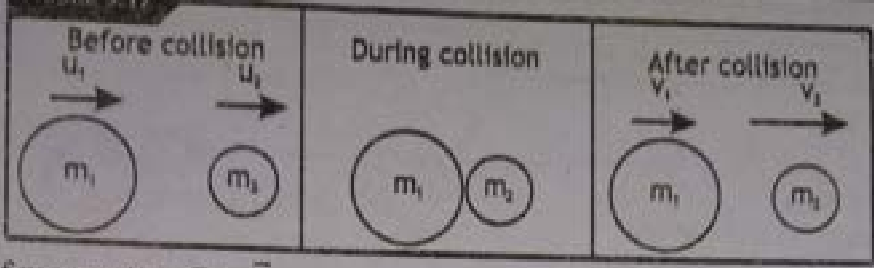
The type of collision in which before and after collision, the motion appears only on straight line is called head-on collision. It is usually known as collision in one dimension.

Q.20 Show that relative speed of approach is equal to relative speed of separation for elastic collision in one dimension.

Ans: **Elastic Collision in One Dimension**

Consider two smooth, non-rotating hard balls of masses m_1 and m_2 moving in such a way so that their centers lie along the same straight line with initial velocities u_1 and u_2 respectively. When they make head on collision with each other their velocities becomes v_1 and v_2 respectively as shown in figure.

FIGURE 3.17



By Law conservation of momentum $P_i = \vec{P}_f$

or $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ________ (1)

rearranging $m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ________ (2)

Since for elastic collision KE is conserved therefore $KE_i = KE_f$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

rearranging
$$\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$$

Or
$$\frac{1}{2}m_1\left(u_1^2 - v_1^2\right) = \frac{1}{2}m_2\left(v_2^2 - u_2^2\right)$$

therefore
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{________ (3)}$$

Dividing equation 1 by equation 2 we get
$$\frac{m_1\left(u_1^2 - v_1^2\right)}{m_1\left(u_1 - v_1\right)} = \frac{m_2\left(u_2^2 - v_2^2\right)}{m_2\left(u_2 - v_2\right)}$$

Or
$$\frac{(u_1^2 - v_1^2)}{(u_1 - v_1)} = \frac{(v_2^2 - u_2^2)}{(v_2 - u_2)} \quad [\text{As } a^2 - b^2 = (a+b)\times(a-b)]$$

Therefore
$$\frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)} = \frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)}$$

Rearranging $u_1 + v_1 = u_2 + v_2$ or $u_1 - u_2 = v_2 - v_1$ ________ (4)

Therefore $\boxed{u_1 - u_2 = -(v_1 - v_2)}$

⇨ $U_{rel} = -V_{rel}$

⇨ Speed of approach = Speed of separation

Negative sign shows that faster body has become slower and slower has become faster.

Q.21 Find the expressions for the velocities of two bodies m_1 and m_2 after elastic collision in one dimension.

Ans: **Determination of velocities after collision**

We can calculate the velocities of the masses after collision by solving equations (2), (3) and (4).

Velocity of mass m_1 (i.e. v_1')

From equation (4), $u_1 + v_1 = v_2 + u_2$

$v_2 = u_1 - u_2 + v_1$ ________ (5)

Using equation (5) in (1), we have

$m_1(u_1 - v_1) = m_2[(u_1 - u_2 + v_1) - u_2]$

⇒ $m_1u_1 - m_1v_1 = m_2u_1 - m_2u_2 + m_2v_1 - m_2u_2$

⇒ $m_1v_1 + m_2v_1 = m_1u_1 - m_2u_1 + m_2u_2 + m_2u_2$

⇒ $(m_1 + m_2)v_1 = (m_1 - m_2)v_1 + 2m_2u_2$

Dividing by $(m_1 + m_2)$ on both sides:

Therefore $\boxed{v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2}{(m_1 + m_2)} u_2}$ ———— (6)

∴ From equation (4),

$$u_1 + v_1 = v_2 + u_2$$

$$\Rightarrow \quad v_1 = v_2 + u_2 - u_1$$

Put this value in eq (1), given by:

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2)$$

$$\Rightarrow \quad m_1(u_1 - \{v_2 + u_2 - u_1\} = m_2 (v_2 - u_2)$$

$$\Rightarrow \quad m_1(u_1 - v_2 - u_2 + u_1) = m_2 (v_2 - u_2)$$

$$\Rightarrow \quad m_1(2u_1 - v_2 - u_2) = m_2 (v_2 - u_2)$$

$$\Rightarrow \quad 2m_1u_1 - m_1v_2 - m_1u_2 = m_2v_2 - m_2u_2$$

$$\Rightarrow \quad 2m_1u_1 + m_2u_2 - m_1u_2 = m_1 v_2 + m_2 v_2$$

$$\Rightarrow \quad 2m_1u_1 + (m_2 - m_1)u_2 = v_2 (m_1 + m_2)$$

$$\Rightarrow \quad \left(\frac{2m_1}{m_1 + m_2}\right) u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 = v_2$$

$$\Rightarrow \quad \boxed{v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)} u_2} \quad \text{———— (7)}$$

Assignment 3.5:

On a highway a car of mass 1500 kg is stopped at traffic signal. A pickup of mass 2000 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0 m/s. How fast was the pickup going just before the collision?

Solution: Mass of Car = mc = 1500 kg
Mass of pickup = mp = 2000 kg
Velocity of car before collision = Uc = 0
Velocity of car after collision = Vc = 10 m/s
Velocity of pickup after collision = Vp = 0
Velocity of pickup before collision = Up = ?
From law of conservation of momentum
mcUc + mpUp = mcVc + mpVp
1500(0) + 2000 (Up) = 1500 (10) + 2000 (0)
0 + 2000 (Up) = 15000 + 0
2000 (Up) = 15000
Up = 15000/2000
Up = 7.5 m/s

Q.22 Is momentum conserved, when momentum changes are produced by explosive forces? Explain.

Ans: Momentum and Explosive Forces

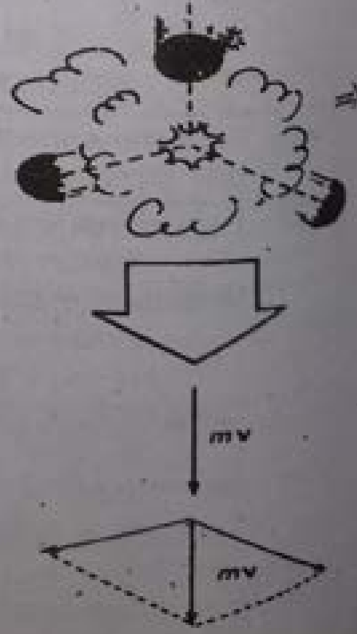
An explosion is a sudden, intense release of energy that often produces a loud noise, high temperature, and flying pieces, and generates a pressure wave. If the system is isolated, its total momentum during the explosion will be conserved.

Mathematically $P_i = P_f$

Examples

1- Explosion of a shell or bomb

Suppose a bomb is falling, in its way it explodes into two pieces. The momentum of the bomb fragments combined by vector addition is equal to the original momentum of falling bomb, as shown in figure.

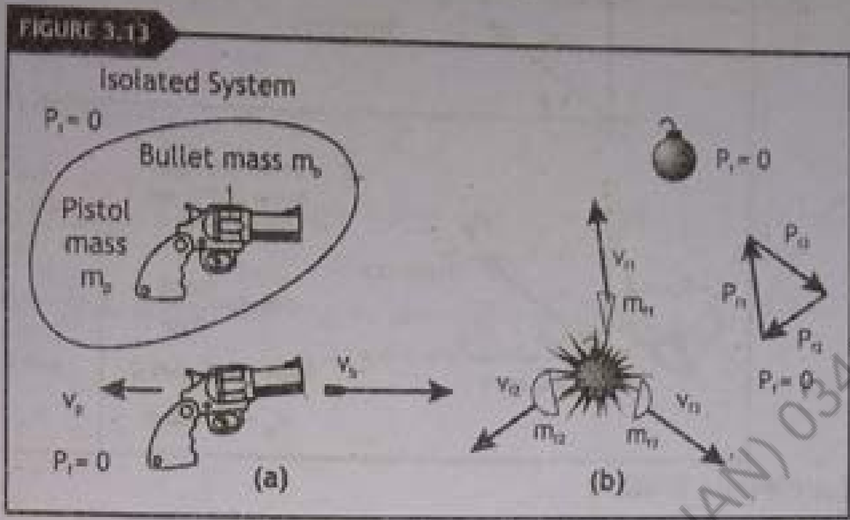


2- Firing of Bullet from Pistol

Consider an isolated system of pistol of mass 'm_P' and bullet of mass 'm_b'. Initial momentum is zero as both bullet and rifle are initially at rest.

$P_i = 0$

FIGURE 3.13



When the bullet is fired from the gun, the gun moves with velocity v_P and bullet moves with velocity v_b in opposite direction such that the total momentum is P_f is again zero.. In this way, momentum of the system is conserved. The velocity with which gun moves in the backward direction is called recoil velocity.

$$P_f = P_i$$

⇨ $$m_P v_P + m_b v_b = 0$$

⇨ $$m_P v_P = -m_b v_b$$

(Momentum of bullet and gun are equal and opposite)

⇨ $$v_P = -\frac{m_b v_b}{m_P}$$

This is formula for recoil velocity of pistol after fire.

MCQ's

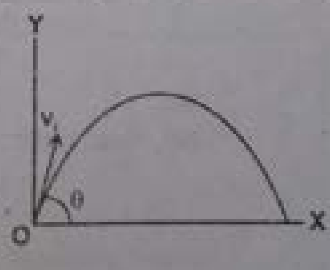
1. If the body of mass 2 kg moving with 15 ms^{-1} collides with stationary body of same mass, then after elastic collision the 2nd body will move with the velocity of ________
   (A) 15m/s (B) 30m/s (C) Zero m/s (D) None of these
2. As the rocket moves upward during its job, its acceleration goes on:
   (A) Increasing (B) Decreasing (C) Remains same (D) It moves with uniform velocity
3. A force of 10 N acts on a body of mass 1 kg for 5 sec. to a distance of 10 m. What is the rate of change of momentum of the body?
   (A) 50 N (B) 25N (C) 20 N (D) 10 N
4. In the absence of external force, the change in momentum is:
   (A) Zero (B) Constant (C) Decreasing (D) Increasing

Answers Key

1. A	2. A	3. D	4. A

Q.23 What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile.

Ans: Projectile Motion

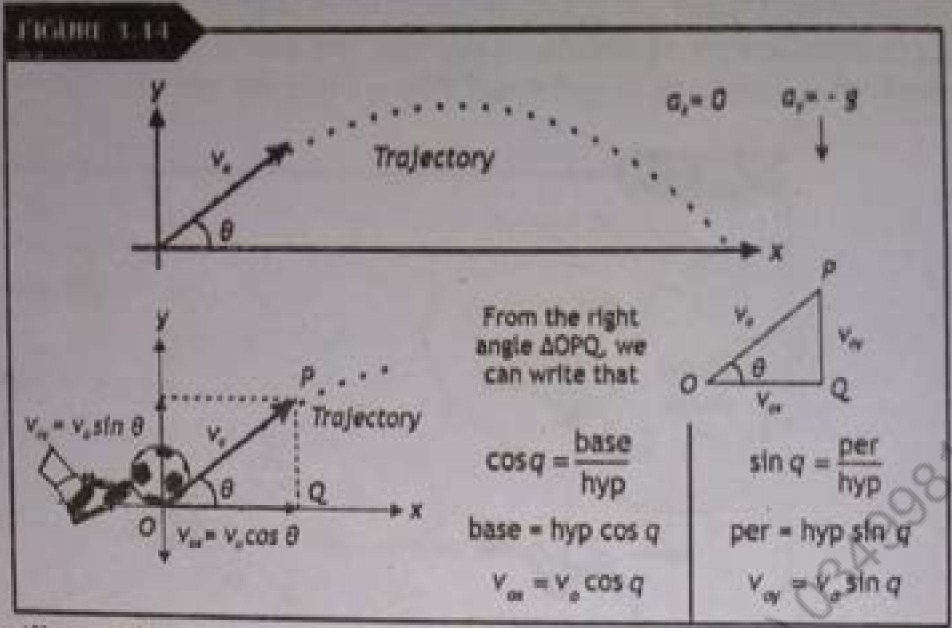


Two dimensional motion of the body under the action of gravity and inertia is called projectile motion.

OR

Two dimensional motion under the constant acceleration due to gravity and inertia is called projectile motion.

FIGURE 3.14



Example of projectile motion

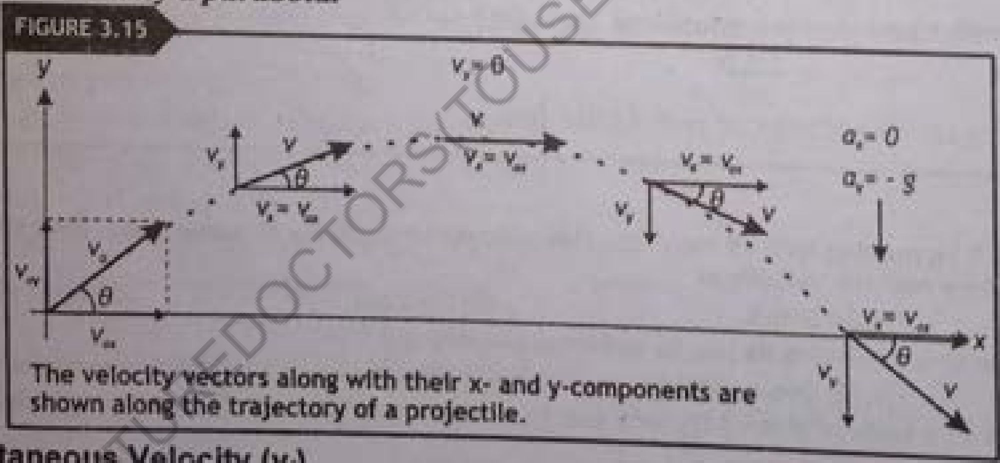
(i) A football kicked off by a player.
(ii) Bullet fired from gun
(iii) A missile fired from a launching pad.
(iv) A ball thrown by a cricketer
(v) A ball thrown from some height

Sign Conventions
In projectile motion a special convention is used i.e. whichever quantity is directed upward is considered positive and whichever quantity is directed downward is considered negative. Therefore, acceleration in projectile motion is equal to (-g) and that is along y-axis

Trajectory

The path followed by the projectile is called its trajectory. The trajectory of a projectile is usually a ***parabola.***

FIGURE 3.15



The velocity vectors along with their x- and y-components are shown along the trajectory of a projectile.

Instantaneous Velocity (v_i)

Suppose a projectile is fired with initial velocity v_i at an angle θ with horizontal ($0 < \theta < 90$).

Let

Horizontal component of initial velocity $= v_{ix} = v_i \cos\theta$

Vertical component of initial velocity $= v_{iy} = v_i \sin\theta$

Horizontal component of velocity

Since there is no horizontal force along horizontal axis so acceleration $a_x = 0$.

$$F_x = 0$$

So, $a_x = 0$

So, final horizontal component of velocity at any instant t is

$$v_{fx} = v_{ix} + a_x t$$

$$v_{fx} = v_{ix} + (0)t \qquad \text{as} \qquad a_x = 0$$

$$v_{fx} = v_{ix}$$

OR $\boxed{v_{fx} = v_{ix} = v_i \cos\theta}$ ________ (1)

So, horizontal component of the velocity remains constant during the whole motion.

Vertical component of velocity

The only force acting on projectile is gravity, so it has gravitational acceleration.

$$F_y = F_g$$

So, $a_y = -g$

Final vertical component of velocity at any instant t is

$$v_{fy} = v_{iy} + a_y t$$

OR $\boxed{v_{fy} = v_i \sin\theta - gt}$ ______ (2) [since $a_y = -g$ and $v_{iy} = v_i \sin\theta$]

Magnitude of velocity

Final velocity of projectile at any instant can be calculated by formula:

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

Putting

$\therefore \quad v_{fx} = v_{ix} = v_i \cos\theta$

$v_{fy} = v_i \sin\theta - gt$

$\Rightarrow \quad v_f = \sqrt{(v_i \cos\theta)^2 + (v_i \sin\theta - gt)^2}$

$\Rightarrow \quad v_f = \sqrt{v_i^2 \cos^2 + \{v_i^2 \sin^2\theta + g^2t^2 - 2(vi \sin\theta)(gt)}$

$\Rightarrow \quad v_f = \sqrt{v_i^2 (\cos^2\theta + \sin^2\theta) + g^2t^2 - 2\, v_i\, gt \sin\theta}$

$\Rightarrow \quad v_f = \sqrt{v_i^2 (1) + g^2t^2 - 2\, v_i\, gt \sin\theta}$

$\Rightarrow \quad v_f = \sqrt{v_i^2 + g^2t^2 - 2\, v_i\, gt \sin\theta}$

Do You Know?

Science plays a gig part in helping javelin throwers to achieve longer distances and the javelin must be thrown at angle of 45° to achieve maximum distance.

Direction of velocity

Let resultant velocity makes an angle ϕ with the horizontal. Then

$$\tan\phi = \frac{v_{fy}}{v_{fx}}$$

$$\phi = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

$$\Rightarrow \quad \emptyset = tan^{-1}\left(\frac{vi \sin\theta - gt}{vi \cos\theta}\right)$$

Q.24 A projectile is thrown with initial velocity v_i making an angle θ with the horizontal. Find its

(a) Maximum height **(b) Time of flight**

(c) Range **(d) Maximum range**

Ans: **Max-Height of the projectile**

The maximum vertical distance covered by the projectile is called the maximum height of the projectile.

For the determination of height we take

Initial vertical velocity of projectile = $v_{iy} = v_i \sin\theta$

Vertical acceleration = $a_y = -g$

Vertical velocity at highest point = $v_{fy} = 0$

Maximum height = y = H = ?

Now according to equation of motion,

FIGURE 3.16

Maximum height *H*

$$2aS = v_f^2 - v_i^2$$

OR $2a_y y = v_{fy}^2 - v_{iy}^2$

$$2(-g)\,H = 0 - (v_i \sin\theta)^2$$

$$-2gH = -v_i^2 \sin\theta^2$$

$$\boxed{H = \frac{v_i^2 \sin^2\theta}{2g}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (1)$$

Time of flight

The time taken by body to cover the distance from the place of projection to the place where it hits the ground at same level is called the time of flight.

OR

It is the time during which projectile remains in air.

As the body goes up and comes back to same level so it covers no vertical displacement.

i.e. $S_y = 0$

Initial vertical velocity of projectile = $v_{iy} = v_i \sin\theta$

acceleration due to gravity = $a_y = -g$

time of flight = T =?

As $\quad S = v_i T + \frac{1}{2} a T^2$

OR $\quad S_y = v_{iy} T + \frac{1}{2} a_y T^2$

OR $\quad 0 = (v_i \sin\theta) T - \frac{1}{2} g T^2$

OR $\quad \frac{1}{2} g T^2 = (v_i \sin\theta) T$

$$\boxed{T = \frac{2 v_i \sin\theta}{g}} \quad \text{\_\_\_\_\_\_\_\_\_\_} (2)$$

Time to reach maximum height (Time of Summit)

It is the time in which projectile reaches maximum height from point of projection.

It is equal to half of the time of flight.

$$T' = \frac{T}{2}$$

$$T' = \frac{2 v_i \operatorname{Sin}\theta}{2g} = \frac{v_i \operatorname{Sin}\theta}{g}$$

2nd method

From 1st equation of motion

$v_f = v_i + at$

$v_{fy} = v_{iy} + a_y t$ -------------------(3)

Put following values in equation (3)

$v_{fy} = 0$ (at highest point)

$v_{iy} = v_i \sin\theta$

$a_y = -g$

$t = T'$

We have, $\quad 0 = v_i \sin\theta - g T'$

$g T' = v_i \sin\theta$

$$T' = \frac{v_i \operatorname{Sin}\theta}{g}$$

FIGURE 3.18

For Your Information

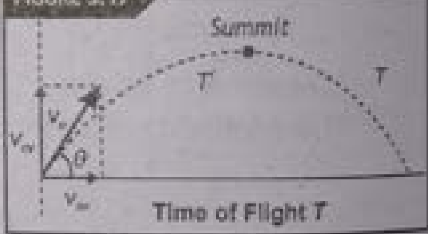
The factor which remains constant during the projectile motion are
* horizontal velocity
* both x and y components of acceleration

For Your Information

$$H = \frac{1}{4} R_{max}$$

Do You Know?

FIGURE 3.17



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projection is larger than 45°, the height attained will be more but the range is again less.

EXPLANATION:

The range is given by,

$$R = \frac{v_i^2}{g} \sin 2\theta$$

The range depends on sin 2θ.

When θ = 45°, the range becomes maximum that is

$$R_{max} = \frac{v_i^2}{g} \sin 90^\circ = \frac{v_i^2}{g}$$

The values sin 2θ for all other angles less or greater than 45° are less than 1, therefore, the range, regarding all other angles is smaller than the range at 45°. The height h is given as,

$$h = \frac{v_i^2 \sin^2\theta}{g}$$

The height depends on angle θ. For greater values of θ, the height will be greater and smaller for smaller values of θ.

Range of projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of projective.

Then $x = v_{ix}t + \frac{1}{2}a_x t^2$ becomes

$$R = v_i\cos\theta \times \frac{2v_i\sin\theta}{g} + 0 \qquad [a_x = 0] \quad \left[t = \frac{2v_i\sin\theta}{g}\right]$$

OR $$R = \frac{v_i^2(2\sin\theta\cos\theta)}{g}$$

$$\boxed{R = \frac{v_i^2\sin 2\theta}{g}} \quad \text{——— (4)} \qquad [\text{Since } 2\sin\theta\cos\theta = \sin 2\theta]$$

The range of the projectile will be maximum when $\sin 2\theta$ has maximum value. i.e.

$\sin 2\theta = 1$

OR $2\theta = \sin^{-1}(1)$

OR $2\theta = 90^\circ$

OR $\theta = 45^\circ$

So, a projectile will have its maximum range when it is thrown at angle of projection of 45°.

Maximum Range

So, equation $R = \frac{v_i^2\sin 2\theta}{g}$ becomes

$$R_{max} = \frac{v_i^2\sin 2(45^\circ)}{g}$$

OR $$R_{max} = \frac{v_i^2\sin 90^\circ}{g}$$

$$\boxed{R_{max} = \frac{v_i^2}{g}} \quad \text{——— (5)} \qquad [\because \sin 90^\circ = 1]$$

Note:

We can express the range of the projectile in terms of maximum range as

$\boxed{R = R_{max}\sin 2\theta}$

Q.25 For which pair of angles, ranges of projectile will be same?

Ans: If initial velocity of projectile and value of g is same then ranges of a Projectile will be same for pair of angles whose sum is 90° i.e. these are Complementary angles of one another e.g. (10°, 80°), (30°, 60°) etc.

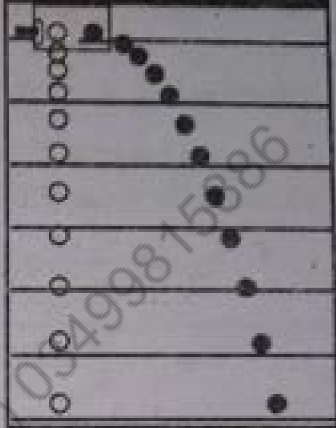
For example the range at 75° & 15° is the same.

$$R_{75^\circ} = \frac{v_0^2}{g}\sin 2(75^\circ)$$

Or $$R_{75^\circ} = \frac{v_0^2}{g}\sin 150^\circ$$

Therefore $$R_{75^\circ} = \frac{v_0^2}{g}(0.5)$$

Interesting Information



A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the Other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

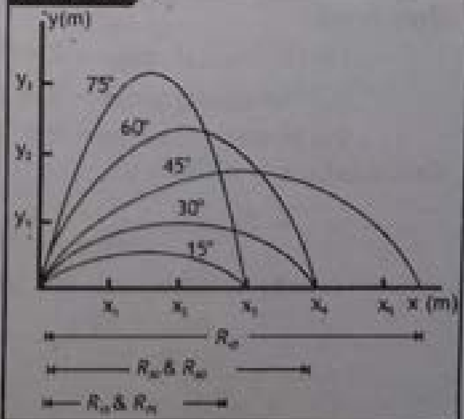
EXPLANATION:

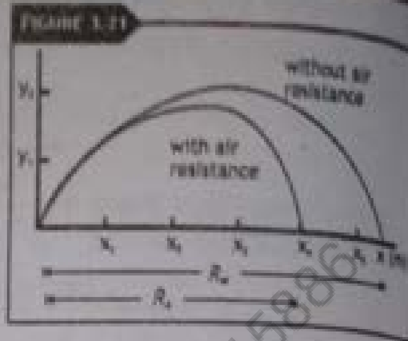
Since vertical acceleration $a_y = g$ in both cases so at any time, their vertical velocities are equal. Hence the two balls are at the same level, i.e., their vertical displacements are equal.

Tid Bit

The ball is hit by a batsman at an angle of 20° and 70° but the ball range is same in both cases its speed is same.

FIGURE 3.19





and $R_{15^\circ} = \frac{v_0^2}{g}\sin 2(15^\circ)$

Or $R_{15^\circ} = \frac{v_0^2}{g}\sin 30^\circ$

Therefore $R_{15^\circ} = \frac{v_0^2}{g}(0.5)$

Hence, the range at 75° & 15° is same.

Similarly the range at 60° & 30° is the same.

$R_{60^\circ} = \frac{v_0^2}{g}\sin 2(60^\circ)$ or $R_{60^\circ} = \frac{v_0^2}{g}\sin 120^\circ$ therefore $R_{60^\circ} = \frac{v_0^2}{g}(0.866)$

$R_{30^\circ} = \frac{v_0^2}{g}\sin 2(30^\circ)$ or $R_{30^\circ} = \frac{v_0^2}{g}\sin 60^\circ$ therefore $R_{30^\circ} = \frac{v_0^2}{g}\sin(0.866)$

Similarly for any two such angles (equal degrees above and below 45°) we can show that the range is same.

Range with Air resistance:

Air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile is reduced as shown in the figure.

POINT TO PONDER

How rockets accelerate in space?

As there is no air in space to push against such that as a reaction rocket is pushed forward. The answer lies in conservation of momentum principle. The rocket ejects gases from its tail at a high velocity, as a result rocket's mass decreases. Thus giving acceleration to the rocket called thrust. Any space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate.

Assignment 3.6:

At Arbab Niaz Cricket Stadium Peshawar a batsman hits the shot at initial velocity of 28 m/s. If the boundary is 72 m from the batsman, will the ball cross the boundary for a six? If the angle with the horizontal is (a) 30° (b) 45° and (c) 70°. (Ignore air resistance)

Solution:

Initial velocity of ball = v_i = 28m/sec

Required:

(a) Horizontal range = R = ? for θ = 30°

(b) Horizontal range = R = ? for θ = 45°

(c) Horizontal range = R = ? for θ = 70°

Calculation:

$$R = \frac{v_i^2 \sin 2\theta}{g} \quad \text{______(1)}$$

Putting v_i = 28m/sec, θ = 30° in equation (1), we get,

$$R = \frac{28^2 \times \sin(2 \times 30^\circ)}{9.8}\text{m} = \frac{784 \times 0.866}{9.8}\text{m} = 69.28$$

⇒ $\boxed{R = 69.28\text{m}}$

As the range is less than the length of boundary [72m], so for $\theta = 30°$, the ball will not cross the boundary.

(b) For $\theta = 45°$, equation (1) becomes

$$R = \frac{28^2 \times \sin(2 \times 45°)}{9.8} = \left(\frac{784 \times 1}{9.8}\right)m$$

⇒ $\boxed{R = 80m}$

In this case the range is greater than the length of the boundary [72m], so the ball will cross the boundary.

(c) Put $\theta = 70°$ in equation (1),

$$R = \frac{282 \times \sin(2 \times 70°)}{9.8} = \left(\frac{784 \times 0.643}{9.8}\right)m$$

⇒ $\boxed{R = 51.44m}$

The range is less than the length of boundary, so in this case, the ball will not cross the boundary.

MCQ's

1. **Motion of projectile is:**
(A) 1 dimensional (B) 2 dimensional (C) 3 dimensional (D) 4 dimensional

2. **The horizontal range is maximum when it is projected at an angle of:**
(A) 0° (B) 30° (C) 45° (D) 60°

3. **For which pair of angles the range of projectile is equal?**
(A) 0°, 45° (B) 35°, 55° (C) 15°, 60° (D) 30°, 75°

4. **The horizontal component of velocity of projectile:**
(A) Increases (B) Decreases (C) Remains same (D) Decreases then increases

5. **The velocity of projectile is maximum:**
(A) At the highest point (B) At the point of launching
(C) At half of the height (D) After striking the ground

6. **The horizontal range of projectile at 30° with horizontal is the same as that at an angle of:**
(A) 45° (B) 60° (C) 90° (D) 120°

7. **A projectile is thrown upward with the velocity "v_i" making an angle θ with the horizontal, the maximum horizontal range is:**
(A) $\frac{v_i^2}{g}$ (B) $\frac{v_i^2}{2g}$ (C) $\frac{v_i^2}{g}\sin 2\theta$ (D) $\frac{v_i^2}{2g}\sin 2\theta$

8. **The time of flight of a projectile, when it is projected from the ground is:**
(A) $\frac{v_i}{g}\cos\theta$ (B) $\frac{v_i}{g}\sin\theta$ (C) $\frac{2v_i}{g}\sin\theta$ (D) $\frac{v_i^2}{g}\sin\theta$

9. **In the projectile motion, the vertical component of velocity:**
(A) Remains constant (B) Varies point to point (C) Becomes zero (D) Increases with time

10. **The angle of projection for which its maximum height and horizontal range are equal:**
(A) 45° (B) 50° (C) 65° (D) 76°

11. **A bomber drops its bomb when it is vertically above the target. It misses the target due to:**
(A) Horizontal component of velocity (B) Vertical component of velocity
(C) Pull of the Earth (D) Acceleration due to gravity

12. **A ball is thrown up with 20 ms^{-1} at an angle of 60° with x-axis, the velocity of the ball at the top position is:**
(A) 0 ms^{-1} (B) 10 ms^{-1} (C) 20 ms^{-1} (D) 16 ms^{-1}

13. **The shape of trajectory of short range projectile is:**
(A) circular (B) parabola (C) ellipse (D) hyperbola

14. **The path followed by a projectile is known as its:**
(A) Range (B) Trajectory (C) Cycle (D) Height

15. **For which pair of angles the range of projectile are equal:**
(A) 20°, 30° (B) 70°, 20° (C) 60°, 40° (D) 80°, 10°

16. **The velocity of projectile at maximum height is:**
(A) $v_i\cos\theta$ (B) Zero (C) Maximum (D) $v_i\sin\theta$

17. **Ranges of projectile are equal for pair of angles:**
(A) 45°, 50° (B) 45°, 60° (C) 40°, 50° (D) 45°, 55°

18. **Horizontal range is equal for the angles:**
(A) 30° and 40° (B) 30° and 50° (C) 30° and 60° (D) 30° and 90°

19. The acceleration along x-axis direction in cases of projectile is:
(A) Zero (B) Equal to gravity (C) Maximum (D) Constant

20. A ball rolls off the edge of a table. The horizontal component of the ball's velocity remains constant during its entire trajectory because:
(A) The ball is acted upon by a force in the only vertical direction
(B) The net force acting on the ball is zero
(C) The ball is not acted upon by a force in the vertical direction
(D) The ball is acted upon by a force in the horizontal direction

Answers Key

1. B	2. C	3. B	4. C	5. B	6. B	7. A	8. C	9. B	10. D	11. A	12. B
13. B	14. B	15. B	16. A	17. C	18. C	19. A	20. A				

FORMULAE

1	Displacement of a particle	$\vec{d} = \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$	
2	Average velocity	$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$	
3	Instantaneous velocity	$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{d}}{\Delta t}$	
4	Average acceleration	$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$	$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$
5	Instantaneous acceleration	$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$	
6	2nd law of motion	$\vec{F} = m\vec{a}$	
7	Linear momentum	$\vec{P} = m\vec{v}$	
8	2nd law of motion in terms of momentum	$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{t}$	
9	Impulse	$\vec{I} = \vec{F} \times t$	$\vec{I} = \vec{F} \times t = m\vec{v}_f - m\vec{v}_i$
10	Law of conservation of linear momentum	$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$	
11	Relation between relative velocity of approach and relative velocity of separation	$\vec{v}_1 - \vec{v}_2 = -(\vec{v}_1' - \vec{v}_2')$	
12	Velocity of mass m_1 after collision in one dimensional elastic collision	$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$	
13	Velocity of mass m_2 after collision in one dimensional elastic collision	$v_2' = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2$	
14	Force due to water flow	$F = \frac{m}{t}v$	

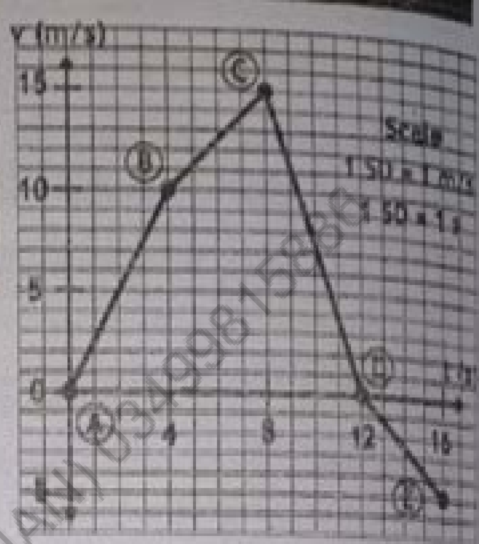
15	Recoil velocity of a rifle	$v' = -\frac{mv}{M}$
16	Acceleration of rocket	$a = \frac{mv}{M}$
17	Horizontal distance of an object thrown horizontally from height h	$x = v_{ix}t$
18	Vertical distance of an object thrown horizontally from height h	$y = \frac{1}{2}gt^2$
19	x-component of instantaneous velocity of a projectile	$v_{fx} = v_{ix} = v_i \cos\theta$
20	y-component of instantaneous velocity of a projectile	$v_{fy} = v_i \sin\theta - gt$
21	Instantaneous velocity of a projectile	$v = \sqrt{v_{fx}^2 + v_{fy}^2}$
22	Height of projectile	$H = \frac{v_i^2 \sin^2\theta}{2g}$
23	Time flight of projectile	$T = \frac{2v_i \sin\theta}{g}$
24	Range of projectile	$R = \frac{v_i^2 \sin 2\theta}{g}$
25	Maximum range of projectile	$R_{max} = \frac{v_i^2}{g}$

Key Points

- A body at rest will remain at rest and a body in motion will remain in uniform motion unless an external resultant force acts upon the body.
- An unbalanced force acting on a body produces an acceleration in the direction of the net force, an acceleration that is directly proportional to the force and inversely proportional to the mass of the body: $F = ma$.
- For every action force there is an equal and opposite reaction force. The action and reaction forces are equal and opposite.
- The linear momentum p of a body is defined as the product of the mass m of that body and its velocity v : p = m v. In terms of momentum, Newton's second law states that the rate of change of momentum of a body is equal to the force acting on the body.
- **Impulse:** There are processes in which momentum changes but the forces are very short-lived, extremely large, varying over wide limits and instantaneously not measureable. The change in momentum is, however, measurable. It is therefore convenient to introduce a physical quantity known as impulse. Impulse J is defined as the product of an average force ΔF and the duration of time Δt during which, the actual force F acts: $J = \Delta F \Delta t$.
- **The principle** of conservation of linear momentum: This principle states that if there is no external force applied to a system, the linear momentum of that system remains constant in time. It is true in propulsion of rockets, decay of nuclei, collision of atoms and molecules etc.
- **Displacement:** Shortest distance between two points is called displacement.
- **Speed:** Rate of change of distance is called speed.
- **Velocity:** Rate of change of displacement is called velocity.
- **Acceleration:** Rate of change of velocity is called acceleration.

Solved Examples

Example 3.1: **The velocity time graph shows the motion of bicyclist in a straight line. (a) From the slope of the graph calculate the acceleration of the bicyclist between segment A and B, B and C, C and D and D and E. (b) Calculate the average acceleration of the bicyclist. Also (c) Plot the acceleration time graph for this motion.**



Solution:

(a) The acceleration from point A to B can be calculated by measuring the slope as

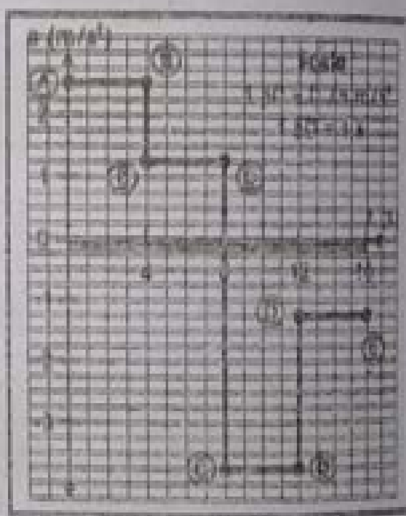
$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$ or $a = \frac{10\ m/s - 0\ m/s}{4s - 0s}$ or $a = \frac{10\ m/s}{4s}$

Therefore $\boxed{a = 2.5\ m/s^2}$

The acceleration from point B to C by measuring the slope is

$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$ or $a = \frac{15\ m/s - 10\ m/s}{8s - 4s}$ or $a = \frac{5\ m/s}{4s}$

Therefore $\boxed{a = 1.25\ m/s^2}$



The acceleration from point C to D can be calculated as

$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$ or $a = \frac{0\ m/s - 15\ m/s}{12s - 8s}$ or $a = \frac{-15\ m/s}{4s}$

Therefore $\boxed{a = -3.75\ m/s^2}$

Similarly the acceleration from point D to E can be calculated as

$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$ or $a = \frac{-5\ m/s - 0\ m/s}{16s - 12s}$ or $a = \frac{-5\ m/s}{4s}$

Therefore $\boxed{a = -1.25\ m/s^2}$

(b) The average acceleration can be calculated by measuring the slope from point A to E as

$\vec{a} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i}$ or $a = \frac{-5\ m/s - 0\ m/s}{16s - 0s}$ or $a = \frac{-5\ m/s}{16s}$

Therefore $\boxed{a = -0.3125\ m/s^2}$

(c) When these data points are plotted on acceeration time graph by choosing suitable scale we get the curve as

Example 3.2: **A (Joint Fighter-17) JF Thunder 17 aircraft takes off at 70.0 m/s (252 km/h). After accelerating uniformly at 3.90 m/s² from rest that lasts 6.5 s during the initial phase of takeoff. The afterburner engines are then turned up to full power for an acceleration of 7.1 m/s². Calculate the length of runway needed and the total time of takeoff.**

Given Data:

acceleration 'a_1' for phase 1 = 3.9 m/s²
acceleration 'a_2' for phase 2 = 7.1 m/s²
time 't_1' for phase 1 = 6.5 s
final velocity 'v_{f2}' for phase 2 = 70.0 m/s

To Find:

(a) length of runway 's' = ?
(b) time of takeoff 't' = ?

Solution:

For first phase of take-off, the distance s can be calculated by using second equation of motion

$$S_1 = v_{i1}t_1 + \frac{1}{2}a_1t_1^2$$

Putting values $S_1 = 0 \times 5 + \frac{1}{2}3.9 \times (6.5s)^2$

Or $S_1 = \frac{1}{2}3.9\ m/s^2 \times 42.25s2$

Therefore $S_1 = 82.3875m$

The final velocity at phase 1, can be calculated by using first equation of motion

$v_{f1} = v_{i1} + a_1t_1$ putting values $v\ v_{f1} = 0\ m/s + 3.90\ m/s^2 \times 6.5s$

therefore $v_{f1} = 25.35\ m/s$

For second phase of take-off, the distance s can be calculated by using third equation of motion

$$2a_2S_2 = v_{f2}^2 - v_{i2}^2 \quad \text{and} \quad S_2 = \frac{v_{f2}^2 - v_{i2}^2}{2a_2}$$

The final velocity 'v_{f1}' at phase 1 which is 25.35 m/s will be initial velocity 'v_{i2}' at phase 2, therefore

$$S_2 = \frac{v_{f2}^2 - v_{i2}^2}{2a_2} \quad \text{Putting values } S_2 = \frac{(70.0\ m/s)^2 - (25.35\ m/s)^2}{2 \times 7.1\ m/s^2}$$

Or $S_2 = \frac{4257.3775\ m^2/s^2}{14.2\ m/s^2}$ and $S_2 = 299.8153\ m$

Therefore $S_2 = 299.8m$

For second phase of take-off, the time 't' can be calculated by using first equation of motion

$v_{f2} = v_{i2} + a_2t_2$ or $v_{f2} - v_{i2} = a_2t_2$

Or $t_2 = \frac{v_{f2} - v_{i2}}{a_2}$ Putting values $t_2 = \frac{70.0\ m/s - 25.35\ m/s}{7.1\ m/s^2}$

Hence $t_2 = 6.3\ s$

The total distance covered is $S = S_1 + S_2$

Putting values $S = 82.4m + 299.8m$

$\boxed{S = 382.2m}$

Hence the minimum runway length under these conditions is 383.2 metres.

The total time taken is $t = t_1 + t_2$

Putting values $t = 6.5\ s + 6.3\ s$

$\boxed{t = 12.8\ s}$

Hence the total time for takeoff under these conditions is 12.8 seconds.

Example 3.3:	**Hassan and Umar are standing face to face on ice wearing ice skates. If Hassan apply a force of 10N [E] on Umar (Assume no other opposing force exists), what are their respective accelerations? If mass of Umar is 80 kg and Hassan is 50 kg.**

Given Data:

Hassan's Mass $m_H = 50$ kg

Umar's Mass $m_U = 80$ kg

Force F = 10 N [E]

To Find:

Hassan's acceleration $a_H = ?$

Umar's acceleration $a_U = ?$

Solution:

When no other opposing force exists, the action for exerted by Hassan on Umar is 10N [E], the acceleration produced in Umar a_U by Newton's second law of motion will be

$$a_U = \frac{F}{m_U} = \frac{10\ N\ [E]}{80\ kg} \quad \text{or} \quad a_U = 0.125\ \frac{kg\ m/s^2}{kg}\ [E]$$

therefore $\boxed{a_U = 0.125\ m/s^2\ [E]}$

The reaction force exerted by Umar on Hassan will be equal and opposite (i.e. -10 N [E], or 10 N [W].

The acceleration produced in Hassan a_H by Newton's second law of motion will be

$$a_H = \frac{F}{m_H} = \frac{-10\ N[E]}{50\ kg} \quad \text{or} \quad a_U = -0.2 \frac{kg\ m/s^2}{kg} [E]$$

or $a_U = -0.2\ m/s^2$ [E] therefore

therefore $\boxed{a_U = 0.2\ m/s^2\ [W]}$

Due to smaller mass Hassan will accelerate more than Umar.

Example 3.4: **A cricket ball of mass 0.163 kg has an initial velocity of – 36 m/s as it approaches a bat. The batsman hits the ball hard and the ball moves away from the bat with velocity of +47 m/s. (a) Determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is 1.6 ms, find the average force exerted on the ball by the bat.**

Given Data:

mass 'm' = 0.163 kg

initial velocity 'v_i' = - 36.2 m/s

final velocity 'v_f' = +47.0 m/s

time of contact 'Δt' = 1.6 ms = 1.6×10^{-3} s

Required:

Impulse applied 'J' = ?

Average force exerted F_{ave} = ?

Solution:

(a) According to the impulse-momentum relation $J = m\vec{v}_f - m\vec{v}_i$

Putting values J = (0.163 kg) (+47.0 m/s) – (0.163 kg) (–36.2 m/s)

J = 7.661 kg m/s + 5.9006 kg m/s = +13.5616 kg m/s

Hence $\boxed{J = +13.6\ Ns}$

(b) The average force can be calculated by using equation $J = \vec{F}_{ave} \times \Delta t$

$$\vec{F}_{ave} = \frac{J}{\Delta t} \quad \text{putting values} \quad \vec{F}_{ave} = \frac{+13.6\ Ns}{0.0016\ s}$$

Hence $\boxed{\vec{F}_{ave} = +8500\ N}$

Example 3.5: **In a nuclear reactor a neutron of mass 1 u (1 u = 1.66×10^{-27} kg) moving a velocity of 2,000 km/s to the right and a heavy water molecule mass 20.0 u moving with a velocity of 0.40 km/s to the left collide head-on. What are the velocities of the neutron and water molecule after the collision?**

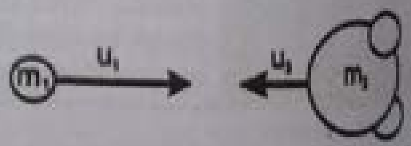
Given Data:

Mass of neutron m_1 = 1u

Mass of water molecule m_2 = 20u

Velocity of neutron before collision u_1 = 2000 km/s

Velocity of water molecule before collision u_2 = 0.40 km/s



To Find:

Velocity of neutron after collision v_1 = ?

Velocity of water molecule after collision v_2 = ?

Solution:

There is no need to convert 'u' into 'kg' as we only want to compare these values.

For head on elastic collision $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2}{(m_1 + m_2)} u_2$$

Putting values $$v_1 = \frac{(1u - 20\ u)}{(1u + 20\ u)} 2000\ km/s + \frac{2 \times 20\ u}{(1u + 20\ u)} 0.40\ km/s$$

Or $v_1 = -1809.52\ km/s + 0.76\ km/s$

Hence $\boxed{v_1 = -1808.76\ km/s}$

The negative sign shows that the neutron rebounds back after head on collision with the water molecule. Also for head on elastic collision

$$v_2 = \frac{2m_1}{(m_1 + m_2)} u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)} u_2$$

putting values $v_2 = \frac{2 \times 1u}{(1u + 20\,u)} 2000 \text{ km/s} - \frac{(1u - 20u)}{(1u + 20u)} 0.40 \text{ km/s}$

or $v_2 = 190.48 \text{ km/s} + 0.38 \text{ km/s}$

therefore $\boxed{v_2 = 190.86 \text{ km/s}}$

Example 3.6: **A cricket ball is hit and moves initially at an angle of 35° above the horizontal ground with a velocity of 25.0 m/s. (a) How high will the ball go? (b) How long will the ball be in the air? (c) What will be the range for this projectile?**

Given Data:

angle 'θ' = 35°
initial velocity 'v_o' = 25.0 m/s
Acceleration due to gravity 'g' = 9.8 m/s²

To Find:

(a) Maximum height 'H' = ?
(b) Time of flight 'T' = ?
(c) Horizontal range 'R' = ?

Solution:

(a) The maximum height H for projectile is mathematically written as

$$H = \frac{v_o^2 \sin^2\theta}{2g} \quad \text{putting values} \quad H = \frac{(25ms^{-1})^2 \times (\sin 35°)^2}{2(9.8ms^{-2})}$$

Therefore $\boxed{H = 10.5m}$

(b) Time of flight for Projectile is mathematically given as

$$T = \frac{2v_o \sin\theta}{g} \quad \text{putting values} \quad T = \frac{2 \times (25ms^{-1}) \times \sin 30°}{9.8ms^{-2}}$$

Therefore $\boxed{T = 2.93s}$

(c) The Horizontal Range R for projectile is mathematically written as

$$R = \frac{v_o^2 \sin^2\theta}{2g} \quad \text{putting values} \quad R = \frac{(25ms^{-1})^2 \times (\sin 2 \times 35°)}{(9.8ms^{-2})}$$

Therefore $\boxed{R = 59.9m}$

Text Book Exercises

Q.1 Select the correct answer of the following questions.

Choose the best possible answer

1. **A ball is thrown vertically upwards at 19.6 m/s. For its complete trip (up and back down to the starting position), its average speed is:**
   A. 19.6 m/s. B. 9.8 m/s. C. 6.5 m/s. D. 4.9 m/s.
2. **If you throw a ball downward, then its acceleration immediately after leaving your hand, assuming no air resistance, is**
   A. 9.8 m/s². B. more than 9.8 m/s²
   C. less than 9.8 m/s² D. Speed of throw is required for answer
3. **The time rate of change of momentum gives**
   A. Force B. Impulse C. Acceleration D. Power

4. **The area between the velocity-time graph is numerically equal to:**
A. Velocity B. Displacement C. Acceleration D. Time

5. **If the slope of velocity-time graph gradually decreases, then the body is said to be moving with:**
A. Positive acceleration B. Negative acceleration C. Uniform velocity D. Zero acceleration

6. **A 7.0-kg bowling ball experiences a net force of 5.0 N. What will be its acceleration?**
A. 35 m/s^2. B. 7.0 m/s^2 C. 5.0 m/s^2 D. 0.71 m/s^2

7. **SI unit of impulse is:**
A. kg ms^{-2} B. N s C. N s^{-1} D. N m

8. **A ball with original momentum +4.0 kg × m/s hits a wall and bounces straight back without losing any kinetic energy. The change in momentum of the ball is:**
A. + 4 N s B. - 4 N s C. + 8 N s^2 D. - 8 N s

9. **A body is traveling with a constant acceleration of 10 m s^2. If S_1 is the distance traveled in 1^{st} second and S_2 is the distance traveled in 2^{nd} second, which of the following shows a correct relation between S_1 and S_2?**
A. $S_1 = S_2$ B. $S_1 = 3\ S_2$ C. $S_2 = 3\ S_1$ D. $2S_2 = 3\ S_1$

10. **During projectile motion, the horizontal component of velocity:**
A. Changes with time B. Becomes zero C. Remains constant D. Increases with time

11. **A projectile is thrown horizontally from a 490m high cliff with a velocity of 100 ms^{-1}. The time taken by projectile to reach the ground is**
A. 2.5 s B. 5.0 s C. 7.5 s D. 10 s

12. **A projectile is launched at 45 to the horizontal with an initial kinetic energy E. Assuming air resistance to be negligible what will be the kinetic energy of the projectile when it reaches its highest point?**
A. 0.50 E B. 0.71 E C. 0.70 E D. E

13. **To improve the jumping record the long jumper should jump at an angle of**
A. 30° B. 45° C. 60° D. 90°

14. **Range of a projectile on a horizontal plane is same for the following pair of angles:**
A. 15° and 18° B. 43° and 47° C. 20° and 80° D. 52° and 62°

No:	Option	ANSWER	EXPLANATION
1.	B	9.8 m/s	$v_f = v_i + gt$ $\Rightarrow 0 = 19.6 - 9.8t$ $\Rightarrow -\frac{19.6}{9.8} = t$ $\Rightarrow t = 2\ s$ So, total time of travel on both sides T = 2 × 2 = 4s Distance to reach maximum height $S = vit + \frac{1}{2}gt^2$ $\Rightarrow S = (19.6)(2) + \frac{1}{2}(-9.8)(2)^2$ $\Rightarrow S = 39.2 - 19.6 = 19.6\ m$ So total distance covered on both sides $\Rightarrow$ S = 2 × 19.6 = 39.2 m Average speed is: $v_{av} = \frac{total\ distance}{total\ time} = \frac{39.2}{4} = 9.8\ m/s$
2.	A	9.8 m/s^2	Because after leaving hand, only force acting on ball is gravity or weight of the ball which produces gravitational acceleration.
3.	A	Force	$F = \frac{\Delta P}{\Delta t}$

4.	B	Displacement	Area under velocity-time graph is equal to displacement covered by body.
5.	Both A & B can be answers		Slope of v-t graph gives acceleration. If slope decreases, then its acceleration will decrease. If slope is positive and decreasing then it has decreasing positive acceleration and if slope is negative and slope is decreasing then it has decreasing negative acceleration.
6.	D	0.71 m/s^2	$a = \frac{F}{m} = \frac{5}{7} = 0.71\, m/s^2$
7.	B	Ns	
8.	D	-8 Ns	$P_i = 4$ kgm/s $P_f = -4$ kgm/s (as no energy is lost i.e. ball bounces back elastically) $\Delta P = P_f - P_i = -4 - 4 = -8$ kgm/s
9.	C	$S_2 = 3\,S_1$	$S_1 = \frac{1}{2}a\,t^2 = \frac{1}{2} \times 10 \times 1^2 = 5\,m$ & $S_{nth} = \frac{1}{2}a\,({t_{nth}}^2 - {t_{nth-1}}^2) \rightarrow S_2 = \frac{1}{2}(10)(2^2 - 1^2)$ $S_2 = 5\,(4-1) = 5 \times 3 = 3 \times 5 = 3S_1$ **Alternate Method** Let $v_i = 0$ (use $S = v_i t + \frac{1}{2}at^2$) $\Rightarrow s = \frac{1}{2}at^2$ $\Rightarrow s_1 = \frac{1}{2}a(t)^2$ $\Rightarrow s_1 = \frac{1}{2}a(1)^2$ $\Rightarrow s_1 = \frac{1}{2}a$ and distant after two seconds. $S' = \frac{1}{2}at_2^2$ $S' = \frac{1}{2}a(2)^2$ $S' = \frac{1}{2}a(4)$ Distance in 2[nd] second is $S_2 = s' - s_1 = \frac{1}{2}a(4) - \frac{1}{2}a$ $S_2 = \frac{1}{2}a(4-1)$ $S_2 = (\frac{1}{2}a)(3)$ $S_2 = S_1(3)$ $S_2 = 3S_1$
10.	C	Remains constant	As no force act on the projectile along horizontal.
11.	D	10 s	Use 2[nd] equation of motion $S = vit + \frac{1}{2}at^2$ For y axis, equation becomes $S = vi_y t + \frac{1}{2}a_y t^2$ ⇨ $S = (0)t + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2$ ⇨ $S = \frac{1}{2}gt^2$ ⇨ $t = \sqrt{\frac{2S}{g}} = \sqrt{\frac{2\,(490)}{9.8}} = \sqrt{\frac{980}{9.8}}$ ⇨ $t = \sqrt{100} = 10\,s$
12.	A	0.5 E	$K.E._{min} = K.E_i \cos^2\theta$ (at highest poinit) ⇨ $E' = E\,Cos(45)^2 = E\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{E}{2} = 0.5\,E$
13.	B	45°	Range is maximum when angle of projection is 45°.
14.	B	43° and 47°	Ranges are equal for angles of projection if their sum is 90° i.e. they are complementary angles

Short Answers of the Exercise

Q.2 Give a short response to the following questions.

Q.1 If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?

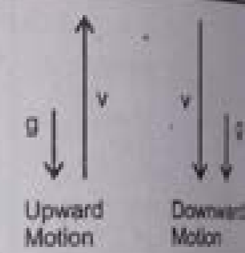
Ans: This is due to relative motion between the two trains.

Explanation:
When you are sitting in faster train then you are in rest w.r.t. your train. So, when you look outside through the window then poles, trees etc will appear to you moving in the backward direction (as they a coming closer to you). Similarly, when you cross the slower train then that train will also appear to you as losing the ground i.e. moving in the backward direction, which will indicate that other train is slower than yours.

Q.2 Can the velocity of a body reverse the direction when acceleration is constant? If you think so, give an example.

Ans: *Yes*, it can be possible.

Example:
When a body is thrown vertically upward, its velocity goes on decreasing due to gravity and becomes zero at the maximum height. After that, it will reverse its direction of velocity, but the magnitude acceleration remains constant during whole flight (*i.e 9.8 m/s^2*).



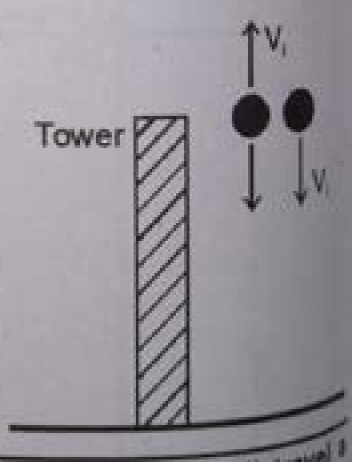
Q.3 When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

Ans: When we stand on the ground then we exert force on ground equal to our weight.

$$F = W = mg$$

This force acts as action on ground. According to newton's 3rd law, ground exerts equal but opposite reaction. But this reaction is just sufficient to keep us standing on ground.
This reaction force can not make us to rise in air. For that purpose, upward force must be greater than our weight whih is acting downward.

Q.4 A man standing on the top of a tower throws a ball vertically up with certain velocity. He also throws another ball vertically down with the same speed. Neglecting air resistance, which ball will hit the ground with higher speed?



Ans: Both the balls hit the ground with ***same speed***.

Explanation:
The ball which is thrown vertically up with velocity v_i will have same velocity v_i when it reaches back to the top of tower. So the two balls have ***same downward velocity*** at top of tower. Hence they hit the ground with same final velocity.

$$V_f^2 = v_i^2 + 2\,gh$$

$$V_f = \sqrt{v_i^2 + 2\,gh}$$

As v_i, h and g is same for two balls therefore v_f is also same.

Q.5 The cricket coach explains that the follow-through with the shot will make the ball travel a greater distance. Explain the reasoning in terms of the impulse-momentum theorem.

Ans: Follow through is advised to increase the momentum and hence increases velocity of the ball to travel a greater distance.

Explanation:
During the follow through, the bat is in contact with the ball for a longer time. According to impulse - momentum theorem, the change in momentum is equal to the impulse.

$$\vec{J} = \vec{F} \times \Delta t \quad \Rightarrow \quad J = \Delta p$$

In order to impart more momentum into the ball so that the ball travels a greater distance, a greater impulse is needed which is achieved by a longer time of contact between bat and ball.

Q.6 When you release an inflated but untied balloon, why does it fly across the room?

Ans: When you release an inflated but untied balloon, it flies across the room to conserve the linear momentum.

Explanation:

In an inflated balloon, air is at higher pressure then atmospheric pressure. When it is released, air escapes from the balloon that carries momentum. To conserve the momentum, the balloon acquires momentum which is exactly opposite to the momentum of the escaping air under the effect of reaction force i.e. $P_{gases} = P_{Balloon}$

Q.7 Modern cars are not rigid but are designed to have 'crumple zones' (irregular fold) that collapse upon impact. What is the advantage of this new design?.

Ans: Crumple zones like bumpers are made in the cars to reduce the force during an accident.

Explanation:

We know that

$$\vec{J} = \vec{F} \times \Delta t$$

$$\Rightarrow \quad F = \frac{J}{\Delta t}$$

During an accident, crumple zones like bumpers collapse, that will increase the collision time (Δt) and hence reduce the impact force. This will make passengers safer.

We can also say that crumple zone absorbs the energy during head-on collision, passenger will feel lesser jerk during collision and remain safe.

Q.8 Why we can hit a long sixer in a cricket match rather than if we toss a ball for our selves?

Ans: it may be due to:

i. In Cricket match, the time of collision between ball and bat decrease which increase the striking force. As a result a ball go larger distance

$$F = \frac{J}{\Delta t}$$

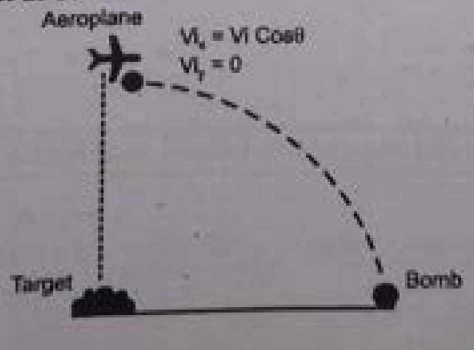
So, when batsman hit pitched ball, Δt is small, so force will be greater.

ii. The motion of the ball when hit for six is like a projectile motion. A projectile has maximum range when it is hit at angle of 45° with the ground. In a cricket match, this angle is possible easily to achieve but it is difficult to achieve when we toss the ball for ourselves.

iii. When opponent bowler throws the ball usually with a greater force and we strike it with proper follow through then it will increase the collision time and hence increases the momentum imparted to ball. This will increase the speed of the ball sufficiently to go for longer distances.

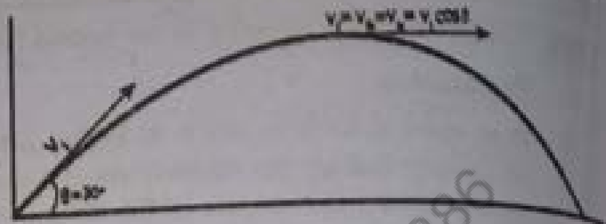
Q.9 An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.

Ans: The bomb has the same velocity as that of the aero-plane when it is dropped. If the bomb is dropped when the aero-plane is vertically above the target, it will strike a point ahead of the target due to constant horizontal velocity component and inertia, the bomb misses the target. The bomb moves like a projectile as shown in fig. It will not hit the target but it will hit at T.



Q.10 Calculate the angle of projection for which kinetic energy at the summit is equal to one-fourth of its kinetic energy at point of projection.

Ans: $K.E._{min}$ of the projectile at highest point can be written as:



$$K.E._{min} = \frac{1}{2}mv_i^2 \cos^2\theta$$

$$\Rightarrow K.E._{min} = K.E_i \cos^2\theta \qquad \left[\because \frac{1}{2}mv_i^2 = K.E_i\right]$$

Given condition:

$$K.E._{min} = \frac{1}{4}K.E_i$$

Put value of $K.E._{min}$, so

$$K.E._i \cos^2\theta = \frac{1}{4}(K.E_i)$$

$$\Rightarrow \quad \cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 60^\circ$$

Q.11 For any specific velocity of projection, the maximum range is equal to four times of the corresponding height. Discuss.

Ans: Maximum range of projectile is at $\theta = 45^\circ$ and it is given by:

$$R_{max} = \frac{v_i^2}{g}$$

Height of projectile is:

$$\boxed{H = \frac{1}{4}R_{max}}$$

At angle of projection $\theta = 45^\circ$

$$H = \frac{v_i^2 \sin^2(45)}{2g}$$

$$\Rightarrow \quad H = \frac{v_i^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad H = \frac{v_i^2}{2g}\left(\frac{1}{2}\right) = \frac{v_i^2}{4g}$$

$$H = \frac{1}{4}\left(\frac{v_i^2}{g}\right) \qquad \left[R_{max} = \frac{v_i^2}{g}\right]$$

$$\Rightarrow \quad H = \frac{1}{4}R_{max}$$

$$\Rightarrow \quad R_{max} = 4 \times H$$

Q.12 What is the angle for which the maximum height reached and corresponding range are equal?

Ans: **Given Data:**

Maximum height = horizontal range

To Find:

Angle of projection = θ = ?

Calculations:

As Maximum height $= H = \frac{v_i^2\sin^2\theta}{2g}$

Range of projectile $= R = \frac{v_i^2\sin 2\theta}{g}$

According to given condition,

$$H = R$$

$$\frac{v_i^2\sin^2\theta}{2g} = \frac{v_i^2\sin 2\theta}{g}$$

OR $\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$

OR $\sin\theta = 4\cos\theta$

OR $\frac{\sin\theta}{\cos\theta} = 4$

OR $\tan\theta = 4$

OR $\theta = \tan^{-1}(4)$

OR $\boxed{\theta = 76^\circ}$

Comprehensive Questions

Q3. Give a short response to the following questions.

1. Explain displacement – time graph and velocity – time graph. In each type give brief details along with appropriate diagram for illustration.

Ans: See Q # 8, 9 and 10

2. Apply Newton's Laws to explain the motion of objects in a variety of context.

Ans: See Q # 12

3. What is linear momentum? Derive and state Newton's second law in terms of linear momentum.

Ans: See Q # 13 and 15

4. State and explain law of conservation of linear momentum for an isolated system of bodies.

Ans: See Q # 17 and 18

5. Define elastic and inelastic collisions. Give examples in each case. Derive mathematical equations for calculating the final velocities of the elastically colliding bodies in one dimension.

Ans: See Q # 19, 20

6. What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile.

Ans: See Q # 23

7. What is maximum height and time of flight for projectile? Derive mathematical equations for Maximum height attained and time of flight.

Ans: See Q # 2

8. What is range of a projectile. State in which condition the range will be maximum if speed of projection is kept constant in a uniform gravitational field. Also show that there are two projection angles for the same range.

Ans: See Q # 24 and 25

NUMERICAL QUESTIONS

1. An object is falling freely under gravity. How much distance will it travel in 2nd and 3rd second of its journey?

Given data:

$S_{2nd} = ?$
$S_{3rd} = ?$

To Find:

To find the distance in a specific second for a freely falling body, we have:

$$S_{nth} = \frac{1}{2}g({t_{nth}}^2 - ({t_{(n-1)\,th}}^2)$$

Distance covered in 2nd second is:

$$S_{2nd} = \frac{1}{2}9.8(2^2 - 1^2)$$
$$S_{2nd} = 4.9(4 - 1) = 4.9\,(3) = 14.7\,m = 15m$$

Distance covered in 3rd second is:

$$S_{2nd} = \frac{1}{2}9.8(3^2 - 2^2)$$
$$S_{2nd} = 4.9(9 - 4) = 4.9\,(5) = 24.5\,m = 25m$$

2. A helicopter is ascending vertically at a speed of 19.6 m s^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

Given data:

Initial velocity of the helicopter = vi = 19.6 ms^{-1} (upward)
Net vertical distance covered by the stone = s = -156.8m (downward)

(There is negative sign, being displacement opposite to initial velocity)

Acceleration due to gravity = g = -9.8 ms^{-2}

To Find:

Time = t = ?

Using the following equation of motion

$S = v_i t + \frac{1}{2} gt^2$, putting the values, we get

$-156.8 = 19.6t + \frac{1}{2}(-9.8)\,t^2$

$-156.8 = 19.6t - 4.9t^2$

$4.9\,t^2 - 19.6\,t^2 - 156.8 = 0$

$4.9(t^2 - 4t - 32) = 0$

$t^2 - 4t - 32 = 0$, making factors, we get

$t^2 - 8t + 4t - 32 = 0$

$t(t - 8) + 4(t - 8) = 0$

$(t - 8)\,(t+4) = 0$

or $t - 8 = 0$, $t = 8$ sec OR $t + 4 = 0$, $t = -4$ sec

But time cannot be negative, thus

t = 8 sec (i.e. time taken by stone)

3. A car moving at 20.0 m/s (72.0 km/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 70 kg in each of the following cases. (a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 2.0 ms. (b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 45 ms, bringing the passenger to rest.

Solution: Final velocity = $v_f = 0$

Mass of passenger = m = 70 kg

To Find:

(a) Average force on passenger for
$\Delta t_1 = 2ms = 2 \times 10^{-3}$ sec

(b) Average force on passenger for
$\Delta t_2 = 45\ ms = 45 \times 10^{-3}$ sec

Calculation:

(a) $J = F_{ave} . \Delta t$

$\Rightarrow \quad F_{ave} = \dfrac{J}{\Delta t}$ ——————(1)

Also $J = \Delta P = mv_f - mv_i = (70 \times 0 - 70 \times 20)$ N.S

$\Rightarrow \quad J = -1400$ N.S

Now putting J = − 1400 N.S and $\Delta t = \Delta t_1 = 2 \times 10^{-3}$ sec in equation (1), we get,

$F_{ave} = \left(\dfrac{-1400}{2 \times 10^{-3}}\right) N = -700 \times 10^3\ N = -7 \times 10^5\ N$ in magnitude, $F_{av} = 7 \times 10^5\ N$

(b) Putting J = -1400 N.S and $\Delta t = \Delta t_2 = 45 \times 10^{-3}$ sec. in equation (1), we get,

$F_{ave} = \left(\dfrac{-1400}{45 \times 10^{-3}}\right) N = -31.1 \times 10^3\ N$

$\Rightarrow$ **$Fave = -3.11 \times 10^4\ N$** In magnitude, $F_{ave} = 3.11 \times 10^4\ N$

4. A 0.4 kg ball traveling with the speed of 15 m s strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s, what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall? (12 kg m s^{-1}, 266.7 N)

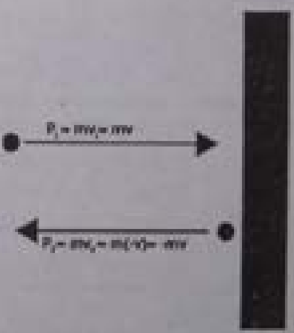
Solution: Mass of ball = m = 0.4 kg.

Velocity of ball = v = 15 m/s

Time interval = Δt = 0.045 s

(a) Momentum imparted to the wall = ΔP = ?

(b) Average force exerted on the wall = F_{av} = ?



(a) For elastic collision, the change in momentum of the ball due to force applied by wall is given by,

$$\Delta P = P_f - P_i = mv - (mv)$$
$$= -mv - mv$$
$$= -2mv$$

So, the momentum imparted to wall by ball is:

$\Delta P = +\ 2mv$

$\Rightarrow \quad \Delta P = 2 \times 0.4 \times 15 = 12$ kg m/s

(b) $F_{av} = \dfrac{\Delta P}{t}$

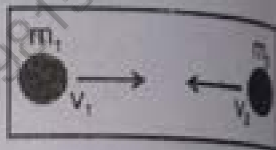
$\Rightarrow \quad F = \frac{16}{0.045} = 266.7\ N$

5. One ball of mass 0.600 kg traveling 9.00 m/s to the right collides head on elastically with a second ball of mass 0.300 kg traveling 8.00 m/s to the left. After the collision, what are the velocities after collision? (– 2.33 m/s (2.33 m/s to right) and 14.67 m/s (14.76 m/s to left))

Solution:

Mass of 1st ball = m_1 = 0.600kg
u_1 = Speed of 1st ball = 9.00m/ sec
Mass of 2nd ball = m_2 = 0.300kg
u_2 = Speed of 2nd ball = – 8mm/sec

The negative sign is used because both bodies moves in opposite direction.



Given Data:

(a) Velocity of 1st ball after collision = v_1'=?
(b) Velocity of 2nd ball after collision = v_2'=?

Calculation:

(a) We know that,

$$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad \text{———} (1)$$

Similarly, $$V_2 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \quad \text{———} (2)$$

Putting the values in equation (1), we get,

$$V_1 = \left[\left(\frac{0.600 - 0.300}{0.600 + 0.300}\right)9.00 + \left(\frac{2 \times 0.300 \times (-8)}{0.600 + 0.300}\right)\right]m/sec$$

$$\Rightarrow \quad V_1 = \left[\left(\frac{0.3 \times 9}{0.9}\right) - \left(\frac{4.8}{0.9}\right)\right]m/sec = (3 - 5.33)m/sec$$

$$\Rightarrow \quad V_1 = -2.33m/sec$$

(b) Putting the values in equation (2), we get,

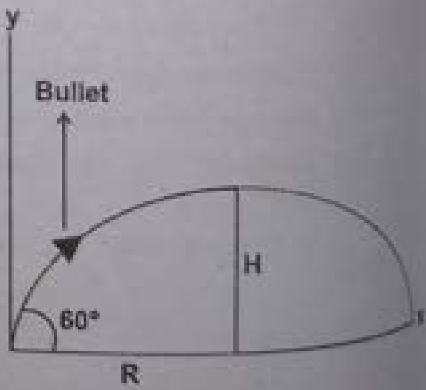
$$V_2 = \left[\left(\frac{0.300 - 0.600}{0.600 + 0.300}\right)(-8) + \left(\frac{2 \times 0.600 \times 9}{0.600 + 0.300}\right)\right]m/sec$$

$$\Rightarrow \quad V_2 = \left[\left(\frac{-0.3 \times (-8)}{0.9}\right) + \left(\frac{10.8}{0.9}\right)\right]m/sec = (2.666 + 12)m/sec$$

$$V_2 = (2.67 + 12)m/sec = 14.67m/sec$$

$$\Rightarrow \quad V_2 = 14.67m/sec$$

6. In a wedding a bullet is fired in air at a speed of 500 m/s making an angle of 60° with horizontal from an AK 47 rifle. (a) How high will the bullet rise? (b) What time would it take to reach ground? (c) How far would it go? (Ignore air resistance) ((a) 9,560 m (b) 88.3 s (c) 22,078 m)



Solution:

Given Data:

Initial speed of bullet in air = v_i = 500 m/sec
Angle of projection = θ = 60°

Required:

(a) Maximum height = H = ?
(b) Time of flight = T = ?
(c) Horizontal range = R = ?

Calculation:

(a) We know that, the maximum height is given by,

$$H = \frac{v_i^2 \sin^2\theta}{2g} \quad \text{———} (1)$$

Putting the values in equation (1), we get,

$$H = \left[\frac{(500)^2 \times (\sin 60^\circ)^2}{2 \times 9.8}\right] m = \left(\frac{250000 \times 0.75}{19.6}\right) m$$

⇒ $\boxed{H = 9566.3\ m}$

(b) We know that, the time of flight is given by,

$$T = \frac{2v_i \sin\theta}{g} \quad \_\_\_\_\_\_\_\_ (2)$$

Putting the values in equation (2), we get,

$$T = \left(\frac{2 \times 500 \times \sin 60}{9.8}\right) sec = \left(\frac{1000 \times 0.866}{9.8}\right) sec$$

⇒ $T = \left(\frac{866}{9.8}\right) sec = 88.36 sec = 88.4\ sec$

⇒ $\boxed{T = 88.4 sec}$

(c) We know that, the horizontal range is given by,

$$R = \frac{v_i^2 \sin^2\theta}{g} \quad \_\_\_\_\_\_\_\_ (3)$$

Putting the values in equation (3), we get,

$$R = \frac{(500)^2 \times \sin(2 \times 60^\circ)}{9.8} = \left[\frac{250000 \times \sin(120^\circ)}{9.8}\right]$$

⇒ $R = \left[\frac{250000 \times 0.866}{9.8}\right] m = 22091.8 m$

⇒ $\boxed{R = 22091.8 m}$

7. The catapult hurls a stone of mass 32.0 g with a velocity of 50.0 m/s at a 30.0° angle of elevation. (a) What is the maximum height reached by the stone? (b) What is its range? (c) How long has the stone been in the air when it returns to its original height? ((a) 31.87 m (b) 5.1 s (c) 220.8 m)

Given Data:

Mass of stone = m = 32 kg

Initial velocity of stone = vi = 50m/sec

Angle of elevation = θ = 30°

Required:

(a) Maximum height = H = ?

(b) Horizontal range = R = ?

(c) Time of glight = T = ?

Calculation:

(a) We know that maximum height is given by,

$$H = \frac{v_i^2 \sin^2\theta}{2g} \quad \_\_\_\_\_\_\_\_ (1)$$

Putting the values in equation (1), we get,

$$H = \left[\frac{(50)^2 \sin(30^\circ)^2}{2}\right] m = \left[\frac{2500 \times (0.5)^2}{19.6}\right]$$

⇒ $H = \left[\frac{2500 \times 0.25}{19.6}\right] m = 31.88 m$

⇒ $\boxed{H = 31.88 m}$

(b) We know that horizontal range is given by:

$$R = \frac{v_i^2 \sin^2\theta}{g} \quad \_\_\_\_\_\_\_\_ (2)$$

Putting the values in equation (2), we get,

$$R = \frac{50^2 \times \sin(2 \times 30^\circ)}{9.8} = \left[\frac{2500 \times 60^\circ}{9.8}\right]$$

$\Rightarrow$ $R = \left[\frac{2500 \times 0.866}{9.8}\right] m = 220.9m$

$\Rightarrow$ $\boxed{R = 220.9m}$

(c) We know that time of flight is given by,

$T = \frac{2v_i \sin\theta}{g}$ ______ (3)

Putting the values in equation (3), we get,

$T = \frac{2 \times 50 \times \sin(30°)}{9.8} = \left(\frac{100 \times 0.5}{9.8}\right) sec$

$\Rightarrow$ $\boxed{T = 5.10 \text{ sec}}$

Additional Conceptual Short Questions With Answers

1. Why does a cricket player retrace his hands backward while catching the ball ?

Ans: Player does so because it will allow him to have greater time for change in momentum.

Since $F = \frac{\Delta P}{\Delta t}$ (Δt becomes greater) $\Rightarrow F \propto \frac{1}{\Delta t}$

$\Rightarrow$ So ball will exert a lesser force on his hands.

2. Show that relation between maximum height and time of flight of projectile is: $H = \left(\frac{gT^2}{8}\right)$?

Ans: Maximum height of projectile is:

$$H = \frac{v_i^2 \sin^2\theta}{2g} \qquad (1)$$

And its time of flight is:

$$T = \frac{2 v_i \sin\theta}{g}$$

$$\Rightarrow (T)^2 = \left(\frac{2v_i \sin\theta}{g}\right)^2$$

$$T^2 = 4\left(\frac{v_i^2 \sin^2\theta}{g^2}\right)$$

$$T^2 = \frac{4}{g} \times 2\left(\frac{v_i^2 \sin^2\theta}{2g}\right)$$

$$T^2 = \frac{8}{g}(H) \qquad \text{[Put value from eq (1)]}$$

$$\Rightarrow \boxed{H = \left(\frac{gT^2}{8}\right)}$$

3. Show that range of projectile on moon is 6 times greater than its range on the earth for same velocity and angle of projection θ?

Ans: Let range of projectile on earth is:

$$R = \frac{v_i^2}{g} \sin 2\theta$$

Let R′ be the range of projectile on the moon,

$$R' = \frac{v_1^2}{g'} \sin 2\theta$$

Since value of gravitational acceleration on the moon is:

$$g' = \frac{g}{6}$$

$$\Rightarrow \quad R' = \frac{v_1^2}{\left(\frac{g}{6}\right)} (\sin 2\theta)$$

$$R' = 6\left[\frac{v_1^2}{g} \sin 2\theta\right]$$

$$\boxed{R' = 6R}$$

So range of projectile on the moon is 6 times greater than its value on the earth.

4. The horizontal range of projectile is 4 times of its maximum height. What is its angle of projection?

Ans: Condition $\Rightarrow R = 4H$

$$\Rightarrow \quad \frac{v_1^2}{g} \sin 2\theta = 4\frac{v_1^2 \sin^2\theta}{2g}$$

$$\Rightarrow \quad \sin 2\theta = 2\sin^2\theta$$

$$2\sin\theta \cos\theta = 2\sin^2\theta$$

$$\Rightarrow \quad \cos\theta = \sin\theta$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 1$$

$$\Rightarrow \quad \tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$

5. In long jump, what factors determine the span of the jump?

Ans: Long jump is the example of projectile motion. So span of the jump is like the range of projectile i.e.,

$$R = \frac{v_1^2}{g} \sin 2\theta$$

So span of jump depends upon angle of projection (i.e. angle with horizontal and initial velocity of the athlete. This is why an athlete starts running before he takes the jump.

6. The angle of projection of a projectile is 30°, then find the ratio between its maximum height and range of projectile?

Ans: If $\theta = 30^\circ$

Then

$$R = \frac{v_1^2}{g} \sin 2(30^\circ)$$

$$R = \frac{v_1^2}{g} \sin 60^\circ$$

$$R = \frac{v_1^2}{g}\left(\frac{\sqrt{3}}{2}\right)$$

$$H = \frac{v_1^2 \sin^2(30)}{2g}$$

$$H = \frac{v_1^2\left(\frac{1}{2}\right)^2}{2g}$$

$$H = \frac{v_1^2}{8g}$$

So ratio between horizontal range and height is:

$$\frac{R}{H}=\frac{\frac{v_1^2}{g}\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{v_1^2}{g}\right)\left(\frac{1}{8}\right)}=\frac{\left(\frac{v_1^2}{g}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{v_1^2}{g}\right)\left(\frac{1}{8}\right)}$$

$$\Rightarrow \quad R:H=\frac{\sqrt{3}}{2}\times\frac{8}{1} \Rightarrow \quad R:H=4\sqrt{3}$$

$$\boxed{R:H=4\sqrt{3}:1}$$

7. Two balls are projected in directions at 30° and 60° with the horizontal. If both attained the same height, then find ratio of their initial speeds?

Ans: We know that maximum height of projectile is:

$$H=\frac{v_i^2\, \text{Sin}^2\theta}{2g}$$

When $\theta = 30°$ then

$$H_1=\frac{v_1^2}{2g}(\text{Sin } 30)^2$$

$$H_1=\frac{v_1^2}{2g}\left(\frac{1}{2}\right)^2$$

$$H_1=\frac{v_1^2}{2g}\left(\frac{1}{4}\right)$$

When $\theta = 60°$ then

$$H_2=\frac{v_2^2}{2g}(\text{Sin } 60)^2$$

$$H_2=\frac{v_2^2}{2g}\left(\frac{\sqrt{3}}{2}\right)^2$$

$$H_2=\frac{v_2^2}{2g}\left(\frac{3}{4}\right)$$

From the given condition

$$H_1=H_2$$

$$\frac{v_1^2}{2g}\left(\frac{1}{4}\right)=\frac{v_2^2}{2g}\left(\frac{3}{4}\right)$$

$$v_1^2=3v_2^2$$

$$\frac{v_1^2}{v_2^2}=\frac{3}{1}$$

$$\frac{v_1}{v_2}=\frac{\sqrt{3}}{1} \Rightarrow \boxed{v_1:v_2=\sqrt{3}:1}$$

8. Can there be any acceleration when a body is moving with constant speed?

Ans: Yes a body can have acceleration when it is moving with constant speed and its direction is changing. For example, when a body is moving along a circular path with constant speed, it will have acceleration due to change of direction of velocity at every point.

9. Newton's first law can be derived from second law of motion. Explain.

Ans: According to Newton's 2nd law of motion

$$F = ma$$

If $\quad F = 0$

Then $\quad ma = 0$

As $\quad m \neq 0 \quad$ Therefore $\quad a = 0$

i.e., In the absence of external force (F = 0), a body at rest remains at rest (a = 0), a body in motion, continues its motion in a straight line with constant velocity (a = 0).

10. At which angle of projection, the range of projectile is half of its maximum range ?

Ans: The range of projectile is given by

$$R = \frac{1}{2}(R_{max})$$

$$\frac{V_i^2 \sin 2\theta}{g} = \frac{1}{2}\left(\frac{V_i^2}{g}\right)$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$2\theta = 30^\circ$$

$$\theta = 15^\circ$$

Hence, the range of projectile is half than its maximum range at an angle of 15°.

11. Two blocks of masses m_1 and m_2 are resting on a frictionless horizontal surface and are in good contact with each other. A force F acts on m_1, as shown in fig 3.17. Derive equation to find the magnitudes of effective forces acting on m_1 and m_2 respectively.

Fig.3.17

Ans: Let "a" be the acceleration produced in the whole system due external applied force "F" and is given by:

$$F = (m_1 + m_2)a \Rightarrow a = \frac{F}{(m_1 + m_2)} \quad \ldots\ldots (i)$$

Since, the masses move together, acceleration is same for both of them. Let F_2 be the effective force acting on m_2 due to m_1 and is given by:

$$F_2 = m_2 a = m_2\left[\frac{F}{(m_1 + m_2)}\right]$$

or $$F_2 = \left[\frac{m_2}{(m_1 + m_2)}\right]F \quad \ldots\ldots (ii)$$

According to Newton's 3rd law, m_1 and m_2 exert the same force in magnitude but opposite in direction as shown. Thus on m_1 now two forces are acting. Let F_1 be the net force acting on m_1 and is given by:

$$F_1 = F - F_2 = F - \left[\frac{m_2}{(m_1 + m_2)}\right]F = \left[\frac{m_1}{(m_1 + m_2)}\right]F \quad \ldots\ldots (iii)$$

Eq. (ii) and Eq. (iii) can be used to find the magnitudes of effective forces acting on m_1 and m_2 in such cases.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **When body is in motion its ____________ always changes.**
A. Velocity B. Acceleration C. Position vector D. Momentum

2. **Instantaneous and average velocities become equal when body__________.**
A. has zero acceleration B. has uniform acceleration
C. has variable acceleration D. moves in a circle

3. **Which of the following pair has same direction always?**
A. force, displacement B. force, velocity
C. force, acceleration D. force, momentum

4. **Area under V-t graph gives _________.**
A. Speed B. Distance C. Acceleration D. Momentum

5. **Rate of change of momentum of a body is equal to _________.**
A. force B. Distance C. Acceleration D. Momentum

6. **Angle between the velocity and acceleration at the maximum height of the projectile is**
A. 0^0 B. 180^0 C. 90^0 D. 45^0

7. **At angle of projection of 45^0, height of projectile will equal to ____times of range**
A. 4 B. 1/4 C. 2 D. 1/2

8. **If velocity of moving body is doubled then stopping distance will become _______.**
A. Two times B. four times C. Three times D. remain same

9. **When velocity of moving object is doubled then which of following quantity becomes double ____.**
A. Acceleration B. Kinetic Energy C. Potential energy D. Momentum

10. **If range of projectile is half of its maximum range then angle of projection is _______.**
A. 30^0 B. 22.5^0 C. 15^0 D. 45^0

11. **The unit of impulse is _______.**
A. N B. Js C. Ns D. Nm

12. **One dyne is equal to _______.**
A. 10^3 N B. 10^5 N C. 10^{-3} N D. 10^{-5} N

13. **If projectile is projected at $\theta = 45^0$ with initial velocity V_i than velocity at highest point is ____**
A. 0 B. $\frac{V_i}{2}$ C. $\frac{V_i}{\sqrt{2}}$ D. $\frac{V_i}{4}$

14. **The direction of acceleration is always along the direction of _______.**
A. velocity B. Momentum C. force D. None of these

15. **Distance covered by a freely falling body in 2 seconds will be _______.**
A. 19.0 m B. 19.2 m C. 19.4 m D. 19.6 m

16. **A car starts from rest and covers a distance of 100 m in one second with uniform acceleration. Its acceleration is _______.** (ANNUAL 2017)
A. 100 m/s^2 B. 200 m/s^2 C. 300 m/s^2 D. 50 m/s^2

17. **A ball rolls off the edge of a table. The horizontal component of the ball's velocity remains constant during its entire trajectory because _______.** (ANNUAL 2017)
A. The net force acting on the ball is zero
B. The ball is not acted upon by a force in the horizontal direction
C. The ball is acted upon by a force in the horizontal direction
D. The ball is acted upon by a force in the only vertical direction

18. Motorcycle safety helmet extends the time of collision and decreases ______. (ANNUAL 2017)

A. Impulse B. Change of collision C. Force acting D. Velocity of Vehicle

19. A brick of mass 2 kg is dropped from a rest position 5 m above the ground. What is its velocity at a height of 3 m above the ground? (ANNUAL 2017)

A. 12.4 m/s B. 6.3 m/s C. 7 m/s D. 1.2 m/s

20. The motion and rest are: (ANNUAL 2018)

A. Discrete B. Random C. Absolute D. Relative

21. The change in position of a body from initial position to final position is called: (ANNUAL 2018)

A. Displacement B. Acceleration C. Position vector D. Velocity

22. The notation delta (Δ) is used to represent a: (ANNUAL 2018)

A. Small change B. Big change C. Zero change D. Very small change

23. When a block of wood of mass 2 kg is pushed along a horizontal flat surface of a bench, the force of friction is 4N. When the block is pushed along the bench with a force of 10N, it moves with a constant: (ANNUAL 2019)

A. Speed of $5ms^{-1}$ B. Acceleration of $3ms^{-2}$ C. Acceleration of $5ms^{-2}$ D. Speed of $3ms^{-1}$

24. A projectile is thrown so that it travels a maximum range of 1000m. How high will it rise? (ANNUAL 2019)

A. 400m B. 500m C. 1000m D. 250m

25. A car takes 1 hour to travel 100 km along a main road and then ½ hour to travel 20 km along a side road. What is the average speed of the car for the Whole Journey? (ANNUAL 2019)

A. $60kmh^{-1}$ B. $70kmh^{-1}$ C. $80kmh^{-1}$ D. $100kmh^{-1}$

26. Rate of change of velocity is called ______. (ANNUAL 2019)

A. Speed B. Distance C. Acceleration D. Displacement

Answers Key

1.	C	2.	A	3.	C	4.	B	5.	A
6.	C	7.	B	8.	B	9.	D	10.	C
11.	C	12.	D	13.	C	14.	C	15.	D
16.	B	17.	B	18.	C	19.	B	20.	D
21.	A	22.	A	23.	B	24.	C	25.	C
26.	C								

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 Choose the correct answer from the given options. (1 x 6 = 6)

SECTION – A

1. **A ball is thrown vertically upwards at 19.6 m/s. For its complete trip (up and back down to the starting position), its average speed is:**
   (A) 19.6 m/s. (B) 9.8 m/s. (C) 6.5 m/s. (D) 4.9 m/s.
2. **A projectile is thrown horizontally from a 490m high cliff with a velocity of 100 ms–1. The time taken by** projectile to reach the ground is
   (A) 2.5 s (B) 5.0 s (C) 7.5 s (D) 10 s
3. **If range of projectile is half of its maximum range then angle of projection is _______.**
   (A) 30^0 (B) 22.5^0 (C) 15^0 (D) 45^0
4. **If the slope of the Velocity Time Graph remains constant then body is moving with:**
   (A) Uniform Velocity (B) Negative Variable Acceleration
   (C) Variable Acceleration (D) Uniform Acceleration
5. **If a force of 10 N acts on a body of mass 5 kg for one second, what is its rate of change of momentum?**
   (A) 10 $kgms^{-2}$ (B) 50 kg ms^{-2} (C) 5 kg ms^{-2} (D) 2 kg ms^{-2}
6. **If the body of mass 2 kg moving with 15 ms^{-1} collides with stationary body of same mass, then after elastic collision the 2nd body will move with the velocity of _______**
   (A) 15m/s (B) 30m/s (C) Zero m/s (D) None of these

Question.No.2 Give short answers of followings: (3 x 7 = 21)

SECTION – B

(i) Show that final velocity of the projectile at any instant is $v_f = \sqrt{v_i^2 + g^2t^2 - 2\, v_i\, gt \sin\theta}$

(ii) On a highway a car of mass 1500 kg is stopped at traffic signal. A pickup of mass 2000 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0 m/s. How fast was the pickup going just before the collision?

(iii) Define impulse how it is related to momentum?

(iv) Modern cars are not rigid but are designed to have 'crumple zones' (irregular fold) that collapse upon impact. What is the advantage of this new design?

(v) An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.

(vi) A 0.4 kg ball traveling with the speed of 15 m s strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s, what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall?

(viii) Differentiate between elastic and inelastic collision. Give example of each. Why most of collisions are inelastic?

Question.No.3 Extensive Questions. (13)

SECTION – C

(a) For elastic collision of two bodies, show that speed of approach of colliding bodies is equal to their speed of separation. (07)

(b) A projectile is thrown with initial velocity v_i making an angle θ with the horizontal. Find its (06)
(a) Maximum height (b) Time of flight (c) Range

The End

CHAPTER
4

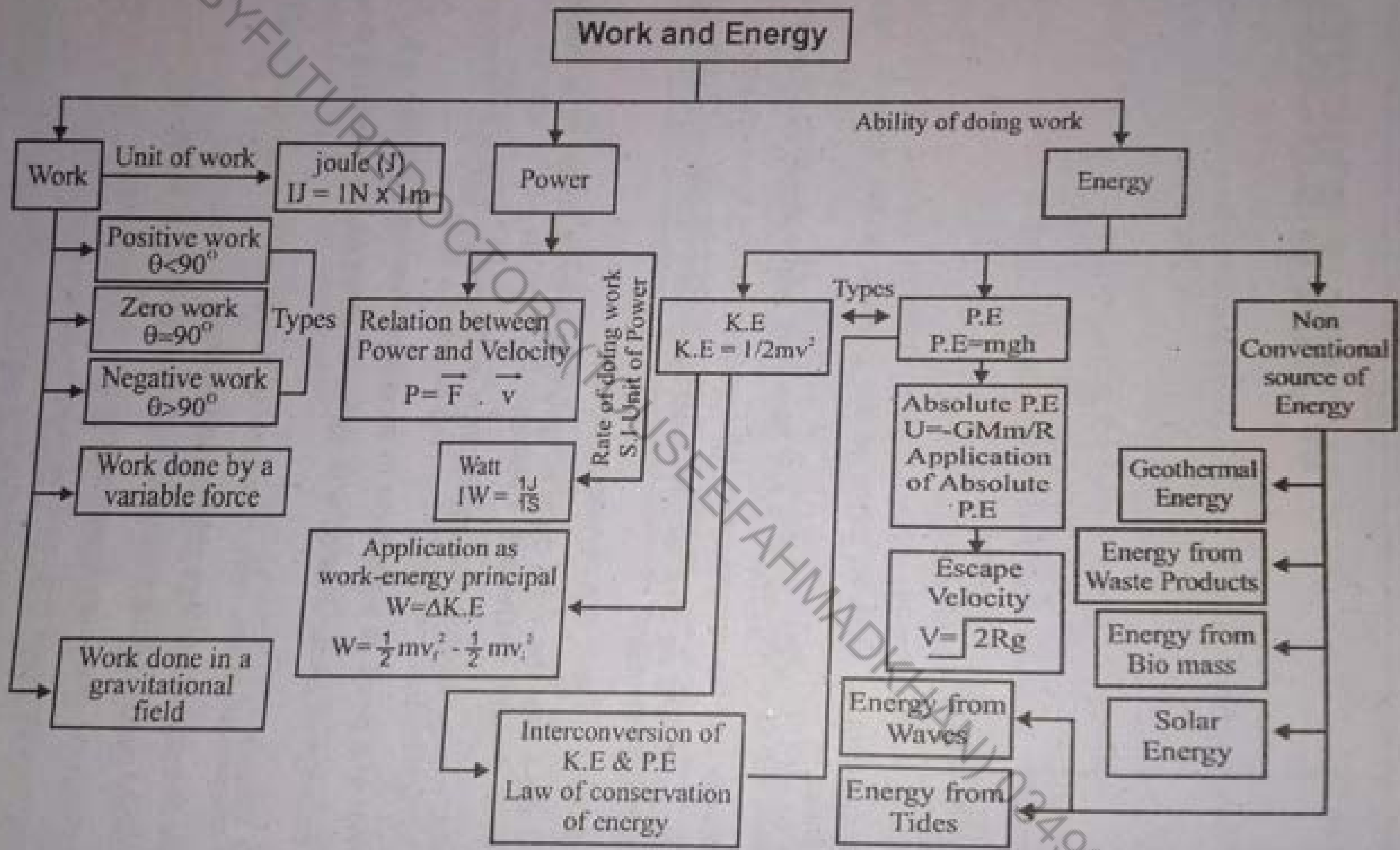
WORK & ENERGY

Learning Objectives

- Describe the concept of work in terms of the product of the force $\vec{F}$ and displacement $\vec{d}$ in the direction of force.
- Distinguish between positive, negative and zero work with suitable examples.
- Describe that work can be calculated from the area under the force-displacement graph.
- Explain gravitational field as an example of field of force and define gravitational field strength as force per unit mass at a given point.
- Prove that gravitational field is a conservative field.
- Compute and show that the work done by gravity is when a mass '*m*' is moved from one given point to another does not depend on the path followed.
- Describe that the gravitational potential energy is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.
- Define potential at a point as work done in bringing unit mass from infinity to that point.
- Explain the concept of escape velocity in terms of gravitational constant *G*, mass *m* and radius of the planet *r*.
- Differentiate conservative and non- conservative forces giving examples of each.
- Express power as scalar product of force and velocity.
- Explain that work done against friction is dissipated as heat in the environment.
- State the implications of energy losses in practical devices and the concept of efficiency.
- Utilize work-energy theorem in a resistive medium to solve problems.
- Discuss and make a list of limitations of some conventional sources of energy.
- Discuss potentials of some non-conventional sources of energy.

Chapter No. 4

CONCEPT MAP



Introduction

The energy of an object changes if an exchange of energy occurs between the object and its environment. Such a transfer can occur due to a **force** or due to an exchange of **heat**.

The transfer of energy via force is a process called doing work. Doing work is the act of transferring the energy. Work, then, is transferred energy. Energy transferred to the object is positive work. Energy transferred from the object is negative work.

Work is often thought in terms of physical or mental effort. In Physics, however, the term involves two things (i) *force* (ii) *displacement*.

Q.1 What do you understand by the term work? Explain.

Ans: **Work Done by Constant Force**

Do You Know?
Work has the same dimension as that of torque.

Definition

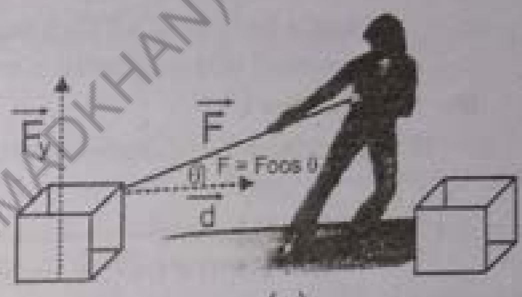
Work is the dot product of force and displacement.

$$W = \vec{F}.\vec{d} \qquad \text{(or)}$$

Work done on a body by a constant force is defined as the product of magnitude of displacement and the component of force in the direction of displacement.

$$W = (F\cos\theta)\,d$$

When a **constant force** is applied on a body and it covers the displacement in the direction of force then work is done.

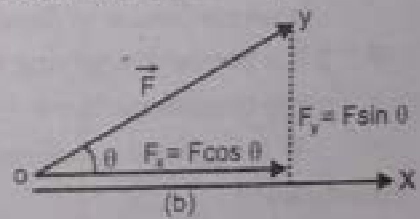


(a)

Definition

Consider a constant force **F** is applied on a body at an angle θ with displacement **d** as shown in fig (4.1)

- Resolve force into its rectangular component $F_X = F\cos\theta$ and $F_Y = F\sin\theta$.
- The horizontal component of force $F\cos\theta$ is along the direction of displacement **d** and is responsible to do work.



(b)

Fig:4.1

Work = (magnitude of displacement) (component of force in the direction of displacement.

$$W = d\,(F\cos\theta)$$

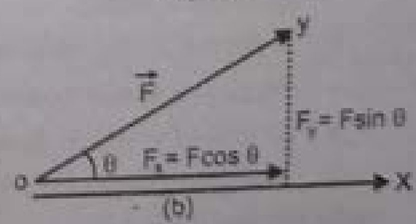
OR $\quad W = F\,d\cos\theta$ ....(1)

OR $\quad W = \vec{F}.\vec{d}$

This is expression for the magnitude of work done.

Work depends upon three factors

- $\vec{F}$ = force applied on a body.
- $\vec{d}$ = displacement covered by the body
- θ = angle between force and displacement



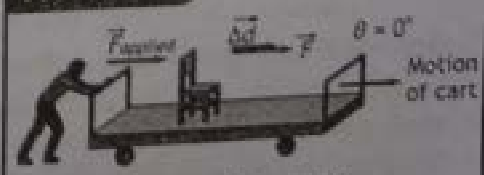
(b)

Fig:4.1

Special Cases:

i- **Maximum Positive Work:**
If applied force **F** and displacement **d** are in the same direction
i.e. $\theta = 0^0$
$W = Fd\cos 0$
$W = Fd(1)$
$W = Fd$ (This is maximum positive work done)

FIGURE 4.2



(a) Positive work

Example: A person is pushing the cart and applied force is in the direction of displacement so it is performing maximum positive work. This positive work will increase its kinetic energy.

ii- **Maximum Negative Work:**

If applied force **F** and displacement **d** are in opposite direction i.e. $\theta = 180^0$

$W = Fd\cos 180^0$

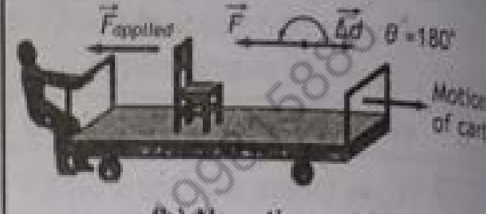
$W = Fd(-1)$

$W = -Fd$ (work done is negative)

Example of negative work

a. Consider the cart is moving forward but the person applies force opposite to its motion to stop it. So work done will be negative. Thus negative work is slows it down and reduces its kinetic energy.

FIGURE 4.2



(b) Negative work

b. Work done by frictional force is negative because direction of frictional force is opposite to displacement.

c. When a body is lifted then work done by gravitational force is negative because gravitational force is downward and its direction is opposite to displacement.

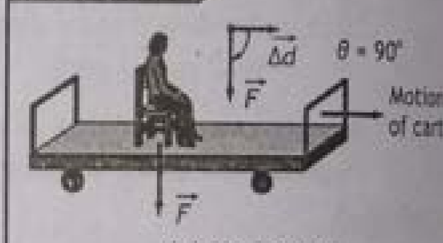
iii- **Zero Work:**

i. If applied force **F** and displacement **d** are in the perpendicular i.e. $\theta = 90^0$

$W = Fd\cos 90^0$

$W = Fd(0) = 0$ (No work is done)

FIGURE 4.2



(c) No Work

Examples:

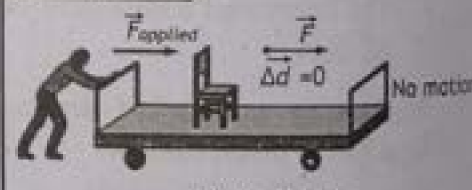
a. A person is sitting on the cart, exerting a downward force i.e. weight while cart has horizontal motion. So, applied force and displacement are perpendicular. Therefore, no work is done.

b. Work done by centripetal force is zero because centripetal force is perpendicular to direction of motion of the body.

ii. If a person is applying force on a body but body is not covering any displacement.

$W = F\,d\cos\theta = F\,(0)\cos\theta = 0$

FIGURE 4.2



(d) No Work

Examples:

a. Person is applying the force on cart but cart is not moving. So, no work is done.

b. A person at rest holding a heavy weight is doing no work.

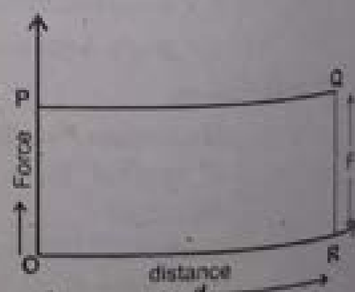
- Work is a ***scalar*** quantity.
- SI unit of work is ***joule*** ($J = Nm = kg\,m^2\,s^{-2}$)
- The dimension of work is $[ML^2T^{-2}]$

cgs unit of work is erg
$1\ J = 10^7$ erg

Definition of joule (J)

$1\ J = 1N \times 1\ m$

When one newton force acts on the body and the body covers a distance of one meter in the direction of force, the work done is said to be one joule.



Graphical representation of Work

Graphically, the area under the force-displacement curve represents the work done by force.

If we plot graph between force and displacement then,

Area under F-d the graph $= (OP)(OR)$

$= F\,d$

Area under F-d the graph $= W$

If force $\vec{F}$ makes an angle θ with horizontal. Then the graph is plotted between $F\cos\theta$ and d.

Q.2 How much work is being done by upward force when a person is holding the bucket and moving forward?

Ans: In this case, ***no work*** is being done

Reason

Because the angle between $\vec{F}$ and $\vec{d}$ is 90°.

So $W = Fd\cos 90^\circ$

$W = 0$

Q.3 Is any work done by centripetal force?

Ans: ***No work*** is being done

Reason

because the force is always perpendicular to the instantaneous displacement of the body in the circular motion.

So $W = F\,d\cos 90^\circ$

$W = F\,d\,(0) = 0$

Q. 4 Give examples of variable force?

Ans: In many cases, force does not remain constant during the process of doing work. For example:

i. As the rocket moves away from earth, work is done against the force of gravity which decreases as inversely proportional to the square of distance of from the earth centre $\left(F \propto \frac{1}{r^2}\right)$ and force does not remain constant.

ii. Force exerted by spring increases with the amount of stretch (x).

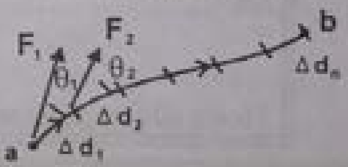
Since: $F = kx \rightarrow F \propto x$

So, spring force is variable force.

Q.5 How can we calculate the work done by variable force?

Ans: **Work Done by a Variable Force (Analytical Approach)**

We follow these steps to



A particle acted upon by a variable force, moves along the path shown from point a to point b.

- Let us consider the path of particle in xy-plane from point a to b as shown in figure.
- Divide the path into n short intervals of displacements $\Delta\vec{d}_1, \Delta\vec{d}_2, \ldots\ldots, \Delta\vec{d}_n$
- The forces acting during these intervals are $\vec{F}_1, \vec{F}_2, \ldots\ldots, \vec{F}_n$ respectively.

The force is considered to be approximately constant for each interval of displacement.

- Then we calculate the work done for the first interval is:

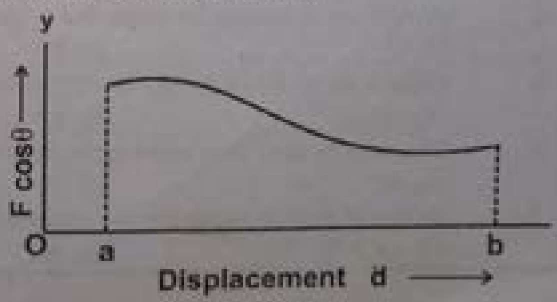
$$\Delta W_1 = \vec{F}_1 . \Delta\vec{d}_1 = F_1\cos\theta_1 \Delta d_1$$

Similarly,

$$\Delta W_2 = \vec{F}_2 . \Delta\vec{d}_2 = F_2\cos\theta_2 \Delta d_2$$

and up to nth interval

$$\Delta W_n = \vec{F}_n . \Delta\vec{d}_n = F_n\cos\theta_n \Delta d_n$$

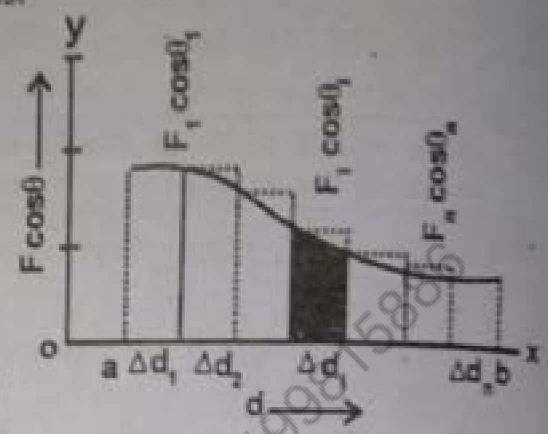


Now the total work done in moving the body from point a to b is.

$$W = (\Delta W_1 + \Delta W_2 + \dots\dots + \Delta W_n)$$

OR $$W = (F_1\cos\theta_1\Delta d_1 + F_2\cos\theta_2\Delta d_2 + \dots\dots + F_n\cos\theta_n\Delta d_n)$$

OR $$W = \sum_{i=1}^{n} F_i\cos\theta_i\Delta d_i$$



How to calculate work graphically

To calculate the work done graphically, follow these steps:

- Plot $F\cos\theta$ versus d.
- Area under the graph is divided into n rectangles from 'a' to 'b'.
- Area of each rectangle represents the work done during that interval.
- The total work done is equal to sum of areas of all the rectangles.

For more accurate calculation of work

The work done can be calculated more accurately, if we further divide the distance into a large number of steps so that each Δd becomes very small. i.e. $\Delta d \rightarrow 0$

So $$W = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i\cos\theta_i\Delta d_i$$

Then we find the total area of rectangles which is equal to area under $F\cos\theta$ verses d graph.

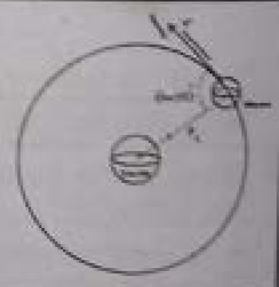
QUIZ

The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (a) positive work, (b) negative work, or (c) no work on the Moon?

Ans: Gravitational force exerted by earth on moon does not perform any work.
Reason: Gravitational force exerted by earth on moon acts as centripetal force along the radius and its instantaneous displacement is tangential. Due to which both force and displacement are perpendicular to each other i.e. $\theta = 90^\circ$.
So,

$$W = F\,d\cos 90^\circ$$
$$W = F\,d\,(0) = 0$$

That's why moon can rotate without the expenditure of any energy.



MCQ's From Past Board Papers

1. **Which of the following have same dimension as that of work?**
   (A) Torque (B) Momentum (C) Velocity (D) Power
2. **The work done is negative when the angle between the force and displacement is:**
   (A) 45° (B) 90° (C) 0° (D) 180°
3. **The area under the curve of force-displacement graph represents:**
   (A) Force (B) Work (C) Power (D) Displacement
4. **Work done is maximum when the angle between force and displacement is:**
   (A) 90° (B) 0° (C) 180° (D) 45°
5. **Mass is considered as highly concentrated form of energy. According to this, 1 kg mass has energy:**
   (A) 1.6×10^{-19} J (B) 9.1×10^{-31} J (C) 9×10^{16} J (D) 6.25×10^{8} J
6. **The work is zero then what is the angle between the force and displacement?**
   (A) 45° (B) 90° (C) 0° (D) 180°
7. **Which of the following are the dimension of work?**
   (A) $[MLT^{-1}]$ (B) $[MLT^{-2}]$ (C) $[ML^2T^{-2}]$ (D) $[MLT]$

When force $\vec{F}$ making angle 60° with displacement $\vec{d}$ then find the work on the body?
(A) Fd (B) $\frac{Fd}{2}$ (C) 0.866 Fd (D) $\frac{Fd}{3}$

Which of the following is the S.I unit of work?
(A) newton (B) watt (C) pascal (D) joule

If a body of mass 5kg is raised vertically through a distance of 1m, then calculate the work done?
(A) 49J (B) 4.9J (C) 490J (D) 0.49J

...s Key

1. A	2. D	3. B	4. B	5. C	6. B	7. C	8. B	9. D	10. A

What is gravitational field and gravitational field strength? Show that gravitational field is a conservative field.
Show that in gravitational field
(1) Work done in gravitational field is independent of path followed.
(2) Work done along a closed path in a gravitational is zero.

Gravitational field

The space around the earth in which its gravitational force acts on a body is called the gravitational field.

We know that a body placed in space surrounding the earth experiences a force which is equal to its weight and is directed towards the center of the earth as shown in the Fig. 4.6. This space around the earth with in which it exerts a force of attraction on other bodies is known as gravitational field.

Fig.4.6: gravitational field around the earth

Gravitational Field Strength

The gravitational force per unit mass on a body is known as gravitational field strength.

$$\text{Gravitational Field Strength} = \frac{\text{Gravitational Force}}{\text{mass}} = \frac{w}{m}$$

Near Earth, we can write as:

$$\text{Gravitational Field Strength} = \frac{mg}{m} = g = 9.8\ \text{N kg}^{-1}$$

Conservative Field

The field, in which work done is independent of the path followed is called conservative field

OR

The field, in which work done along a closed path is zero, is called conservative field.

Examples

- Gravitational field
- Electric field

(1) Work done along a closed by Gravitational Field is zero.

Let us consider a closed path 'ABCA' in gravitational field, as shown in Fig. A body of mass m and weight w is carried along the closed path from A to B and then from B to C. Now, we have to calculate the work done along different parts of the closed path:

(i) *Work done between 'A' and 'B' will be:*

$$W = F\,d\cos\theta$$
$$\Delta W_{A\to B} = wd_2\cos 90° = wd_1(0)$$
$$\Delta W_{A\to B} = 0$$

(ii) *Work done between 'B' and 'C' will be:*

$$\Delta W_{B\to C} = wd_1 \cos 180^\circ = wd_2(-1)$$

or $\Delta W_{B\to C} = -wd_1$

(iii) *Work done between 'C' and 'A' will be:*

$$\Delta W_{C\to A} = wd\cos\theta = w(d\cos\theta) \quad \ldots(i)$$

From figure ΔABC is right angle triangles

$$\frac{base}{hypotenuse} = \frac{d_1}{d} = Cos\theta$$

$$d_1 = d\,Cos\theta$$

Then (i) becomes

$$\Delta W_{C\to A} = w(d\cos\theta) = wd_1$$

$$\Delta W_{C\to A} = wd_1$$

Total work done in closed path 'ABCA' will be:

$$W_T = \Delta W_{A\to B} + \Delta W_{B\to C} + \Delta W_{C\to A}$$

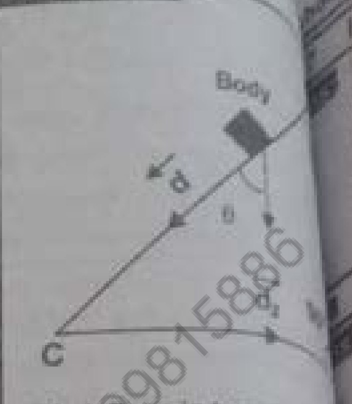
Putting the values, then

$$W_T = 0 + (-wd_1) + wd_1 = 0$$

Thus $W_T = 0$

Thus, total work done in a closed path 'ABCA' in gravitational field is zero, so gravitational field is a conservative field.

FIGURE 4.7



Work Done in two differ...

Work Done by Gravitational Field is independent of the path followed

The work done is independent of path followed by a body between two points in the gravitational...

Now consider another fig. 4.8.

Let the body be displaced in gravitational field by two different paths.

(i) **Direct path from 'A' to 'C'**

In this case:

$$W_T = wd\cos\theta,$$

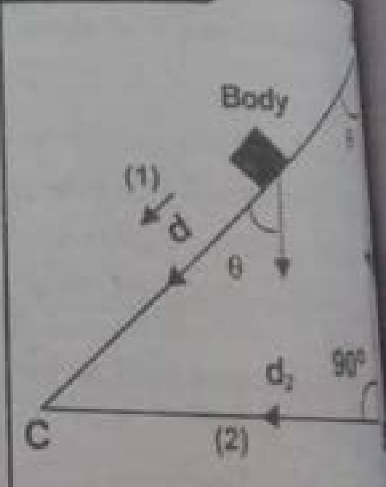
from figure, $d\cos\theta = d_1$

So $W_T = w(d\cos\theta) = wd_1$

$$W_T = wd_1 \quad \ldots\ldots(a)$$

Since $\frac{base}{hypotenuse} = \frac{d_1}{d} = Cos\theta$
$\Rightarrow d_1 = d\,Cos\theta$

FIGURE 4.8



Work Done in two different p...

(ii) **For In direct path ABC:**

From 'A' to 'B', work done will be:

$$\Delta W_{A\to B} = wd_1\cos 0^\circ = wd_1$$

And from B to C, $\Delta W_{B\to C} = wd_2\cos 90^\circ = 0$

So total work done using indirect path will be:

$$W_T = \Delta W_{A\to B} + \Delta W_{B\to C} = wd_1 + 0 = wd_1$$

So $W_T = wd_1 \quad \ldots..(b)$

Comparing (i) and (ii), we conclude that work done on the body in between any two po... gravitational field is independent of the path followed by the body. Thus, gravitational fie... conservative field.

Electrostatic field is also a conservative field.

Note: strictly speaking, magnetic field is not a conservative field because it does not satisfy all the b... conditions of being conservative field.

But it can be considered as conservative field especially when a charged particle is moving in a circle in t...

Q.7 Is frictional force conservative force? Explain

Ans: The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done along different paths cannot be same i.e. the work done against the frictional force certainly depends on the path followed.

Also remember non conservative forces like friction do not store energy but dissipate energy (in the form of heat, sound etc.) that's why energy wo rk done by such forces depends upon path followed and total work done cannot be zero.

Q.8 Define power and instantaneous power. Give its unit.

Ans: **Power**

Power is defined as work done per unit time.

OR

Power is defined as the rate of doing work.

OR

Rate of transfer of energy is called power.

If W is the work done in unit time t then power of the body is:

$$P = \frac{W}{t}$$

For Your Information

Conservative Forces
- Gravitational force
- Elastic spring force
- Electric force

Non Conservative Forces
- Frictional force
- Air resistance
- Tension in a string
- Normal force
- Propulsion force of a rocket
- Propulsion force of a motor

Average Power

It is defined as ***total work done divided the total time taken.***

Mathematically

$$P_{av} = \frac{\Delta W}{\Delta t}$$

Where

ΔW = the work done and Δt = time taken

Instantaneous Power

Power of body at any instant is called instantaneous power.

OR

Instantaneous power is defined as the limiting value of $\frac{\Delta W}{\Delta t}$ *as time Δt, following the time Δt, approaches zero.*

So, $$P_{ins} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Unit of Power

Power is a scalar quantity. SI unit of power is ***joule/second,*** called watt (W).

Definition of watt

The power is said to be one watt if *one joule of work is done in one second.*

$$1000\,W = 1\,kW$$

- In British Engineering System of units, unit of power is horse power (hp)

$$1\ hp = 746\ W$$

- **Dimensions of power are $[ML^2T^{-3}]$.**

Information:

1. Watt is commonly used in the power of bulbs, electric fans and other for other home appliances.
2. The power of grid stations and power generating stations in taken in mega watts (MW).
3. Power of TV set is 120 watts 4. Power of calculator is 7.5×10^{-4} watts.

Q.9 Show that instantaneous power, $P = \vec{F} \cdot \vec{v}$

Ans: **Proof**

Let $\vec{F}$ is the constant force acting on a moving body and $\vec{v}$ is constant velocity of the body. Then the power delivered to the body at any instant is given by

$$P = \frac{\Delta W}{\Delta t}$$

$$\Rightarrow \quad P = \frac{\vec{F} \cdot \Delta \vec{d}}{\Delta t}$$

$$\Rightarrow \quad P = \vec{F} \cdot \left(\frac{\Delta \vec{d}}{\Delta t}\right)$$

$$\Rightarrow \quad P = \vec{F} \cdot \vec{V}$$

$$\Rightarrow \quad P = F\, v \cos\theta$$

Another Definition of power

Power can also be defined as the *scalar product of force applied on the body and its velocity.*

For your Information

Approximate Power

Device	Power (W)
Jumbo Jet Aircraft	1.3×10^{8}
Car at 90 kmh^{-1}	1.1×10^{5}
Electric heater	2×10^{3}
Colour T.V	120
Flash Light (two cells)	1.8
Pocket calculator	7.5×10^{-4}

Q.10 Define commercial unit of electrical energy.

Ans: **Commercial Unit of electrical energy**

The commercial unit of electrical energy is **kilowatt-hour.**

Kilowatt-hour

Kilowatt-hour is *the work done in one hour by an agency whose power is one kilowatt.*

OR *If 1kilowatt power is maintained by an agency for 1 hour than energy consumed will be 1kwh.*

So,

$$1kWh = 1000\ W \times 3600\ sec$$
$$= 1000\ J/sec \times 3600\ sec$$
$$= 3600000\ J$$
$$= 3.6 \times 10^{6} J$$
$$1kWh = 3.6\ M\ J$$

Tit Bit

The food we eat in one day has about the same energy as $1/3^{rd}$ or 0.33 liters of petrol.

1 liter of petrol has the same energy as the food we eat in 3 days.

Experiments shows that the average power of a man walking upstairs at an ordinary speed is only about 0.33 kW.

Assignment 4.1:

What is the power of an airplane of mass 3000 kg if when on the runaway it is capable of reaching a speed of 80 ms^{-1} from rest in 4.0 s?

Solution: Power = P = ?

Mass = m = 3000 kg

Final velocity = v_f = 80 ms^{-1}

Initial velocity = v_i = 0 ms^{-1}

Time = t = 4 s

Its acceleration is:

$$a = \frac{v_f - v_i}{t}$$

$$\Rightarrow \quad a = \frac{80 - 0}{4} = \frac{80}{4} = 20$$

Power of airplane can be calculated by:

$$P = F\,v$$
$$P = (ma)\,v$$
$$P = 3000 \times 20 \times 80$$
$$P = 4800000\ W = 4.8 \times 10^{6} = 4.8\ MW$$

MCQ's

1. **What is the ratio of dimensions of K.E and power?**
   (A) 1:1 (B) [T] : 1 (C) 1 : [T] (D) [M]:[T]
2. **What are the dimensions of power?**
   (A) $[ML^{2}T^{1}]$ (B) $[ML^{2}T^{-2}]$ (C) $[ML^{2}T^{-3}]$ (D) $[ML^{2}T^{0}]$
3. **Power is equal to the dot product of force and ________.**
   (A) Displacement (B) Acceleration (C) Velocity (D) Position vector
4. **6 joule of work is done in 3 seconds then find its power?**
   (A) 6 watt (B) 3 watt (C) 18 watt (D) 2 watt

5. **Which one is non-conservative force?**
(A) Electric force (B) Magnetic force (C) Gravitational force (D) Frictional force

6. **The ratio of dimensions of power to work is:**
(A) 1: T (B) T: 1 (C) 1: T^2 (D) T^2: 1

7. **The scalar product of force and velocity is _______.**
(A) Work (B) Power (C) Momentum (D) Energy

8. **Which one is a conservative force?**
(A) Elastic Spring Force (B) Frictional Force (C) Air Resistance (D) Tension in the string

9. **Kilowatt hour is the unit of _________.**
(A) Work (B) Force (C) Power (D) Momentum

10. **Which of the following is the example of conservative force?**
(A) Restoring force in compressed spring (B) Tension in the string
(C) Propulsion force of rocket (D) Gravitational field

Answers Key

1. B	2. C	3. C	4. D	5. D	6. A	7. B	8. A	9. A	10. D

Q.11 Define energy. Give the two types of mechanical energy.

Ans: Energy

The ability of a body to do work is called energy.

OR

Energy is the agent which causes some changes in the state of the system.

Types of Mechanical energy

1. Kinetic Energy 2. Potential energy

Kinetic Energy

Energy possessed by a body due to its motion is called kinetic energy.

Mathematically

$$K.E. = \frac{1}{2} mv^2$$

Where m is the mass of body moving with velocity v.

Kinetic energy may be translational, rotational etc.

Relation between K.E. and Momentum of the body

$$K.E. = \frac{1}{2} mv^2$$

Multiplying and dividing R.H.S by m

$$K.E. = \frac{m^2v^2}{2m}$$

$$K.E. = \frac{p^2}{2m}$$

Potential Energy

The energy possessed by a body because of its position in a force field or because of its physical condition i.e. position *is called potential energy.*

Gravitational Potential Energy

The potential energy due to gravitational field near the surface of the earth at a height h is

$$P.E. = mgh$$

Elastic Potential Energy

The energy stored in a compressed / stretched spring is called elastic potential energy.

Do You Know

It takes about 9×10^7 J to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

For Your Information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30 x 10[illegible]
Burning 1 litre petrol	5 x 10[illegible]
K.E. of car at 90 km h^{-1}	1 x 10[illegible]
Running Person at 10 km h^{-1}	3 x 10[illegible]
Fission of one atom of uranium	1.8 x 10[illegible]
K.E. of a molecule of air	6 x 10[illegible]

For Your Information

If p is the momentum and E is the energy of mass m then

$$E = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

POINT TO PONDER

The pyramids in Egypt are thought to have been built by slaves hauling loads to height by inclined planes. It was the first use of inclined plane in construction happened around 2600BC.A pyramid at Egypt, believed to have been built by hauling stones up inclined planes.

Elastic potential energy $= \frac{1}{2}kx^2$

Where k spring constant and x is the extension.

➢ *The units of energy are the same as those of work.*

FIGURE 4.11

Crane

The crane is a machine that is in our daily life to lift heavy loads. The crane uses three simple machines.

1. Pulley
2. Lever
3. Wheel & Axle

Q.12 Define and explain the terms: Input, Output, efficiency for a machine. What is implication of energy losses in practical devices?

Ans: Machine:

"A machine is a device for multiplying forces or simply changing the direction of forces."

A machine helps to do different types of works easily, it reduces the human effort.

The principle underlying every machine is the conservation of energy.

(i) Input of Mechanical Machines:

Amount of work done on a machine by a given effort (force) is called input on a machine.

If an effort/ force 'F_{in}' acts through a distance 'D_{in}' then work done on the machine is called input.

Input = effort × distance through which effort acts

Or

$$\text{Input} = F_{in} \times D_{in}$$

(ii) Output of Mechanical Machines:

Amount of work done by a machine on the load (weight) is called output of the machine.

If the machine moves/lifts the load 'W' through the distance 'h', then work done by the machine is called output.

Output = load (weight) x distance covered by the load

Or

$$\text{Output} = F_{out} \times D_{out}$$

OR $\quad \text{Output} = W \times h$

(iii) Efficiency:

The ratio of output of a machine to the input applied on machine, is called its efficiency.

$$\eta = \frac{\text{Output}}{\text{Input}}$$

$$\eta = \frac{\text{Fout} \times \text{Dout}}{\text{Pin} \times \text{Din}}$$

$$\text{Efficiency in percentage} = \frac{\text{Fout} \times \text{Dout}}{\text{Fin} \times \text{Din}} \times 100\%$$

It has no unit.

In physics, ***mechanical efficiency is the effectiveness of a machine.***

FOR YOUR INFORMATION

Efficiency of some electrical equipments: LED light bulbs have been introduced to replace ordinary light bulbs, as they are much more efficient. Let's take a look at a standard 50-watt. The energy consumption to use a light bulb like this would cost about 1278Rs in a year. An LED, running over the course of 1 year would cost only 260Rs to operate. Using these causes less energy to be wasted as heat. Recently developed, AC/DC fans can operate on less energy while producing a high airflow. In fact, they can cut down your power consumption by up to 65% and can operate on solar penal. AC/DC fans are designed to run on 12V and consume around 26-35W. Ordinary Fan consume 75watt while AC/DC fans consume about 35 to 40watt so AC/DC fans are more efficient.

Implication of Energy Losses in Practical Devices

The efficiency of an ideal machine is 100% but an actual machine's efficiency will always be less than 100%. This means that some of the work put into the system is dissipated in the form of thermal energy (heat). In a mechanical system, friction is the most common cause of the energy lost to heat.

The actual Mechanical advantage of a system is always less than the ideal mechanical advantage due to these losses.

Q.13 Explain the work-energy principle in case of change in K.E. of the body.

Ans: **Work-Energy Principle**

Statement

Work done on a body is equal to the change in its kinetic energy.

Proof

Let

m = mass of body

v_i = initial velocity of the body

F = force applied on the body

d = distance covered by the body

v_f = final velocity of the body

The work done on the body is

$$\text{Work done} = F\,d \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

Now, according to equation of motion

$$2ad = v_f^2 - v_i^2 \qquad (\text{as } S = d)$$

OR
$$d = \frac{1}{2a}(v_f^2 - v_i^2) \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

And according to Newton's second law of motion

$$F = ma \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

Using equations (2) and (3) in (1), we get

$$\text{Work done} = ma\frac{1}{2a}(v_f^2 - v_i^2)$$

$$\text{Work done} = \frac{1}{2}m(v_f^2 - v_i^2)$$

OR
$$\text{Work done} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

OR Work done = final K.E. – initial K.E. = change in K.E.

Tid-bits

All the food you eat in one day has about the same energy as 1/3 litre (0.33 litre) of petrol.

Table: 4.1

Practical devices	Efficiency
Petrol heat engine	(25-30)%
Diesel engine	(34-40)%
Steam locomotive	(35-40)%
Incandescent lamp	5%
Fluorescent lamp	20%
Steam turbine	(34-46)%
Air craft gas turbine	36%
Nuclear power plant	(30-35)%
Fossil fuel power plant	(30-40)%
Electric generator	(70-94)%
Electric motor	(50-93)%
Dry cell battery	90%
Battery	90%
*Home coal furnace	55%

Note:

- If a body is raised from the surface of earth, the work done changes its gravitational P.E.
- If a spring is a compressed, the work done on it is equal to the increase in elastic potential energy.

MCQ's From Past Board Papers

1. If velocity 'v' of an object is double, then K.E becomes ________.
   (A) double (B) remains same (C) four times (D) sixteen times
2. What is the K. E of bullet of mass 500 gm moving at a speed of 200 ms^{-1}?
   (A) 250J (B) 125J (C) 2500J (D) 10,000J
3. kilowatt hour is a unit of:
   (A) Energy (B) Power (C) Pressure (D) Force
4. K.E can be defined as the dot product of
   (A) Momentum and force (B) Force and velocity
   (C) Average momentum and velocity (D) none

5.	Which one is the biggest unit of energy			
	(A) erg	(B) joule	(C) watt-hour	(D) kilo-watt hour
6.	Earth receives larger amount of energy directly from			
	(A) Wind	(B) Water	(C) Sun	(D) Moon
7.	A body of mass 1kg drops from the top of tower of height 50m, what will be its K.E 10m below the top			
	(A) 490 J	(B) 49 J	(C) 98 J	(D) 980 J

Answers Key

1. C	2. D	3. A	4. C	5. D	6. C	7. C

Q.14 Define absolute potential energy. Derive relation for absolute P.E. of body of mass m at distance r from the center of earth.

Ans: **Absolute Potential Energy**

The absolute gravitational potential energy of an object at a certain position is the work done by gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero.

OR

The amount of work done in moving the body at a certain point in the gravitational field to a position of zero potential such that body is never accelerated is called absolute P.E.

Calculation of Absolute P.E

- Let a body of mass m is displaced in space from point 1 to N in the gravitational field. The gravitational force does not remain constant during this displacement.
- As gravitational force varies inversely to the square of the distance from the surface of earth $\left(F_{av} = \frac{GmMe}{r_{av}^2} \Rightarrow F_{av} \propto \frac{1}{r_{av}^2}\right)$, so its magnitude keep on decreasing as we move away from surface of earth.
- In order to calculate the work done by gravitational force, the distance between 1 to N is divided into equal small steps of length Δr, so that the value of force remains constant for each step.

Work done during 1st step (1 to 2)

Suppose

m = mass of the body

M = mass of earth

r_1 = distance of point 1 from the center of the earth

r_2 = distance of point 1 from the center of the earth

Calculation of r

The distance between the center of this step and center of the earth is

$$r_{av} = \frac{r_1 + r_2}{2} \quad \text{(1)}$$

Also displacement of body from point 1 to 2 is

$$\Delta r = r_2 - r_1 \quad \text{(2)}$$

OR

$$r_2 = \Delta r + r_1 \quad \text{(3)}$$

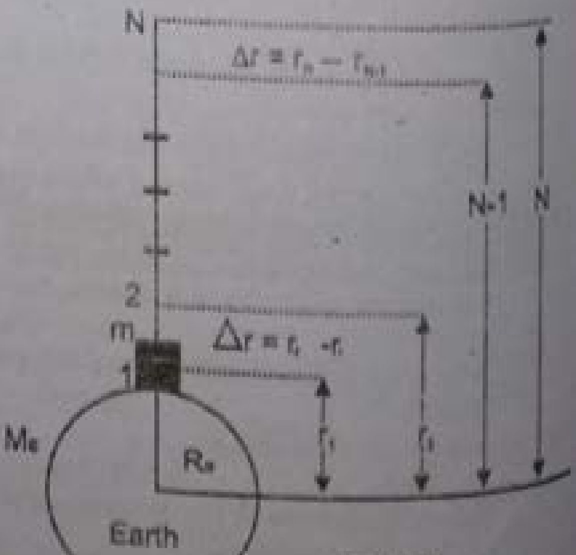


Fig:4.10

Using equation (3) in (1)

$$\Rightarrow \quad r_{av} = \frac{r_1 + \Delta r + r_1}{2}$$

Using equation (3) in (1)

$$\Rightarrow \quad r_{av} = \frac{r_1 + \Delta r + r_1}{2}$$

$$\Rightarrow \quad r_{av} = \frac{2r_1 + \Delta r}{2}$$

$$\Rightarrow \quad r_{av} = \frac{2r_1}{2} + \frac{\Delta r}{2}$$

$$\Rightarrow \quad r_{av} = r_1 + \frac{\Delta r}{2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (4)}$$

Squaring both sides, we have

$$\Rightarrow \quad r_{av}^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$\Rightarrow \quad r_{av}^2 = r_1^2 + \frac{\Delta r^2}{4} + 2r_1\frac{\Delta r}{2}$$

$$\Rightarrow \quad r_{av}^2 = r_1^2 + \frac{\Delta r^2}{4} + r_1\Delta r$$

Since $(\Delta r)^2 << r_1^2$, so this term can be neglected as compared to r_1^2

$$\Rightarrow \quad r_{av}^2 = r_1^2 + r_1\Delta r$$

$$\Rightarrow \quad r_{av}^2 = r_1^2 + r_1(r_2 - r_1)$$

$$\Rightarrow \quad r_{av}^2 = r_1^2 + r_1r_2 - r_1^2$$

$$\boxed{r_{av}^2 = r_1 r_2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (5)}$$

Now, the gravitational force F at the center of this step is

$$F_{av} = \frac{GmM_e}{r_{av}^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (6)}$$

Putting value from equation (5) in (6), we have

$$F_{av} = \frac{GmM_e}{r_1r_2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (7)}$$

Thus the work done during 1[st] step is

$$W_{1\to2} = \vec{F}\cdot\Delta\vec{r}$$

$$= F_{av}\,\Delta r\cos180^\circ \quad (\text{Since } \vec{F} \text{ is opposite to } \Delta\vec{r})$$

$$= F\Delta r(-1) = -F\Delta r$$

$$= -G\frac{Mm}{r_1r_2}(r_2 - r_1) \quad [\text{using equations (2) \& (7)}]$$

$$W_{1\to2} = -GMm\left[\frac{r_2}{r_1r_2} - \frac{r_1}{r_1r_2}\right]$$

$$W_{1\to2} = -GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Work done during 2[nd] step (2 to 3)

$$W_{2\to3} = -GMm\left[\frac{1}{r_2} - \frac{1}{r_3}\right]$$

Work done during last step (N-1 to N)

For last step work done is,

$$W_{N-1\to N} = -GMm\left[\frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

Do You Know?

There more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

Practical Devices	Efficiency
Petrol heat engine	(25 - 30) %
Diesel engine	(34 - 40) %
Steam locomotive	(35 - 40) %
Incandescent lamp	5 %
Fluorescent lamp	20 %
Steam turbine	(34 - 46) %
Air craft gas turbine	36 %
Nuclear power plant	(30 - 38) %
Fossil fuel power plant	(30 - 40) %
Electric generator	(70 - 99) %
Electric motor	(80 - 92) %
Dry cell battery	90 %
Electric heater	100 %
Hair drier	100 %
Battery	90 %
Home coal furnace	55 %

Total work done from point 1 to N

$$W_{total} = W_{1\to2} + W_{2\to3} + ... + W_{N-1\to N}$$

Putting values

$$\Delta W_{total} = -GmM_e\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - GmM_e\left(\frac{1}{r_2} - \frac{1}{r_3}\right) .... GmM_e\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$= -GmM_e\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + ... + \frac{1}{r_{N-1}} - \frac{1}{r_N}\right]$$

$$\Delta W_{1\to N} = -GmM_e\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

As point N is at infinity

$$\Delta W_{1\to\infty} = -GmM_e\left[\frac{1}{r_1} - \frac{1}{r_\infty}\right]$$

$$\frac{1}{r_\infty} = 0$$

$$\Delta W_{1\to\infty} = -GmM_e\left[\frac{1}{r_1} - 0\right]$$

$$\Delta W_{1\to\infty} = \frac{-GmM_e}{r_1}$$

$\Delta W_{1\to\infty}$ is called absolute P.E therefore

$$\text{Absolute P.E} = -\frac{GmM_e}{R_e}$$

Where R_e is radius of earth

$$W_{total} = -GM_em\left[\frac{1}{r_1} - \frac{1}{r_N}\right]$$

> **Tit-bits**
> More coal has been used since 1945 than was used in the whole of history before that.

Absolute P.E

If N lies at infinity, then

$r_N = \infty$ and $\frac{1}{r_N} = \frac{1}{\infty} = 0$

So $\quad W_{total} = -GM_em\left[\frac{1}{r_1} - \frac{1}{\infty}\right] = -GM_em\left[\frac{1}{r_1} - 0\right]$

OR $\quad W_{total} = \frac{-GM_em}{r_1}$ ________________ (8)

In general, the absolute gravitational potential of body at distance r from the center of the earth is

$$\boxed{U = \frac{-GM_em}{r}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (9)}$$

Absolute P.E on the surface of Earth

When the body lies at the surface of the earth then, r = R. So, equation (9) becomes

$$\boxed{U_g = \frac{-G_eMm}{R}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (10)}$$

Where R is the radius of the earth and the negative sign represents that gravitational field of earth for mass m is attractive.

Note:

- When the body moves away from earth's surface, r increases, U increases. (i.e. it becomes less negative.)
- When the body falls towards the earth's surface, r decreases, U decreases. (i.e. it becomes more negative.)

Note: To calculate P.E. choice of zero reference point

We can take the ***surface*** of the Earth or the point at ***infinity*** as zero P.E reference.

As we go away from earth, gravitational force decreases and it becomes zero at a very far point from earth surface (where earth cannot attract anybody towards itself), at this point P.E. becomes zero, so, in case of absolute P.E. zero reference point lies at infinity.

Q.15 Define gravitational potential. Write its formula.

Ans: **Gravitational Potential:**

It is gravitational potential energy per unit mass of the body in the gravitational field.

OR

The potential energy per unit mass at that point which is at distance r from the centre of earth and it is shown as:

$$V(r) = \frac{P.E. \, at \, distance \, r}{mass}$$

$$V(r) = \frac{-\frac{GmM_e}{r}}{m}$$

$$V(r) = \frac{-GM_e}{r}$$

At distance r = Re (on the surface of earth)

$$V(r) = \frac{-GM_e}{Re}$$

Q.16 Define escape velocity and derive the mathematical expression for escape velocity.

Ans: **Escape Velocity**

The initial velocity of a body with which it goes out of the earth's gravitational field is called escape velocity

OR

The initial velocity, which a projectile must have at earth surface in order to go out of earth's gravitational field.

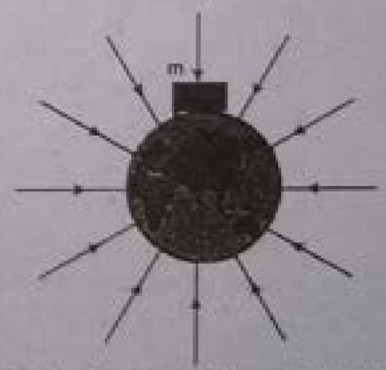


Fig:4.11: Gravitational field around the earth

Explanation

When a body is thrown upward, it returns back after reaching a certain height. That is due to gravitational force acting downward. If we increase the initial velocity of the body then it gains more height. If we go on increasing the initial velocity, then at certain velocity, the body it will not return back to the ground. This particular velocity is called *escape velocity.*

Expression for escape velocity

We know that the absolute P.E. of a body of mass m on the surface of earth is

$$P.E._i = U_g = \frac{-GMm}{R} \quad \text{------------} \quad (1)$$

And P.E. of the body at infinity is:

$$P.E._f = 0$$

As the body goes out of gravitational field, it P.E. becomes zero.

So, $Change \, in \, P.E. = P.E._f - P.E._i = 0 - \left(\frac{-GmM_e}{R_e}\right) = \frac{GmM_e}{R_e}$

From law of conservation of energy, The increase in P.E. is equal to initial K.E.

i.e. Initial K.E. = Change in P.E.

$$\text{Initial K.E} = \frac{GmM_e}{R_e}$$

OR $$\frac{1}{2}m v_{esc}^2 = \frac{GmM_e}{R_e}$$

(Where m = the mass of the body; Me = mass of the earth and, Re= radius of earth.)

OR $$v_{esc}^2 = \frac{2GM_e}{R_e}$$

OR $$\boxed{v_{esc} = \sqrt{\frac{2GM_e}{R_e}}} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

This is expression for the escape velocity of a body.

Another expression for escape velocity

As the gravitational force for a mass m placed on the surface of the earth of mass is

$$F = \frac{GMm}{R^2}$$

But $F = mg$

So $$mg = \frac{GmM_e}{Re_e^2}$$

OR $$g = \frac{GM_e}{R_e^2}$$

OR $GM = gR_e^2$

Thus equation (2) becomes,

$$v_{esc} = \sqrt{\frac{2gR_e^2}{R_e}}$$

OR $$v_{esc} = \sqrt{2gR_e} \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

For Your Information	
Some Escape speeds (km^{-1})	
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Nepyune	22.4
Uranus	25.4
Saturn	37.0
Jupiter	61

Factors:

It depends upon:

a. Mass of planet **b.** Radius of planet **c.** gravitational acceleration g at that planet

Note: escape velocity does not depend upon mass of body and angle of projection.

Value of escape velocity on Earth

As $g = 9.8 m/s^2$ and $R_e = 6.4 \times 10^6 m$

So $v_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$

$= 11.2 \times 10^3$ m/s

OR $= 11.2 \times 10^3$ m/s

OR $\boxed{v_{esc} = 11.2 \text{ km/s}}$

Assignment 4.2:

How fast would the moon need to travel in order to escape the gravitational pull of Earth, if Earth has a mass of 5.98×10^{24} kg and the distance from Earth to the moon is 3.84×10^8 m?

Solution: Mass of earth = M = $5.98 \times 10^{24}\ kg$

Distance from earth to moon = r = R + h = $3.84 \times 10^8\ m$

Escape velocity of moon = V_{esc} = ?

Formula: $V_{esc} = \sqrt{\frac{2GM}{r}}$

By Putting values $V_{esc} = \sqrt{\frac{2\times(6.67\times10^{-11})\times(5.98\times10^{24})}{3.84\times10^{8}}}$

$V_{esc} = \sqrt{20.78 \times 10^{5}}$

$V_{esc} = 1441\ ms^{-1}$

Q.17 Discuss inter-conversion of potential energy and kinetic energy.

Ans: Inter Conversion of Potential Energy and Kinetic Energy

Let us consider an object of weight 'w' lying at B at height 'h' as shown in Fig.

At Point B,

It possesses only potential energy. Its P.E. at point B w.r.t. point A is:

$(P.E.)_B = w \times h$

Its K.E. is zero at point B as it is in rest.

$(K.E.)_B = \frac{1}{2} mv^2 = \frac{1}{2} m (0)^2 = 0$

So, total energy at point B is in the form of potential energy.

At Point A,

- Gravitational P.E. at point B can be used to drive the nail into wood.
- If hammer is released from point B, it will do work on the nail by driving it into the wood block at point A.

FIGURE 4.14

Hammer falling on the nail by driving it into the wood

Hence,

Work done on the nail to push it = P.E. it possesses at point B = Force x displacement = w x h

- Under the action of gravitational force, the hammer loses its potential energy and acquiring Kinetic energy while falling downward. Just before hitting the ground the potential energy of the hammer is minimum and kinetic energy maximum. Thus, if there is no friction:

Kinetic energy at A = loss in potential energy = w × h = mgh

This means that when potential energy of a body decreases there is an equal increase in its kinetic energy.

Thus, Loss of potential energy = Gain in kinetic energy ------------------- (1)

- **Work- Energy Theorem in Resistive Medium**

When Friction is present:

In many situations, friction cannot be ignored. Because frictional forces reduce the mechanical energy (but not the total energy), they are called dissipative forces, because some part of mechanical energy is used against the friction and dissipates as heat and sound energy.

$\Rightarrow$ (w × h) = (K.E. gained) + (f × h)

OR **Loss in P.E. = k.E gained + (Work done against friction)** ------------------ (2)

Since, (w x h = loss of potential energy) : (f x h = work done against friction)

This is a very important work-energy equation.

- Similarly, when the body moves up, then:

Loss in K.E at A = Gain in P.E at B + work done against friction.

This is law of conservation of energy in terms of K.E and P.E.

Do you know?

The pull of the moon does not only pull the sea up and down but it pulls land up and down by as much as 0.25 m.

Q.18 State law of conservation of energy. Why new sources of energy has to be developed if energy is conserved?

Ans: **Conservation of Energy**

Statement

Energy cannot be destroyed. It can be transformed from one form into another, but the total amount of energy remains constant.

Conservation of mechanical energy

The K.E and P.E are the different forms of mechanical energy.

The total mechanical energy of the body is equal to the sum of K.E and. P.E.

P.E may change into K.E. Similarly, the K.E may also be Chang into P.E, but total energy remains constant.

Mathematically,

Total energy = P.E. + K. E.= constant

This is the special case of conservation of energy.

Need of New Energy Sources

In daily life we observe many energy changes from one form to another. At last all energy transfers heat the environment which is useless. So, useful energy is *decreasing* though total energy is conserved. That is why we need to develop new sources of energies.

Logo of UN, which shows save energy from all sides.

For Your Information

Source of energy	Original Source
Source	Sun
Bio mass	Sun
Fossil fuels	Sun
Wind	Sun
Waves	Sun
Hydro electric	Sun
Tides	Moon
Geothermal	Earth

Assignment 4.3:

Consider a person on a sled sliding down a 100 m long hill on a 30° incline. The mass is 20 kg and the person has a velocity of 2 ms^{-1} down the hill when they're at the top. (a) How fast is the person traveling at the bottom of the hill? (b) If, the velocity at the bottom of the hill is 10 ms^{-1} because of friction. How much work is done by friction?

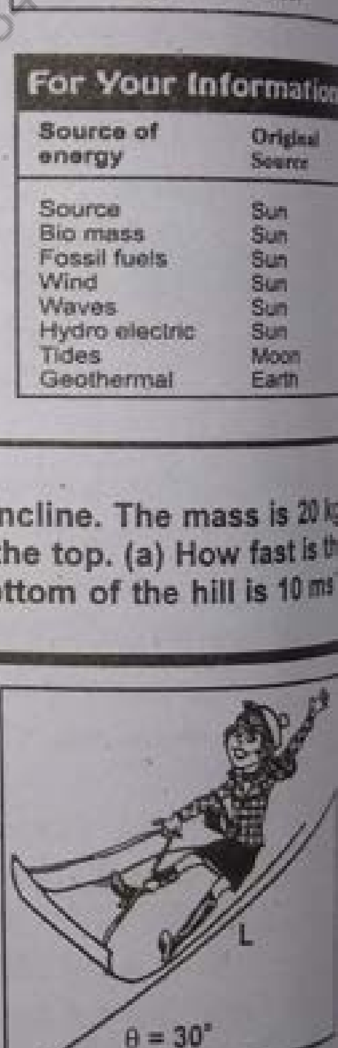
Solution:

(a) Velocity at the bottom v_b = ?

Length of hill L = 100 m then height of the hill can be calculated as follow;

$$h = L \sin\theta = 100 \sin 30^\circ = 100 \times 0.5 = 50 \text{ m}$$

Now calculating velocity at the bottom;



Sled Sliding

$$K.E_b = P.E + K.E_t$$
$$\tfrac{1}{2} m v_b^2 = m g h + \tfrac{1}{2} m v_t^2$$
$$\tfrac{1}{2} m v_b^2 = m (g h + \tfrac{1}{2} m v_t^2)$$
$$v_b^2 = 2(g h + \tfrac{1}{2} m v_t^2)$$
$$v_b^2 = 2 g h + v_t^2 = 2 \times 9.8 \times 50 \times 2^2 = 9800 + 40 = 9840$$
$$v_b = \sqrt{9840} = 31.368 \text{ m/s} = 31.4 \text{ m/s}$$

(b) Work done against friction = ?

If velocity at the bottom v_b = 10 m/s

Total energy at top = K.E at bottom + Work against friction

$$P.E + K.E_t = K.E_b + W_t$$
$$\Rightarrow W_1 = P.E + K.E_t - K.E_b = m g h + \tfrac{1}{2} m v_t^2 - \tfrac{1}{2} m v_b^2$$
$$\Rightarrow W_1 = 20 \times 9.8 \times 50 + \tfrac{1}{2} \times 20 \times 2^2 - \tfrac{1}{2} \times 20 \times 10^2$$
$$\Rightarrow W_1 = 9800 + 40 - 1000 = 8840 \text{ j}$$

Q.19 What are nonrenewable energy sources? Shortly brief different nonrenewable energy sources

Ans: Sources of Energy

Non-Renewable Energy Sources

"These are the natural energy sources from earth that exist in limited amount and cannot be replaced if it is used up. These are such sources which cannot be refilled by natural means at the same rate at which it is consumed."

Energy Sources

Renewable	Nonrenewable
Hydroelectric	Coal
Wind	Natural Gas
Tides	Oil
Geothermal*	Uranium
Biomass	Oil shale
Sunlight	Tar sands
Ethanol/Methanol**	

* Individual fields may run off
** Renewable when made from bio mass

- Energy can be generally classified as non-renewable and renewable. **Over 85% of the energy used in the world is from non-renewable supplies.**
- Most developed nations are dependent on non-renewable energy sources such as fossil fuels (coal, oil and gas) and nuclear power. These sources are called non-renewable because they cannot be renewed.
- **Fossil fuels** are the most commonly used types of non-renewable energy. They were formed when incompletely decomposed plant and animal matter was buried in the earth's crust. This process occurred over millions of years. The three main types of fossil fuels are coal, oil, and natural gas.

a. Coal:

- Coal is the most abundant fossil fuel in the world with an estimated reserve of one trillion metric tons.
- Coal formed slowly formed over millions of years from the buried remains of ancient swamp plants.
- Different types of coal resulted from differences in the pressure and temperature that prevailed during formation.
- The softest coal (about 50% carbon), which also has the lowest energy output, is called **lignite**.
- Currently, the world is consuming coal at a rate of about **5 billion metric tons per year**.
- The main use of coal is for:
  - power generation,
  - heating
  - cooking.
- If consumption continues at the same rate, the current reserves will last for more than 200 years.
- The burning of coal results in significant atmospheric pollution.

b. Oil:

- Oii is available in abundance in most of the middle east countries such as Saudi Arabia, Kuwait, Iran, Iraq and UAE etc.
- Like coal, it was also made out of dead plants and animals that had lived millions of years ago. When plants and animals died they were covered with thick layer of mud and sand which created huge pressure and temperature.
- Most known oil reserves are already being exploited, and oil is being used at a rate that exceeds the rate of discovery of new sources.
- If oil consumption rate remains same, oil supplies may be exhausted in another 50 years or so.

c. Natural Gas:

- Natural Gas is the gaseous form of fossil fuels.
- It is a mixture of several gases including methane, ethane, propane and butane.
- It burns completely and leaves no ashes.
- It causes almost no pollution and is one the cleanest form of fossil fuel.
- **The natural gas is made into liquefied petroleum *gas (LPG).***

Energy From Fossil fuels

Uses:

- In developed countries, natural gas is used primarily for heating, cooking, and powering vehicles.
- It is also used in a process for making ammonia fertilizer.

➢ The current estimate of natural gas reserves is about **100 million metric tons.**

➢ At current usage levels, this supply will last an estimated 100 years.

d. Nuclear Energy

➢ When atoms are split apart, this energy can be used to make electricity. This process is called nuclear fission.

➢ In a nuclear power plant, fission takes place inside a reactor.

➢ Nuclear power in Pakistan makes a small contribution to total electricity production and requirements, supplying only 6.1terawatt hour(s) (5.5%) of the electricity in 2015.

Q.20 What are renewable energy sources? Shortly brief different renewable energy sources.

OR Describe briefly various non-conventional sources of energy.

Ans: **Renewable Energy Sources:**

The resources which are being continuously renewed by nature constantly are called as Renewable Resources.

➢ Non-Conventional' sources of energy are also called Renewable resources.

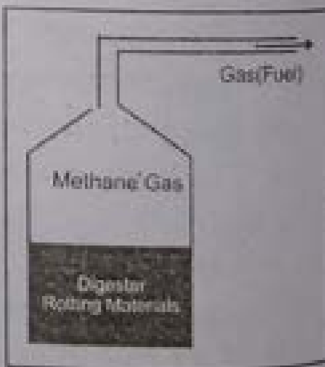
➢ The sun, the winds, tides, geothermal energy, biomass, farm and animal waste including human excreta are the non-conventional sources of energy.

➢ They are inexpensive in nature.

➢ These are such energy sources (The sun, the winds, tides, geothermal energy, biomass, farm and animal waste including human excreta) not very common these days. However, it is expected, that these sources will contribute substantially to the world energy demand of the future. These are called **non conventional energy sources.**

➢ Some of these are introduced briefly here.

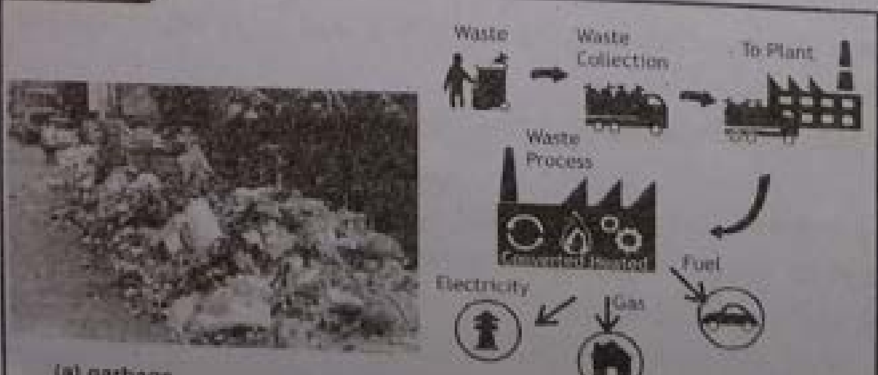
1. Energy from Biomass:



➢ Biomass energy is energy generated from plants and animals, and it is a renewable source of energy. Biomass is matter usually thought of as garbage. Some of it is just stuff lying around -- dead trees, tree branches, yard clippings, left-over crops, wood chips, and bark and sawdust, animal dunk etc.

➢ The trucks bring the waste from factories and from farms to a biomass power plant. This is then fed into a furnace where it is burned. The heat is used to boil water in the boiler, and the energy in the steam is used to turn turbines and generators.

FIGURE 4.16



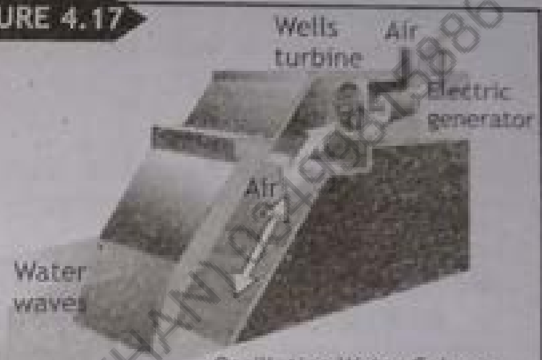
(a) garbage (b) energy from waste.

- Technologies today are able to recover the energy contained in plastics. Plastics have a high energy content that can be converted to electricity, synthetic gas, fuels and recycled feed stocks for new plastic
- About 81 million ton/annum biomass production has a huge potential to produce enough bio-energy by employing different technologies viz. combustion, gasification, pyrolysis, process etc.
- Similarly, available dung from 72 million animals (cows and buffalos) and available poultry droppings from 785 million poultry birds can produce considerable biogas to produce heat and electricity.

3. Energy from waves:

- Ocean waves are caused by the wind as it blows across the sea.
- The tides and winds blow across the surface of ocean.
- These waves are powerful source of renewable energy.
- There are many devices which are designed to efficiently convert wave power into electricity.
- **Ocean Water Column (OWC)** is one of them as shown.
    - **An Ocean Water Column (OWC)** consists of partially submerged structure that opens to the ocean below the water surface.
    - When these waves come through the structure, it cause the water column to rise and fall with the wave.
    - Due to which the air in the top structure to pressurize and depressurize.
    - This in turn pushes and pulls air through the connected air turbine at the top of the structure, that generates electricity.

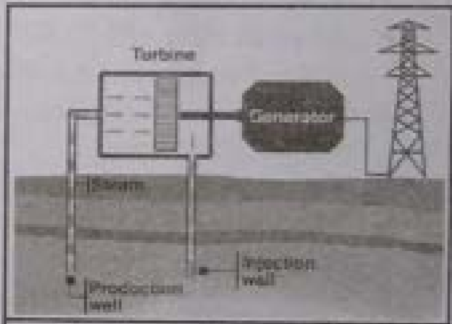
FIGURE 4.17



Oscillating Water Columns (OWC) device.

4. Geothermal energy:

- The inside of the Earth is full of heat which can be converted into different forms of energy therefore it is called geothermal energy.
- Geothermal power plants, which uses heat from deep inside the Earth to generate steam to make electricity as shown in Figure 4.18.
- At a **geothermal power plant**, wells are drilled 1 or 2 miles deep into the Earth to pump steam or hot water to the surface.

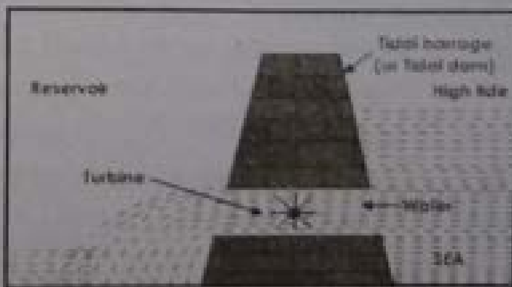


Geothermal Power Plant

- Hot water is pumped from deep underground through a well under high pressure.
- When the water reaches the surface, the pressure is dropped, which causes the water to turn into steam.
- The steam spins a turbine, which is connected to a generator that produces electricity.
- The steam cools off in a cooling tower and condenses back to water.
- The cooled water is pumped back into the Earth to begin the process again.
- You're most likely to find one of these power plants in an area that has a lot of hot springs, geysers, or volcanic activity, because these are places where the Earth is particularly hot just below the surface.

5. Tidal Energy:

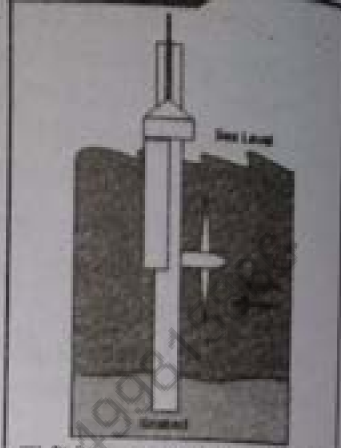
- Using the power of the tides, energy is produced from the gravitational pull from both the moon and the sun, which pulls water upwards, while the Earth's rotational and gravitational power pulls water down, thus creating high and low tides.
- This movement of water from changing tides is a natural form of K.E. therefore tidal energy is called renewable energy.
- Tidal barrages are the most efficient tidal energy sources as shown in Figure.



Tidal Barrges

- A tidal barrage is a dam that utilizes the potential energy generated by the change in height between high and low tides.
- This energy turns a turbine or compresses air, which generates electricity.
- The Oxford University engineers calculated that underwater turbines strung across the entire width of the narrow inlet of the sea could generate a maximum 1.9 GW (giga watt) of power, averaged across the fortnightly tidal cycle. That is equivalent to 16.5 TW/h (terawatt/hour) of electricity a year, almost half Scotland's entire annual electricity consumption in 2011.

FIGURE 4.20



Tidal energy technologies

7. **Solar Energy:**

- Solar energy is the radiant light and heat from the Sun that has been harnessed by humans since ancient times using a range of ever evolving technologies.
- According to the International Energy Agency, global capacity of solar PV had reached 402 gigawatts (GW) at the end of 2017.
- The research findings indicate that solar energy is the best renewable energy option for Pakistan due to many factors such as price, operation and maintenance costs and life span.
- Pakistan is blessed with 5.5 Wh m^{-2} d^{-1} solar exposure with annual mean sunshine duration of 8–10 h d^{-1} throughout the country.

8. **Wind Energy:**

- Wind energy describes the process which wind is used to generate electricity.
- Wind turbines convert the kinetic energy into mechanical power.
- An equivalent of 100 billion watts per year of power in the shape of wind energy is available on the earth.
- In the windy regions, wind mills are installed to produce mechanical energy.
- This mechanical energy may be used in tube wells or flour mills.
- Wind speed 5-7 ms^{-1} persists in the coastal regions of Sindh and Baluchistan provinces with more than 20,000 MW of economically feasible wind power potential.

FIGURE 4.21

Wind energy

MCQ's From Past Board Papers

1. **Biomass is the potential source of________.**
   (A) Renewable energy (B) Non- renewable energy (C) Both A and B (D) Tidal energy
2. **A solar cell converts the light energy into ________.**
   (A) Heat energy (B) Chemical energy (C) Electrical energy (D) Atomic energy
3. **Work done by gravitational field in displacing object up to certain height is ________.**
   (A) Negative (B) Zero (C) Minimum (D) Virtual
4. **Absolute P.E of an object at an infinite height w.r.t earth is taken as:**
   (A) Negative (B) Zero (C) Minimum (D) Virtual
5. **Escape velocity on the surface of earth is 11.2 km s^{-1} the escape velocity on the surface of another planet of same mass as that of earth but of $\frac{1}{4}$ times of the radius of earth is:**
   (A) 5.6 km s^{-1} (B) 11.2 km s^{-1} (C) 22.4 km s^{-1} (D) 44.8 km s^{-1}
6. **Escape velocity of a body of mass 1000 kg is 11 kms^{-1}. If the mass of the body is doubled then its escape velocity will be:**
   (A) 5.5 km s^{-1} (B) 11 km s^{-1} (C) 22 km s^{-1} (D) 44 km s^{-1}
7. **What is the value of escape velocity on earth?**
   (A) 1.1 km/s (B) 11 km/s (C) 1.1 km s (D) 1.1 cm/s

8. What is the required power to lift a mass of 5000g to height of 1 m in 2 second?
(A) 2.45 watt (B) 24.5 watt (C) 245 watt (D) 2.45 k watt

9. Which one is non-renewable source of energy?
(A) Wind (B) Biomass (C) Coal (D) Sunlight

Answers Key

1. A	2. C	3. A	4. B	5. C	6. B	7. B	8. B	9. C

FORMULAE

1	Work done by constant force	$W = \vec{F} \cdot \vec{d}$	
2	Work done by variable force	$W = \sum_{i=1}^{n} F_i \Delta d_i \cos\theta_i$	
3	Power	$P = \vec{F} \cdot \vec{v}$	$P = \frac{W}{t}$
4	Average Power	$P_{av} = \frac{\Delta W}{\Delta t}$	
5	Instantaneous Power	$P_{ins} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$	
6	Kinetic Energy	$K.E = \frac{1}{2} mv^2$	$K.E = \frac{P^2}{2m}$
7	Gravitational Potential Energy	$P.E = mgh$	
8	Work Energy Principle	$W = \frac{1}{2} mv_f^2 - \frac{1}{2} v_i^2$	
9	Absolute P.E	$U = \frac{-GMm}{r}$	$U = \frac{-GMm}{R}$ (at surface of earth)
10	Escape Velocity	$v = \sqrt{2gR}$	$v = \sqrt{\frac{GM}{R}}$
11	Conservation of Energy	$mg(h_1 - h_2) = \frac{1}{2} m (v_2^2 - v_1^2)$	$mg = \frac{1}{2} mv^2 + fh$

Key Points

- The work done on a body by a constant force is defined as the product of the displacement and the components of the force in the direction of the displacement.

  $W = \vec{F} \cdot \vec{d} = F d \cos\theta$, where 'θ' is angle between $\vec{F}$ and $\vec{d}$.
- When an object is moved in the gravitational field of the Earth, the work is done by the gravitational force. The work done in the Earth's gravitational field is independent of the path followed, and the work done along a closed path is zero. Such a force field is called conservative field.
- Power is defined as the rate of doing work and is expressed as $P = \frac{W}{t}$.

- Energy of a body is its capacity to do work. The kinetic energy is $K.E = \frac{1}{2}mv^2$
- The potential energy is possessed by a body because of its position in a force field.
- The absolute P.E of a body on the surface of Earth is

  Absolute Potential Energy $= \frac{-GM_e m}{R_e}$
- The initial velocity of a body with which it should be projected upward so that it does not come back, is called escape velocity.

  $v_{esc} = \sqrt{2gR} = 11\ km\ s^{-1}$.
- Some of the non-conventional energy sources are

(a)	Energy from the tides	(b)	Energy from waves
(c)	Solar Energy	(d)	Energy from biomass
(e)	Energy from waste products	(f)	Geothermal energy

Solved Examples

Example 4.1: **Calculate the power required of a 1400-kg car under the following circumstances: (a) the car climbs a 10" hill (fairly steep hill) at a steady and (b) the car accelerates along a level road from 90 kmh^{-1} to 100 kmh^{-1} in 6.0 s to pass another car. Assume the average retarding force on the car is throughout.**

Given Data:

Mass m = 1400kg, angle of climb = 10", initial speed v_i = 90kmh^{-1}, final speed v_f = 110 kmh^{-1}, time t = 60s

Required:

power P = ?

Solution:

(a) To move at a steady speed up the hill, the car must, by Newton's second law, exert a force F equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, mg sin 10". Thus

$\vec{F}$ = 700 N + mg sin 10"

= 700 N + (1400 kg) (9.80ms^{-2}) (0.174) = 3100 N.

Since v = 80 kmh^{-1} = 22 ms^{-1} and is parallel to $\vec{F}$ then the power is P = Fv = (3100 N) (22 ms^{-1}) = 6.8 × 10^4 W = 68 kW = 91 hp.

91 hp.

(b) The car accelerates from 25 ms^{-1} to 30.6 ms^{-1} (90 kmh^{-1} to 110 kmh^{-1}) on the flat. The car must exert a force that overcomes the 700-N retarding force plus that required to give it the acceleration a_x = (30.6 ms^{-1} – 25.0 ms^{-1})/ 6.0s = 0.93 ms^{-2}

We apply Newton's second law with x being the horizontal direction of motion (no component of gravity):

$ma_x = \Sigma\vec{F}_x = \vec{F} - {}_R$.

We solve for he force required, F

$\vec{F} = ma_x + \vec{F}_R$

= (1400 kg) (0.93 ms^{-2}) + 700N = 1300N + 700N = 2000 N.

Since the required power increases with speed and the motor must be able to provide a maximum power output in this case of

P = (2000 N) (30.6 ms) = 6.1 × 10^4 W = 61 kW = **82 hp.**

FIGURE 4.9

Example 4.2: **Find the work require to lift a mass of 5 tones to a height of 30m. If this is done in 2 minutes, what power is being used?**

Given Data:

Mass m = 500 kg, height h = 30 m, time t = 2 mint = 2 × 60s = 120 s

Required:

power P = ?, work w = ?

Solution:

W = Fd = mgh = (5000kg) (9.81ms-2) (30m) = 1471500J

$$P = \frac{W}{t} = \frac{1471500J}{120s}$$

P = 12262.5 Watt

Example 4.3: **A machine needed 1000J of energy to raise a 10 kg block at distance of 6.0m. What is the machine efficiency?**

Given Data:

Input work = 1000J, mass m = 10kg, distance d = 6.0m

Required:

Machine efficiency h = ?

Solution:

First, find the work done to raise the block: W = mgh

= 10kg × 9.8 m/s² × 6.0m = 588J

$$\text{Efficiency} = h = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{588}{1000} \times 100\% = 58.8\% = 59\%$$

Pulleys are machines used to lift heavy loads. Modern cranes are complicated form of pulley system.

h = 59%

Example 4.4: **Block and tackle system of pulleys is used to raise a load of 500N through a height of 20m. The work done against friction is 2000J. Calculate the (a) work done by the effort (b) the efficiency of the system.**

Given Data:

Load = w = 500N, height h = 20m, work against friction = 2000J

Required:

Worked done by effort = ?, efficiency h = ?

Solution:

(a) Work done by effort = work done in raising load + work done against friction

= 500 × 20 + 2000 = 12000J

(b) $$\text{Efficiency} = h = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{500 \times 20}{12000} \times 100\% = 83\%$$

12000J, & h = 83%

Example 4.5: **The moon's radius is 1.74×10^6m and the acceleration due to gravity, $g = 1.6\ ms^{-2}$ on its surface. Find out the escape velocity from moon's surface.**

Given Data:

Radius of moon = $R_m = 1.74 \times 10^6$m

Acceleration due to gravity = $g_m = 1.6ms^{-2}$

Required:

Escape velocity = V_{esc} = ?

Solution:

$$V_{esc} = \sqrt{\frac{2GM_m}{R_m}}$$

$V_{esc} = \sqrt{2g_mR_m}$ putting the values

$$V_{esc} = \sqrt{2 \subset 1.6 \times 1.74 \times 10^6}$$

$$\boxed{V_{esc} = 2.360 \times 10^3\ ms^{-1}}$$

Example 4.6: **Compare the escape speed of a rocket launched from the mmon with Earth. The mass of the moon is 7.35×10^{22} kg and the radius is 1.74×10^6 m.**

Given Data:

Mass of moon = $M_m = 7.35 \times 10^{22}$ kg

Radius $R = 1.74 \times 10^6$m

Required:

Speed v = ?

Solution:

$$v = \sqrt{\frac{2GM_m}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11} N.m^2/kg^2)}{1.74 \times 10^6 m}} = 2370 ms^{-1}$$

Notice that you can escape from the moon by traveling much more slowly than you must travel to escape the gravitational pull of Earth. This is why launching a Lunar Module from the moon's surface was so much easier than lunching an Apollo spacecraft from Earth.

$$\boxed{v = 2.370 \times 10^3\ ms^{-1}}$$

Example 4.7: **A ball of mass 100 g is thrown vertically upward at a speed of 25 m s^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction?**

Given Data:

(i) Height = h = ? Friction = f = ?

Mass = m = 100 g = 0.1 kg Speed = v = 25 m s^{-1}

Required:

(ii) Height = h = 25 m

Work done against air resistance = fh = ?

Solution:

(i) As f = 0so

Loss in K.E = Gain in P.E

$\frac{1}{2} mv^2 = mgh$

$\Rightarrow$ $h = v^2/2g$

Putting the values then:

$h = (25 \times 25) / (2 \times 9.8) = 31.9$ m

(ii) Loss in K.E = Gain in P.E + gh

$\frac{1}{2} mv^2 = mgh + fh$ $\Rightarrow fh = \frac{1}{2} mv^2 - mgh$

$fh = 0.5 \times 0.1 \times 25 \times 25 - 0.1 \times 9.8 \times 25 = 6.75$ J

Putting the value of h we get $f = \frac{6.75}{25} = 0.27$ N

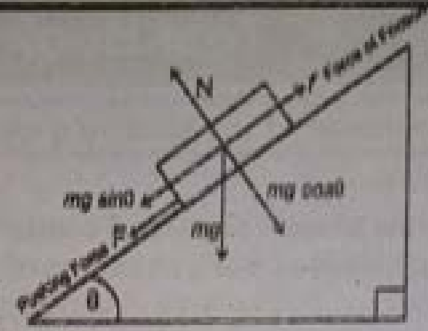
Text Book Exercises

Q.1 Select the correct answer of the following questions.

Choose the best possible answer

1. **You push a heavy crate down a ramp at a constant velocity. Only four forces act on the crate. Which force does the greatest magnitude of work on the crate?**
   A. The force of friction. B. The force of gravity C. The normal force. D. The force you pushing.
2. **The force constant of a wire is k and that of another wire is 3k when both the wires are stretched through same distance, if work done are W_1 and W_2, then:**
   A. $W_2 = W_1$ B. $W_2 = 9W_1$ C. $W_1 = 3W_2$ D. $W_2 = 3W_1$
3. **Escape velocity on the surface of the earth is $11.2kms^{-1}$. If the mass of the earth increases to twice its value and the radius of the earth becomes half, the escape velocity is:**
   A. $5.6kms^{-1}$ B. $11.2kms^{-1}$ C. $22.4kms^{-1}$ D. $33.6kms^{-1}$
4. **An example of non-conservative force is:**
   A. Electric force B. Gravitational Force C. Frictional force D. Magnetic force
5. **When the speed of your car is doubles, by what factor does its kinetic energy increase?**
   A. $\sqrt{2}$. B. 2. C. 4. D. 8.
6. **One horse power is given by:**
   A. 746 W B. 746 KW C. 746 MW D. 746 GW
7. **Work is said to be negative when $\vec{F}$ and $\vec{d}$ are:**
   A. Parallel B. Anti-parallel C. Perpendicular D. at 45°
8. **Two bodies of masses m_1 and m_2 have equal momentum their kinetic energies E_1 and E_2 are in the ratio:**
   A. $\sqrt{m_1} : \sqrt{m_2}$ B. $m_1 : m_2$ C. $m_2 : m_1$ D. $\sqrt{m_1^2} : \sqrt{m_2^2}$
9. **The atmosphere is held to the earth by.**
   A. Winds B. Gravity C. Clouds D. The rotation of earth
10. **If momentum is increased by 20% then K.E increases by:**
    A. 44% B. 55% C. 66% D. 77%
11. **If the K.E of a body becomes four times of the initial value, then new momentum will:**
    A. Become twice its initial value B. Become three times, its initial value C. Become four times, its initial value D. Remains constant.
12. **Two bodies with kinetic energies in the ratio of a 4 : 1 are moving with equal linear momentum. The ratio of their masses is:**
    A. 1 : 2 B. 1 : 1 C. 4 : 1 D. 1 : 4
13. **A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. A force of 0.2 N acts on it in the direction of motion of the body for 10s. The increase in its kinetic energy is.**
    A. 2.8 J B. 3.2 J C. 3.8 J D. 4.4 J
14. **If force and displacement of particle in the direction of force are doubled. Work would be:**
    A. Double B. 4 times C. Half D. $\frac{1}{4}$ times

No.	Option	ANSWER	EXPLANATION
1.	(A)	Force of Friction	• Since normal force and mg cosθ forces are perpendicular to motion of crate, so they donot do any work. Other forces do work on crate. • Crate is moving with constant velocity, so net force is zero. Along the ramp, two forces mg sinθ and Pushing force F is balanced by frictional force f (i.e. f = mg sinθ + F)

			 So, friction is greatest force, • Therefore, it performs greater work than other forces acting on block
2.	(D)	$W_2 = 3W_1$	Work done in stretching the first spring is $W_1 = E.P.E. = \frac{1}{2}kx^2$ For second spring, we can write as: $W_2 = \frac{1}{2}(3k)x^2 = 3\left(\frac{1}{2}(k)x^2\right)$ $\Rightarrow W_2 = 3W_1$
3.	(C)	22.4 km s^{-1}	$v_{esc} = \sqrt{\frac{2GM}{R}}$ Since $M' = 2M$ and $R' = \frac{R}{2}$ So, $v'_{esc} = \sqrt{\frac{2G(2M)}{\frac{R}{2}}}$ $\Rightarrow v'_{esc} = \sqrt{\frac{4(2GM)}{R}}$ $\Rightarrow v'_{esc} = 2\sqrt{\frac{2GM}{R}} = 2\ v_{esc}$ $\Rightarrow v'_{esc} = 2\,(11.2\ km\ s^{-1}) = 22.4\ km\ s^{-1}$
4.	(C)	Frictional Force	Frictional force is non conservative. Magnetic force under certain conditions can be considered as conservative. So, best option is (c).
5.	(C)	4	$K.E. = \frac{1}{2}mv^2$ $\Rightarrow K.E.' = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4\,K.E.$
6.	(A)	746 watts	1hp = 550 foot pound per second (550 ft lb /s) Since 1 pound (1lb)= 4.44822 N 1 foot (1ft) = 0.3048 m So, 1hp = 550 x 0.3048 m x 4.44822 N /s ⇨ 1hp = 745.69 Nm = 746 J/s 1hp = 746 W
7.	(B)	Anti-parallel	W = Fd (cos180) = Fd (-1) = - Fd
8.	(C)	$m_2 : m_1$	$\left[\because E = \frac{p^2}{2m} \Rightarrow p^2 = 2mE\right]$ $\because P_1 = P_2$ $\sqrt{2m_1E_1} = \sqrt{2m_2E_2}$

			$m_1E_1 = m_2E_2$ $\frac{E_1}{E_2} = \frac{m_2}{m_1}$
9.	(B)	Gravity	Atmosphere consists of gases attracted by earth
10.	(A)	44%	Initial K.E. $= \frac{P^2}{2m}$ New momentum $= P + \frac{P}{5} = \frac{6P}{5}$ **Remember** Initial momentum = P Increase in momentum $= 20\% = \frac{20}{100}P = \frac{1}{5}P$ Final K.E. $= \frac{\left(\frac{6P}{5}\right)^2}{2m} = \frac{36P^2}{25 \times 2m}$ Increase in K.E. $= K.E._f - K.E._i = \frac{36P^2}{25 \times 2m} - \frac{P^2}{2m}$ $= \frac{P^2}{2m}\left(\frac{36}{25} - 1\right)$ Increase in K.E. $= \frac{11}{25}\left(\frac{P^2}{2m}\right)$ % increase in K.E. $= \frac{\text{Increase in K.E.}}{\text{Initial K.E.}} \times 100$ $= \frac{\frac{11}{25}\left(\frac{P^2}{2m}\right)}{P^2/2m} \times 100$ % increase in K.E. $= 44\%$
11.	(A)	Becomes twice of its initial value	$P = \sqrt{2m\,K.E.} \longrightarrow (1)$ So, $P' = \sqrt{2m\,(4K.E.)}$ $\Rightarrow P' = 2\sqrt{2m\,(K.E.)}$ Putting value from Eq (1) $P' = 2P$
12.	(D)	1: 4	$K.E. = \frac{p^2}{2m}$ $K.E. \propto \frac{1}{m}$ (Since P is same) $\frac{m_1}{m_2} = \frac{K.E._2}{K.E._1}$ $\frac{m_1}{m_2} = \frac{1}{4}$ $m_1 : m_2 = 1 : 4$
13.	(D)	4.4 J	$K.E._i = \frac{p^2}{2m} = \frac{10^2}{2 \times 5} = 10\,J$ $\Delta P = F \times \Delta t = 0.2 \times 10 = 2\,Ns$ $P_f = P + \Delta P = 10 + 2 = 12$

			$K.E_f = \frac{P_f^2}{2m} = \frac{12^2}{2\times 5} = \frac{144}{10} = 14.4\,J$ $\Delta K.E. = K.E._f - K.E._i = 14.4 - 10 = 4.4\,J$
14.	(B)	4 times	W = Fd When force and displacement are doubled W' = (2F) (2d) = 4 Fd = 4 W

Short Answers of the Exercise

Q.2 Write short answers of the following questions.

1. A bucket is taken to the bottom of a well, does the bucket possess any P.E Explain?

Ans: Yes bucket possess P.E. when a bucket is taken to the bottom of a well.

- Bucket possess P.E. when a bucket is taken to the bottom of a well because some work is done against the up thrust of water. This work done is stored in the form of potential energy in the bucket.
- We can also say that the bucket possesses some negative potential energy with reference to the ground level.
- Also we can say that bucket has P.E. with respect to centre of earth.

2. When an arrow is shot from its bow, it has K.E. From where does it get the K.E?

Ans: When bow is stretched backward with arrow in it then elastic potential energy stores in the bow and arrow.

- Elastic P.E. stored in bow and arrow is equal to the work done to stretch the bow.
- When arrow is shot then its stored elastic P.E. converts into its K.E of arrow due to which the arrow moves in the forward direction.

3. Does a hydrogen filled balloon possess any P.E? Explain

Ans: Yes a hydrogen filled balloon possesses potential energy. The capacity of a body to do work is called energy. Hydrogen is the lightest gas and its density is less than air. Therefore the up thrust force acting on hydrogen filled balloon is greater than the weight of the balloon and the net force acting on the balloon is:

F_{net} = upthrust force − weight

$F_{net} = F_u - W$

$F_{net} = F_u - mg$

This net force acting on hydrogen filled balloon moves it upward.

Alternate Answer:

When hydrogen filled balloon moves up against the gravity its position change w.r.t ground and P.E increases.

Since P.E. = mgh

- P.E. $\propto$ h (height from ground)

4. Is K.E a vector quantity?

Ans: No, K.E. is a scalar quantity.

Reason: K.E. is scalar quantity as it can be expressed in term of scalar product.

$K.E. = \frac{1}{2} mv^2$

$K.E. = \frac{1}{2} m (\vec{v} . \vec{v})$ Since $(\vec{v} . \vec{v}) = v^2$

As mass of body is scalar and $(\vec{v} . \vec{v}) = v^2$ also provide scalar, K.E. of body is scalar quantity.

5. What happens to K.E. of a bullet when it penetrates into a target?

Ans: Possible Energy changes:

Moving bullet possess K.E. when it hits the target and penetrate, some part of its K.E. is used against the friction provided by target material and it also dissipates energy in the form of heat and sound.

Mathematically:

According to work energy principle

$$W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

When bullet comes to rest $v_f = 0$

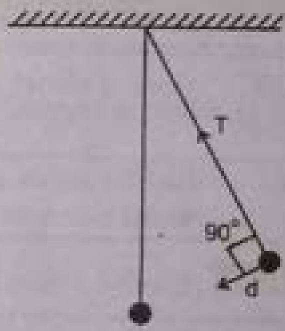
$$W = \frac{1}{2} m(0)^2 - \frac{1}{2} mv_i^2$$

$$W = -\frac{1}{2} mv_i^2$$

−ve sign shows that K.E is lost.

This loss of K.E is equal to work done to penetrate the bullet in to target.

6. Does the tension in the string of a swinging pendulum do any work? Explain.



Ans: No, it does not do any work.

Reason:

The tension in the string does not do work because it is always perpendicular to the curved path (i.e. instantaneous displacement).

Since, $W = F\ d \cos 90^\circ$

$\Rightarrow W = F\ d\ (0) = 0$

7. A meteor when enters into the earth's atmosphere burns. What happens to its energy?

Ans: When meteor enters the earth atmosphere then air friction acts on it. The huge amount of energy is used against the resistive force of the atmosphere, in bringing the meteor to rest. The loss of K.E appears in the form of heat energy which burns the meteor to ashes.

8. What type of energy is stored in the spring of watch?

Ans: A compressed spring of watch stores elastic P.E. in it, which is given by formula

$$E.P.E. = \frac{1}{2} K x^2$$

This elastic P.E stored in the spring is equal to work done to compress the spring.

This elastic P.E stored in the spring again convert into mechanical energy to move the arms of the watch.

9. A man drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

Ans: Energy changes

A cup thrown from certain height losses its gravitational potential energy and gain its K.E. When it strikes the ground then a part of this kinetic energy is used to break the cup and rest of the energy converts into;

(i) Sound energy

(ii) K.E of scattered moving pieces

(iii) Heat energy dissipated against friction.

10. A man rowing boat upstream is at rest with respect to shore, is he doing work?

Ans: No work is done as boat is at rest with respect to shore.

Reason:

As boat is at rest with respect to shore so it covers no displacement with respect to shore.

Therefore $d = 0$

Since $W = F\,d\cos\theta$

$\rightarrow$ $W = F\,(0)\cos\theta = 0$

11. Why energy savers are instead of normal bulbs?


Energy Savers

Ans: Energy savers are used instead of normal bulb due to the following reasons:

(i) The consumption of energy in energy savers is far less than normal light bulbs.

(ii) Energy savers produces light of high power and intensity as compared to normal light bulbs.

(iii) Energy savers produce very little amount of heat, while 98% of input is converted into heat in light bulbs.

Comprehensive Questions

Q3. Give a short response to the following questions.

1. Define work and show that it is the dot product of force and displacement. At what conditions work done will be maximum or minimum?

Ans: See Q # 1 from book.

2. Define power and show that power is the dot product of force and velocity. What are the different units of power used in our daily life?

Ans: See Q # 8, 9 from book.

3. Prove that Absolute P.E = $\frac{GmM_e}{R_e}$

Ans: See Q # 15 from book.

4. Calculate the values of the escape velocity of a body and show that it is equal 11.2 km s^{-1}.

Ans: See Q # 17 from book.

5. Describe briefly various non-conventional sources of energy.

Ans: See Q # 21 from book.

Numerical Problems

1. A 70 kg man runs up a long flight of stairs in 4 s. the vertical height of the stair is 4.5m. Calculate his power.

Given Data:

Mass of the man = $m = 70\ kg$
Time taken = $t = 4\ s$
Height of the stairs = $h = 4.5\ m$

To Find:

Power output = P = ?

Calculation:

As
$$power = P = \frac{W}{t}$$
or
$$P = \frac{mgh}{t}$$
Putting values, we get
$$P = \frac{70 \times 9.8 \times 4.5}{4}$$
$$P = \frac{3087}{4}$$
$$P = 771.5\ watt.$$
Or
$$P = 7.7 \times 10^2\ watt$$

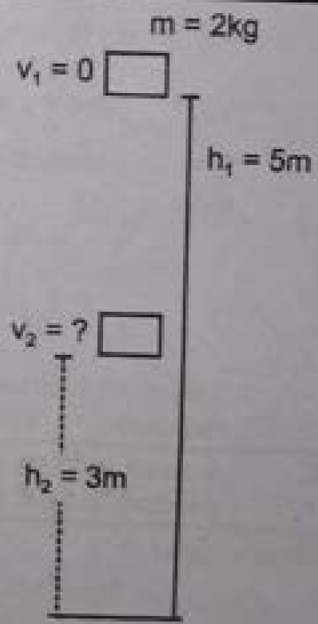
2. A body of mass 2.0 kg is dropped from a rest position 5m above the ground. What is its velocity at height of 3.0 m above the ground?

Given Data:

Mass of brick = $m = 2\ kg$
Initial velocity of brick = $v_1 = 0$
Initial height = $h_1 = 5m$
Final height = $h_2 = 3m$

To Find:

Velocity at height 3m above the ground = v_2 = ?



Calculation:

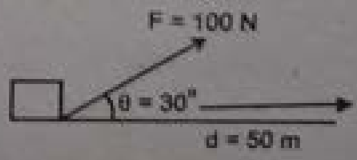
Loss of P.E = Gain in K.E.
$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2)$$
Putting values, we get
$$2 \times 9.8(5-3) = \frac{1}{2} \times 2(v_2^2 - 0^2)$$
$$2 \times 9.8 \times 2 = v_2^2$$
Or $v_2^2 = 39.2$ (As $v_2 = v$)

So $v = 6.3\ m\ s^{-1}$

3. A man pulls a trolley through a distance of 50m by applying a force of 100N which makes an angle of 30° with horizontal. Calculate the work done by the man.

Given Data:

Displacement = $d = 50\ m$
Applied Force = $F = 100N$
Angle = $\theta = 30°$



To Find:

Work done = W = ?

Calculation:

$W = F\,d\cos\theta$

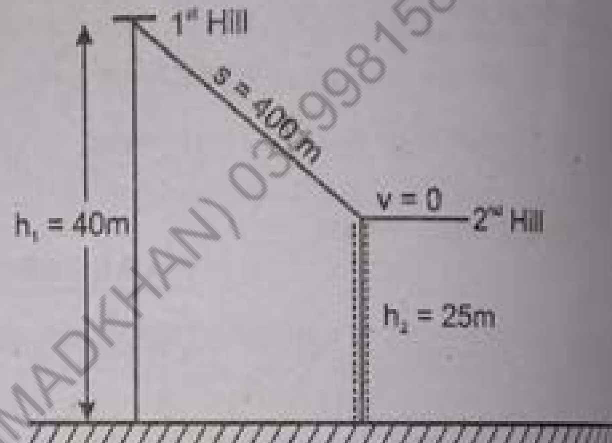
$\Rightarrow \quad W = 100 \times 50 \times \cos 30^\circ$

$\Rightarrow \quad W = 100 \times 50 \times 0.866$

$\Rightarrow \quad W = 4330\ J$

4. The roller-coaster car starts its journey from a vertical height of 40m on the first hill and reaches a vertical height of only 25 on the second hill, where it slows to a momentary stop. It travelled total distance of 400m. Determine the thermal energy produced and estimate the average friction force on the car whose mass is 1000 kg.

Given Data:



1st Hill = h_1 = 40m

2nd Hill = h_2 = 25m

At 2nd Hill velocity is equal to zero

$v = 0$

$g = 9.8\ ms^{-2}$

$m = 100\ kg$

$s = 400\ m$

To Find:

(a) $f = ?$

(b) Thermal Energy = ?

Solution:

According to Question

Law of Consecration of Energy

$(P.E)_{1st\ Hill} = (K.E)_{2nd\ Hill}$ + Work Done against friction

$mg(h_1 - h_2) = \frac{1}{2}mv^2 + w_f$

$\Rightarrow \quad 1000 \times 9.8\,(40 - 25) = \frac{1}{2} \times 1000 \times (0) + w_f$

$\Rightarrow \quad 147000\ J = w_f$

Now

$w_f = fh = 147000$

$fs = 147000$

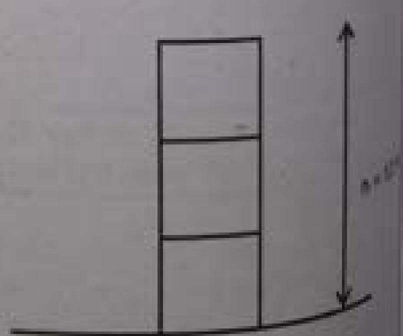
$f = \frac{147000}{s} = \frac{147000}{480} \quad \Rightarrow \quad \boxed{f = 367.5\ N}$

This work done against friction proudness same amount of thermal energy (Heat Energy)

Heat energy = W_f = 147000J

5. A man whose mass is 70kg walks up to the third floor of a building which is 12m above the ground in 20s. Find his power in watts and hp.

Given Data:



Mass of man = m = 70kg,

Height of building = h = 12m,

Time taken = t = 20 sec,

To Find:

(a) power in watt = ?

(b) power in horse power = ?

Solution:

(a) We know that,

Calculation:

$P = \frac{W}{t} \qquad [\because\ W = P.E. = mgh]$

$\Rightarrow \quad P = \frac{P.E.}{t}$

$\Rightarrow \quad P = \frac{mgh}{t}$

$\Rightarrow \quad P = \frac{(70 \times 9.8 \times 12)}{20}$

$\Rightarrow \quad \boxed{P = 411.6 \text{ watt}}$ [1 hp = 746 watts]

(b) $P = \frac{411.6}{746} \text{ hp}$

$\Rightarrow \quad \boxed{P = 0.55 \text{ hp}}$

6. To what height can a 400W engine lift a 100 kg mass in 3s?

Given Data:

Power = P = 400 w
mass = m = 100 kg
time = t = 3 s
g = 9.8 ms^{-2}

To Find:

h = ?

Solution:

$P = \frac{mgh}{t}$

$P \times t = mgh$

$h = \frac{P \times t}{mg} \Rightarrow \frac{400 \times 3}{100 \times 9.8}$

$\boxed{h = 1.22 \text{ m}}$

7. A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction.

Given Data: Mass of ball = m = 100gm = 0.1kg
Speed of ball = v = 25 m/sec.

To Find: (a) Height to which the ball would reach = h = ?
(b) If height = h′ = 25m, then
(i) work done = ? (ii) Force of friction = F = ?

Calculation: (a) If we ignore the air resistance, then,
Loss in K.E = Gain in P.E

$\Rightarrow \quad \frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v^2/2 = gh$

$\Rightarrow \quad gh = \frac{v^2}{2}$

$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{(25)^2}{2} \times 9.8 \quad \Rightarrow \quad h = 31.9 \text{ m}$

(b) (i) h′ = 25m, In case of air resistance, we have,
Loss in K.E = gain in P.E + work done against friction

$\Rightarrow$ work done = loss in K.E − gain in P.E

$\Rightarrow \quad W = \frac{1}{2}mv^2 - mgh'$

$\Rightarrow \quad W = \frac{1}{2} \times 0.1 \times (25)^2 - 0.1 \times 9.8 \times 25 \quad = (31.25 - 24.5)\ J$

$\Rightarrow \quad \boxed{W = 6.7\ J}$

(ii) we know that, work = Fd

$\Rightarrow \quad W = Fh' \qquad [d = h']$

$\Rightarrow \quad F = \frac{W}{h'} = \frac{6.7}{25}$

$\Rightarrow \quad F = 0.26\ N \qquad \Rightarrow \quad \boxed{F = 0.3\ N}$

8. An object of mass 1000 g falls from a height of 30m on the sand below. If it penetrates 4cm in the sand, what opposing force is exerted on it by the sand? Neglect air friction.

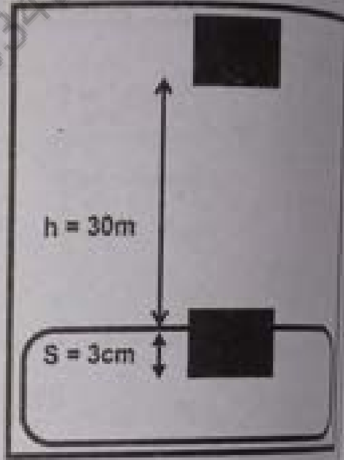
Data: m = 1000 g = 1 kg

Height = h = 30 m

Distance through which object penetrate in sand = S = 4 cm

S = 0.04 m

Opposing force = f = ?



Solution:

Loss of P.E = work done

$mgh = fS$

$f = \frac{mgh}{S}$

$= \frac{1 \times 9.8 \times 30}{0.04}$

$= 7350\ N$

9. A body of mass 'm' drops from Bridge into water of the river. The bridge is 10m high from the water surface.

(a) Find the speed of the body 5m above the water surface.

(b) Finds the speed of the body before it strikes the water.

Data: Mass = m

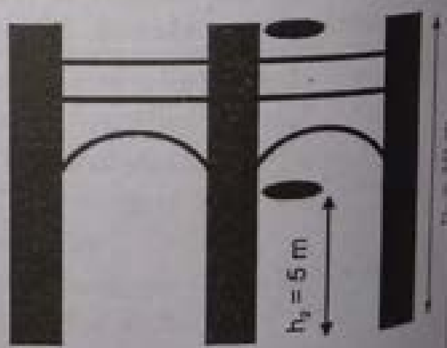
Height of bridge from water = h_1 = 10 m

Initial velocity = v_i = 0

(a) h_2 = 5m

v_f = ?

(b) final velocity at surface of water = v_f' = ?



Solution:

Given of K.E = Loss of P.E

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = mgh_1 - mgh_2$

$\frac{1}{2}mv_f^2 - \frac{1}{2}m(0)^2 = mg(h_1 - h_2)$

$\frac{1}{2}mv_f^2 = mg(h_1 - h_2)$

$v_f^2 = 2g(h_1 - h_2)$

Taking square root of both sides

$v_f = \sqrt{2g(h_1 - h_2)}$

$= \sqrt{2 \times 9.8\,(10 - 5)}$

$v_f = \sqrt{2 \times 9.8 \times 5}$

$\boxed{v_f = 9.9\ m/s}$

(b) $\frac{1}{2}mv'^2_f - \frac{1}{2}mv^2_i = mgh_1$

$\frac{1}{2}mv'^2_f - \frac{1}{2}m(0)^2 = mgh_1$

$\frac{1}{2}mv'^2_f = mgh_1$

$v'^2_f = 2gh_1$

Taking square root of both sides

$v'_f = \sqrt{2gh_1}$

$= \sqrt{2 \times 9.8 \times 10}$

$\boxed{v'_f = 14\ m/s}$

10. The engine of a JF – Thunder fighter develops a thrust of 3000N. What horse power does it at a velocity of 600 m/s.

Solution:

Given Data: Thrust of engine = F 3000N
Velocity = v = 600 m/sec

To Find: Power in horse power = P = ?

Calculation:

We know that, $P = \vec{F} \cdot \vec{v}$

⇒ $P = Fv \cos\theta$

⇒ $P = 3000 \times 600 \times \cos 0°$ [$\theta = 0°$ and $\cos 0 = 1$]

⇒ $P = 1800000 \times 1 = 1800000$ watt

⇒ $P = 1800000$ watt [1h.p = 746 watt]

⇒ $P = (1800000/746)$ h.P

⇒ $P = 2412.9$ h.p

⇒ $\boxed{P = 2413\ h.p}$

11. The mass of the moon 1/80 of the mass of the earth and corresponding radius is 1/4 of the earth. Calculate the escape velocity on the surface of moon.

Solution:

Given Data:

Mass of moon = $M_m = \frac{1}{80} \times M_e$

Radical of moon = $R_m = \frac{1}{4} \times R_e$

To Find:

$V_{esc} = ?$ (at moon)

Solution:

$V_{esc} = \sqrt{\frac{2GM_m}{R_m}} \Rightarrow \sqrt{\frac{2G \times M_e}{80 \times \frac{1}{4} \times R_e}}$

$V_{esc} = \sqrt{\frac{2 \times 4GM_m}{80 \times R_e}} \Rightarrow \sqrt{\frac{8 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{80 \times 6.4 \times 10^6}}$

$V_{esc} = 2500\ ms^{-1}$

$\boxed{V_{esc} = 2.5\ km\ s^{-1}}$

Additional Conceptual Short Questions With Answers

1. A car is accelerated from rest to a speed of $10ms^{-1}$. Let the energy spent be E. Then if we accelerate the car from $10ms^{-1}$ to $20ms^{-1}$. How much energy will be spent?

Ans: For 1st case

From work - Energy theorem

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$E = \frac{1}{2} m \left[v_f^2 - v_i^2\right]$$

$$E = \frac{1}{2} m \left[(10)^2 - (0)^2\right]$$

$$= \frac{1}{2} m \left[100 - 0\right]$$

$$\boxed{E = \frac{1}{2} m (100)} \quad (1)$$

Now for 2nd case

$$W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$E' = \frac{1}{2} m \left[(20)^2 - (10)^2\right]$$

$$E' = \frac{1}{2} m \left[400 - 100\right]$$

$$E' = \frac{1}{2} m (300)$$

$$E' = \left(\frac{1}{2} m (100)\right) 3 = 3\left[\frac{1}{2} m (100)\right]$$

$\Rightarrow$ $\boxed{E' = 3E}$ Energy will increase by 3 times.

2. Show that $K.E. = \frac{P^2}{2m}$ where P is momentum and m is mass of body.

Ans:

$$K.E. = \frac{1}{2} mv^2$$

Multiplying and dividing R.H.S by m

$$K.E. = \frac{1}{2} mv^2 \times \frac{m}{m}$$

$$K.E. = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m}$$

Putting P = mv

$$K.E. = \frac{P^2}{2m}$$

3. A light body and a heavy body have equal momenta. Which of the two has larger K.E? Also find ratios of their energies?

Ans: We know that

$$E = \frac{P^2}{2m}$$

$$\Rightarrow \quad E \propto \frac{1}{m}$$

⇒ Energy and mass are inversely proportional to each other if momentum is kept constant. So lighter body will have greater energy.

Let for lighter body

$$E_1 = \frac{P^2}{2m_1}$$

Let for heavier body

$$E_2 = \frac{P^2}{2m_2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{\frac{P^2}{2m_1}}{\frac{P^2}{2m_2}}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$$

It also shows that energy and mass of the body are inversely proportional if P is constant.

4. What is the velocity of the particle if momentum and K.E. are numerically equal?

Ans: From the given condition

$$P = K.E$$

$$mv = \frac{1}{2}mv^2$$

$$1 = \frac{1}{2}v$$

$$\Rightarrow \boxed{v = 2ms^{-1}}$$

5. A heavy body and a light body have equal K.E. which of the two has larger momentum ?

Ans: Let two bodies of masses m_1 and m_2 and momentum P_1 and P_2 respectively.

$$K.E_1 = \frac{P_1^2}{2m_1}$$

$$K.E_2 = \frac{P_2^2}{2m_2}$$

As their K.E. is equal

$$K.E_1 = K.E_2$$

$$\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$$

$$\frac{P_1^2}{m_1} = \frac{P_2^2}{m_2}$$

$$\frac{P_1^2}{P_2^2} = \frac{m_1}{m_2}$$

or

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$$

If $m_1 > m_2$ then $P_1 > P_2$ i.e., the heavier body has larger momentum.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **SI unit of energy is:**
   A. erg B. calorie C. joule D. all of these

2. **1 Kwh = _______:**
   A. 3.6 MJ B. 36×10^5 J C. 3.6×10^6 J D. all of these

3. **If body of mass of 2 kg is raised vertically through 2m, then the work done will be:**
   A. 4 J B. 38.2J C. 39.2 J D. 40 J

4 **Two masses 1g and 4g are moving with K.E the ratio of their momenta are:**
   A. 1: 16 B. 1:2 C. 1 : 4 D. None

5. **1 kg block is dropped from a height of 5m, its velocity just before hitting the ground is:**
   A. $\sqrt{9.8}$ B. 5 C. 9.8 D. $7\sqrt{2}$

6. **Brick of mass 2kg dropped rest from height of 4m, then its velocity at height 2m above ground is:**
   A. 3.6 m/s B. 8.6.m/s C. 6.26 m/s D. 7.8 m/s

7. **KWH is unit of:**
   A. Energy B. Power C. Force D. None

8. **Energy released by burning of 1 liter of petrol:**
   A. 100 J B. 7×10^3 J C. 5×10^7 J D. 4×10^3 J

9. **On a clear day intensity of solar energy reaching earth surface is about:**
   A. 1.4 kwm^{-2} B. 1 kwm^{-2} C. 1.2 kwm^{-2} D. 1.6 kwm^{-2}

10. **If K.E of moving body is doubled then momentum becomes:**
    A. $\sqrt{2}$ times B. 2 times C. 3 times D. 4 times

11. **Which of the following is the example of conservative force:** (FBISE
    A. Tension in the string B. Propulsion force of rocket
    C. Gravitational field D. Restoring force in compressed spring

12. **Anybody requires ____ escape velocity, to escape from the gravitational pull of the mars:**
    A. 2.4 km/s B. 4.3 km/s C. 5 km/s D. 10.4 km/s (FBISE

13. **A brick of mass 2 kg is dropped from a rest position 5 m above the ground. What is its velocity at a height of 3 m above the ground?** (FBISE
    A. 12.4 m/s B. 6.3 m/s C. 7 m/s D. 1.2 m/s

14. **Anybody requires __________ escape velocity, to escape from the gravitational pull of the Venus:** (FBISE
    A. 5.5 km/s B. 2.4 km/s C. 4.3 km/s D. 10.4 km/s

15. **When the angle between force and displacement is greater than 90°, the work done is:** (FBISE
    A. Negative B. Positive C. Maximum D. Zero

16. **Which of the following is non-conservative force?** (FBISE
    A. Electric force B. Elastic spring force C. Gravitational force D. Normal force

17. **1kWh is equal to:** (FBISE
    A. 3.6×10^6 J B. 3.6×10^{-6} J C. 3.60×10^9 J D. 3.6×10^{-9} J

18. **If angle 'θ' is greater than 90°, the work done is:** (FBISE
    A. Maximum B. Positive C. Zero D. Negative

19. **One horse power is equal to:** (FBISE 2019)

A. 746 Joules B. 746 KW C. 746 N D. 746 Watt

20. **An example of non-conservative force is:** (FBISE 2019)

A. Electric force B. Magnetic force C. Gravitational force D. Frictional force

21. **The expression for escape velocity is given by:** (FBISE 2019)

A. $2gR^2$ B. $\sqrt{2gR}$ C. $\frac{gR^2}{2}$ D. $2gR$

Answers Key

1.	C	2.	D	3.	C	4.	B	5.	D
6.	C	7.	A	8.	C	9.	B	10.	A
11.	C	12.	C	13.	B	14.	D	15.	A
16.	D	17.	A	18.	D	19.	D	20.	D
21.	B								

SELF – ASSESSMENT PAPER

Total Marks: 40

Question.No.1 Choose the correct answer from the given options. (1 x 6 = 6)

SECTION – A

1. **The area under the curve of force-displacement graph represents:**
   (A) Force (B) Work (C) Power (D) Displacement
2. **Which one is a conservative force?**
   (A) Elastic Spring Force (B) Frictional Force (C) Air Resistance (D) Tension in the string
3. **Escape velocity on the surface of the earth is $11.2kms^{-1}$. If the mass of the earth increases to twice its value and the radius of the earth becomes half, the escape velocity is:**
   (A) $5.6kms^{-1}$ (B) $11.2kms^{-1}$ (C) $22.4kms^{-1}$ (D) $33.6kms^{-1}$
4. **One horse power is given by:**
   (A) 746 W (B) 746 KW (C) 746 MW (D) 746 GW
5. **If momentum is increased by 20% then K.E increases by:**
   (A) 44% (B) 55% (C) 66% (D) 77%
6. **Which one is the biggest unit of energy?**
   (A) erg (B) joule (C) watt-hour (D) kilo-watt hour
7. **A body of mass 1kg drops from the top of tower of height 50m, what will be its K.E 10m below the top:**
   (A) 490 J (B) 49 J (C) 98 J (D) 980 J

Question.No.2 Give short answers of followings. (3 x 7 = 21)

SECTION – B

(i) Define kilo watt hour (kWh). Show that 1kWh = 3.6 M J
(ii) Show that in gravitational field Work done along a closed path in a gravitational is zero.
(iii) Show that instantaneous power, $P = \vec{F} \cdot \vec{v}$?
(iv) Is K.E a vector quantity?
(v) A man drops a cup from a certain height, which breaks into pieces. What energy changes are involved?
(vi) A man whose mass is 70kg walks up to the third floor of a building which is 12m above the ground in 20s. Find his power in watts and hp.
(vii) Show that escape velocity of the body from earth is $v_{esc} = \sqrt{\frac{2GM_e}{R_e}}$

Question.No.3 Extensive Questions. (13)

SECTION – C

(a) Define absolute Potential energy. Derive its formula (07)
(b) A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction. (06)

The End

CHAPTER

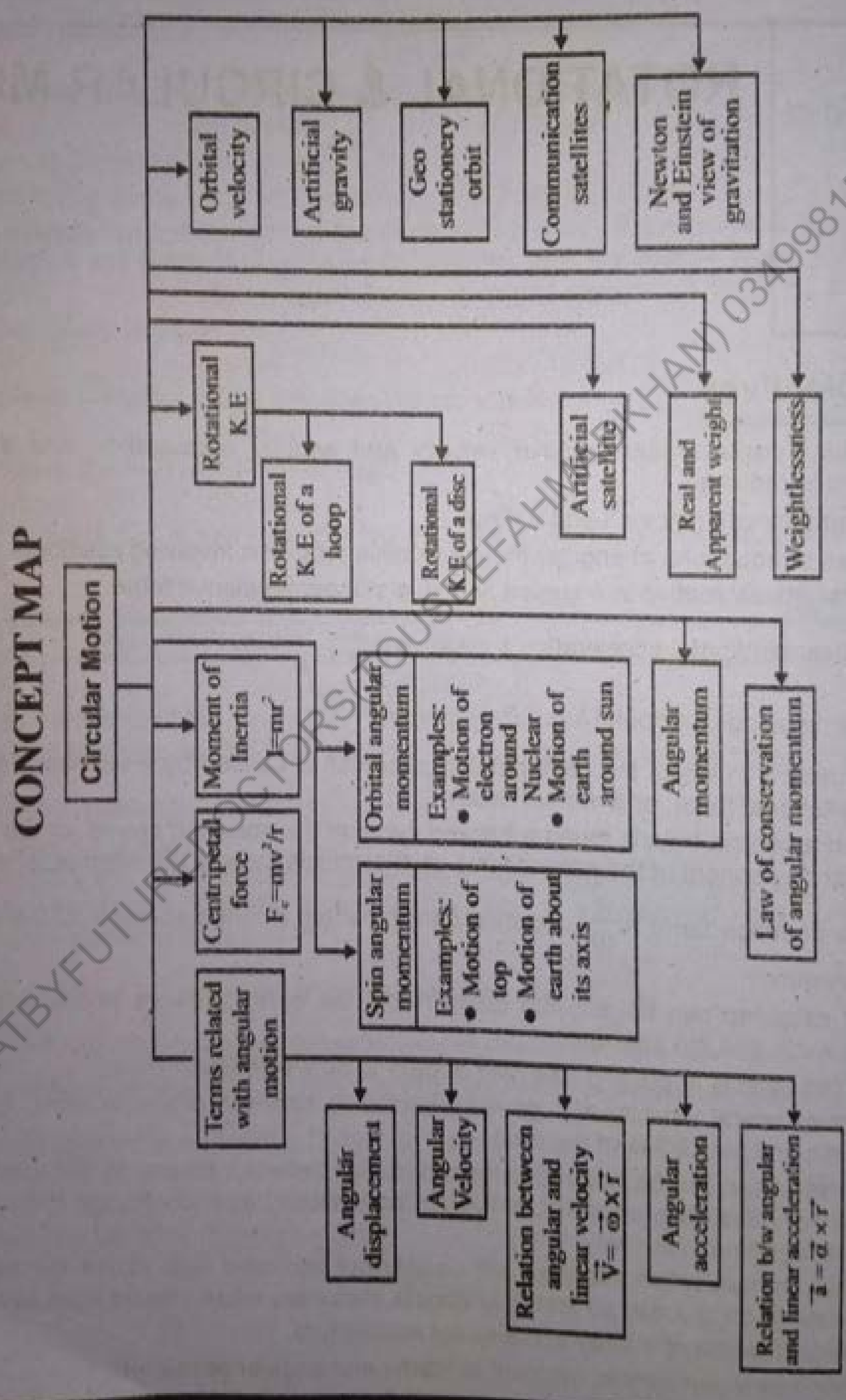
5

ROTATIONAL & CIRCULAR MOTION

Learning Objectives

- Define angular displacement, angular velocity and angular acceleration and express angular displacement in radians.
- Solve problems by using $S = r\theta$ and $v = r\omega$.
- State and use of equations of angular motion to solve problems involving rotational motions.
- Describe qualitatively motion in a curved path due to a perpendicular force.
- Derive and use centripetal acceleration $a = r\omega^2$, $a = \frac{v^2}{r}$.
- Solve problems using centripetal force $F = mr\omega^2$, $F = \frac{mv^2}{r}$.
- Describe situations in which the centripetal acceleration is caused by a tension. Force, a frictional force, a gravitational force, or a normal force.
- Explain when a vehicle travels round a banked curve at the specified speed for the banking angle, the horizontal component of the normal force on the vehicle causes the centripetal acceleration.
- Describe the equation $\tan\theta = \frac{v^2}{rg}$, relating banking angle θ to the speed v of the vehicle and the radius of curvature r.
- Explain that satellites can be put into orbits round the earth because of the gravitational force between the earth and the satellite.
- Explain that the objects in orbiting satellites appear to be weightless.
- Define the term orbital velocity and derive relationship between orbital velocity, the gravitational constant, mass and the radius of the orbit.
- Analyze that satellites can be used to send information between places on the earth which are far apart, to monitor conditions on earth, including the weather, and to observe the universe without the atmosphere getting in the way.
- Describe that communication satellites are usually put into orbit high above the equator and that they orbit the earth once a day so that they appear stationary when viewed from earth.
- Define moment of inertia of a body and angular momentum.
- Derive a relation between torque, moment of inertia and angular acceleration.
- Explain conservation of angular momentum as a universal law and describe examples of conservation of angular momentum.
- Use the formulae of moment of inertia of various bodies for solving problems.

Chapter No. 5


CONCEPT MAP
Circular Motion
Terms related with angular motion
Angular displacement
Angular Velocity
Relation between angular and linear velocity $\vec{V} = \vec{\omega} \times \vec{r}$
Angular acceleration
Relation b/w angular and linear acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$
Centripetal force $F_c = mv^2/r$
Moment of Inertia $I = mr^2$
Spin angular momentum
Examples: • Motion of top • Motion of earth about its axis
Orbital angular momentum
Examples: • Motion of electron around Nuclear • Motion of earth around sun
Angular momentum
Law of conservation of angular momentum
Rotational K.E
Rotational K.E of a hoop
Rotational K.E of a disc
Artificial satellite
Real and Apparent weight
Weightlessness
Orbital velocity
Artificial gravity
Geo stationery orbit
Communication satellites
Newton and Einstein view of gravitation

Q.1 Define and explain angular displacement.

Ans: **Angular Displacement**

The angle subtended at the center of a circle by an arc along which a body moves on the circumference in a given time is called angular displacement.

OR

When a rigid body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.

It is denoted by $\Delta\theta$.

For small value of $\Delta\theta$, the angular displacement is a vector quantity.

This angle $\theta = \angle AOB$ is the angular displacement of wheel after given a small push.

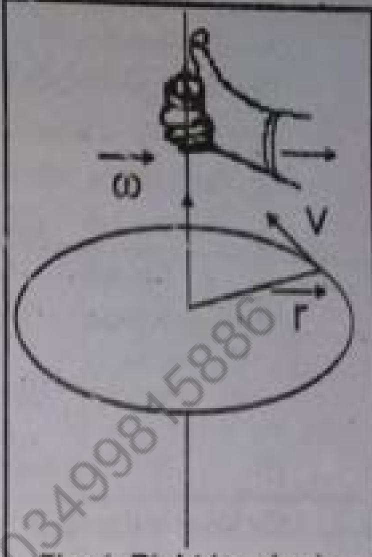


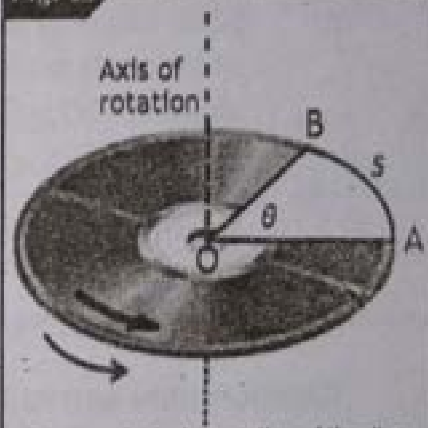
Fig: 1. Right hand rule.

Sign Convention:

For anticlockwise rotation of a body between two points on circumference, the angular displacement $\Delta\theta$ is positive.

For the clock-wise rotation, the angular displacement $\Delta\theta$ is negative.

Fig: 2.



Counter clockwise rotation of the disc.

Direction of Angular Displacement:

In order to determine the direction of angular displacement, we use the '*right hand rule*'

Right hand Rule:

"Grasp the axis of rotation in right hand with fingers curling in the direction of rotation then the erect thumb indicates the direction of angular displacement."

Units:

Angular displacement is measured in degrees, or revolutions or radians.

Radian: The SI unit of angular displacement is radian.

"It is the angle subtended by an arc at the center of the circle whose length is equal to the radius of circle."

Its other its famous units are degrees and revolution.

Degree:

"It is the angle subtended at centre of circle by $\frac{1}{360}$ th part of its circumference."

Revolution:

The angle subtended by a complete round trip of the body along the circumference of the circle is called one revolution.

Dimension: Angular displacement has *no* dimensions.

Q.2 Derive the relation between linear displacement and angular displacement OR prove that $S = r\theta$

Ans: **Relation between linear displacement and angular displacement**

Consider a particle that is moving in a circle of radius 'r' with center at O. Let particle moves from point "A" to point "B" in a circle such that

$\angle AOB = 1$ rad. $\quad\therefore\quad$ arc $AB = r =$ radius of circle.

We take point "D" very near to 'B', so that arc DB = S (approximately). Angle corresponding to arc DB is $\angle DOB = \theta$.

By basic geometry, we know that:

Arc length ∝ angle subtended

So, their ratios will be equal.

$$\frac{\text{Arc DB}}{\text{Arc AB}} = \frac{\angle DOB}{\angle AOB}$$

In other words:

$$\frac{S}{r} = \frac{\theta}{1\text{rad}} \text{ (using diagram)}$$

If $\Rightarrow$ $S = r\theta$ (θ is in radian).

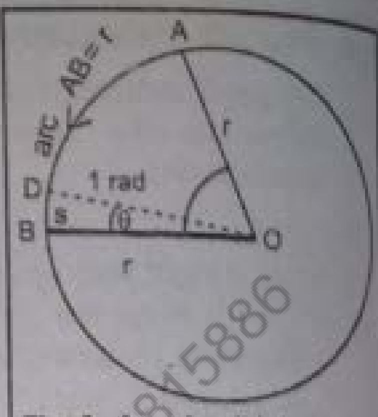


Fig: 3. Angular didplacement

Q.3 Define different units of angular displacement, radian, degree and revolution. Also derive different relations between them.

Ans: **Relation between radian, degree and revolution**

(i) *If the length of the arc is equal to the radius of the circle then the angle subtended at the centre of the circle is one radian (from Fig. 3), it is seen that:*

arc AB = AO = BO = r = radius of circle.

Thus angle $\angle AOB = \theta$

By definition $\theta = \frac{S}{r} = \frac{\text{arc AB}}{r} = \frac{r}{r} = 1 \text{ radian}$

(iii) Number of radians in one revolution $= \frac{S}{r} = \frac{\text{Circumference}}{\text{radius of circle}} = \frac{2\pi r}{r} = 2\pi$ rad

Relation between radian and degree

For one revolution $\theta = 360^\circ$

So, 2π radian $= 360^\circ = 1$ revolution

Or $1 \text{ rad} = \frac{360^\circ}{2\pi}$

Or $1 \text{ rad} = \frac{360^\circ}{2 \times 3.14}$

Or $\boxed{1 \text{ radian} = 57.3^\circ}$

Since, $1 \text{ rad} = \frac{360^\circ}{2\pi}$

So, $1^\circ = \frac{2\pi}{360}$ rad

Or $1^\circ = \frac{2 \times 3.14}{360}$ rad

Or $1^\circ = 0.0174$ rad

Q.4 Define and explain the term angular velocity.

Ans: **Angular Velocity**

Time rate of change of angular displacement is called as angular velocity.

$$\vec{\omega} = \frac{\overrightarrow{\Delta\theta}}{\Delta t}$$

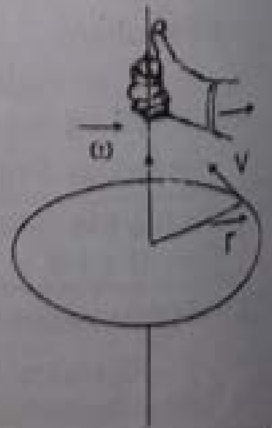


Fig:5.1(a) Right hand rule

Average Angular velocity:

Total change in angular displacement in total time taken is called average angular velocity. Suppose $\Delta\theta$ is the angular displacement during the time Δt. So the average angular velocity can be expressed as,

$$<\vec{\omega}_{av}> = \frac{\overrightarrow{\Delta\theta}}{\Delta t}$$

Instantaneous Angular Velocity:

It is the angular velocity of the body at any instant.

OR

The instantaneous angular velocity can be defined as the limiting value of $\Delta\theta/\Delta t$ as the time interval Δt, following the time t, approaches to zero.

$$\vec{\omega}_{ins} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta\theta}}{\Delta t}$$

Note

If a body moves with uniform angular motion, its average angular velocity will be equal to its instantaneous angular velocity.

Direction:

Angular velocity is a vector quantity. Its direction is along the axis of rotation and can be determine by **right hand rule.**

Unit:

The SI unit of angular velocity is rad/s (radian per second). it is also measured in revolution/minute and degree/second. The dimension of angular velocity is $[T^{-1}]$.

Assignment 5.1:

A rotating pulley completes 12rev in 4s. Determine the average angular velocity in rev/s, rpm, and rad/s?

Given Data:

$\theta = 12\ rev = 12 \times 2\pi\ rad = 24\ \pi\ rad$

$t = 4\ s$

$\omega = ?$

$\omega = \frac{\theta}{t}$

⇨ $\omega = \frac{12}{4} = 3\ rev/s$

⇨ $\omega = \frac{12}{4} = 3 \times 60\ rev/\min = 180\ rev/min = 180\ rpm$

⇨ $\omega = \frac{\theta}{t} = \frac{24\pi}{4} = 6\pi \frac{rad}{s} = 6 \times 3.14 \frac{rad}{s} = 18.84\ rad\ s^{-1}$

Q.5 Define and explain the angular acceleration?

Ans: Angular Acceleration

The time rate of change of angular velocity is called angular acceleration.

Average Angular Acceleration

When we switch on electric fan, the angular velocity goes on increasing. If ω_i is the initial angular velocity and ω_f is the final angular velocity at time t_i and t_f respectively. Then the average angular acceleration during time $t_f - t_i$ is can be defined as ***"the ratio of total change in angular velocity to the total time interval"***

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i}$$

Or $$\vec{\alpha} = \frac{\overrightarrow{\Delta\omega}}{\Delta t}$$

FOR YOUR INFORMATION

When angular velocity of the body is increasing then angular acceleration is along the direction of angular velocity and if angular velocity is decreasing then 'd' is opposite to the direction of angular velocity.

Instantaneous angular acceleration

The instantaneous angular acceleration can be defined as the limiting value of $\frac{\Delta\omega}{\Delta t}$ as the time interval Δt approaches to zero, is called instantaneous angular acceleration.

So, $$\vec{\alpha}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t}$$

Direction:

It is a vector quantity. The direction of angular acceleration is along the axis of rotation.

Axis of Rotation

All particles of a rotating body moves in circles. The line joining the centres of these circles is called axis of rotation.

Unit:

The SI unit of angular acceleration is rad/s^2. Its dimension are $[T^{-2}]$.

MCQ's From Past Board Papers

1. Which of the following is equal to one radian?
(A) 0° (B) 90° (C) 57.3° (D) 180°
2. 30° is equal to _____ radian
(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{5}$ (D) $\frac{\pi}{2}$
3. One radian equal to
(A) 2π rev (B) $\frac{\pi}{4}$ rev (C) $\frac{\pi}{2}$ rev (D) $\frac{1}{2\pi}$ rev
4. The angle subtended at the center by circumference of a circle is
(A) π radian (B) π radian (C) 2π radian (D) $\frac{\pi}{2}$ radian
5. 100 radians equals to
(A) 57.3° (B) 573° (C) 5730° (D) 5.73°
6. Which of the following are the dimensions of angular acceleration?
(A) $[T^{-1}]$ (B) $[LT^2]$ (C) $[T^{-2}]$ (D) $[T^{-3}]$
7. The angle through which a body moves is:
(A) Angular velocity (B) Angular acceleration (C) Angular displacement (D) Angular momentum
8. The ratio of circumference of a circle to its diameter is equal to
(A) 2π (B) π (C) $\frac{\pi}{2}$ (D) One Steradian
9. How many degrees are in 1 revolution?
(A) 57° (B) 90° (C) 180° (D) 360°
10. π-radian is equal to:
(A) 0° (B) 90° (C) 57.3° (D) 180°
11. Which of the following quantity is Dimensionless?
(A) Angular velocity (B) Angular acceleration (C) Centripetal force (D) Angular displacement
12. A wheel of radius 2 m turns through an angle of 57.3°. It lays out a tangential distance:
(A) 2 m (B) 4 m (C) 57.3 m (D) 114.6 m
13. 2 radian = _______
(A) 114.6° (B) 57.3° (C) 75.3° (D) 37.5°
14. Direction of angular acceleration is always along:
(A) x-axis (B) y-axis (C) z-axis (D) the axis of rotation

Answers Key

1. C	2. B	3. D	4. C	5. C	6. C	7. C	8. B	9. D	10. D	11. D	12. A
13. A	14. D										

Q.6 Define and explain angular displacement. Show that 1 radian = 57.3°

Ans: **Relation between Angular and Linear motions**

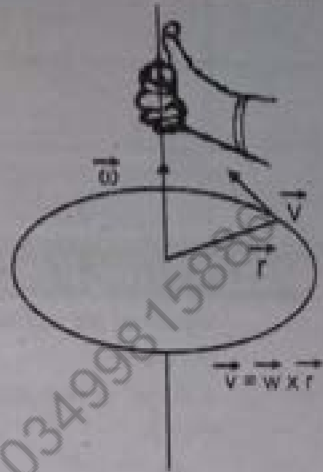
Relation between Angular and Linear velocities:

Consider a particle that is moving in a circle of radius 'r' with center at O. Let particle move from point "A" to point "B" in a circle such that it

If we take 'θ' in radians, then $S = r\theta$ ----------(1)

Similarly, in linear, motion, when a body moves with uniform velocity $\vec{v}$, in time 't', its linear displacement will be:

$$S = vt \quad \text{------------(2)}$$



Comparing the above equations, we can derive

$$vt = r\theta$$

$$v = r\left(\frac{\theta}{t}\right)$$

Putting $\frac{\theta}{t} = \omega$

$$v = r\omega$$

In vector form, we can write $\vec{v} = \vec{\omega} \times \vec{r}$

In magnitude form, $v = r\omega \sin\theta$

{For circular motion, $\theta = 90^\circ$ i.e. r is perpendicular to v}

Above vector form shows that $\vec{\omega}$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{v}$ (it is always along axis of rotation.)

$$\Rightarrow \quad v = r\omega \sin 90 = r\omega\,(1) = r\omega$$

Relation between Angular and Linear accelerations:

By definition of acceleration of the body is:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

Where $\vec{v}_i$ is initial linear velocity, $\vec{v}_f$ is final linear velocity, this change in velocity occurs in time 't'.

Putting $\vec{v}_f = \vec{\omega}_f \times \vec{r}$ and $\vec{v}_i = \vec{\omega}_i \times \vec{r}$

$$\vec{a} = \left(\frac{\vec{\omega}_f \times \vec{r} - \vec{\omega}_i \times \vec{r}}{t}\right)$$

$$\Rightarrow \vec{a} = \left(\frac{\vec{\omega}_f - \vec{\omega}_i}{t}\right) \times \vec{r}$$

Putting $\frac{\vec{\omega}_f - \vec{\omega}_i}{t} = \alpha$

$$\Rightarrow \vec{a} = \vec{\alpha} \times \vec{r}$$

Or $a = \alpha r \sin\theta\,\hat{n}$

Which relates linear acceleration and angular acceleration.

In special case, when angle between r and α is 90° in case of circular motion, then:

$$a = r\alpha, \qquad \therefore \sin 90^\circ = 1$$

The direction of $\vec{a}$ will be along thumb using right hand rule when $\vec{r}$ and $\vec{\alpha}$ are multiplied.

This is the tangential acceleration a_t.

Q.7 How can we write the equations of motion in case of angular motion?

Ans: Equation of Angular Motion

Equation of angular motion are similar to those in linear motion except that S, v and a have been replaced with θ, ω and α respectively. Thus

Equations for Linear Motion	Equations for Angular Motion
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$S = v_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$

Equation of angular motion hold only if axis of rotation is fixed. In this case all the angular vectors have the same direction. So they can be treated as scalars.

MCQ's From Past Board Papers

1. Which of the following is correct?
   (A) $\omega = vr$ (B) $v = r/\omega$ (C) $v = r\omega$ (D) $\omega = r/v$
2. A wheel of radius 50 cm having the angular speed of 5 rad/s will have linear speed in m/s:
   (A) 1.5 (B) 2.5 (C) 3.5 (D) 4.5
3. $1\frac{rev}{min}$ is equal to:
   (A) $\frac{\pi}{6}$ rad s^{-1} (B) $\frac{\pi}{15}$ rad s^{-1} (C) $\frac{\pi}{20}$ rad s^{-1} (D) $\frac{\pi}{30}$ rad s^{-1}
4. The dimensions of angular velocity are:
   (A) $[T^{-1}]$ (B) $[LT^{-1}]$ (C) $[LT^{-2}]$ (D) $[L^{-1}T]$
5. The rate of change of angular velocity is called:
   (A) Angular velocity (B) Angular accelerartion (C) Angular displacement (D) Angular speed

Answers Key

1. C	2. B	3. D	4. A	5. B

Q.8 Define centripetal force and centripetal acceleration. Derive their equations

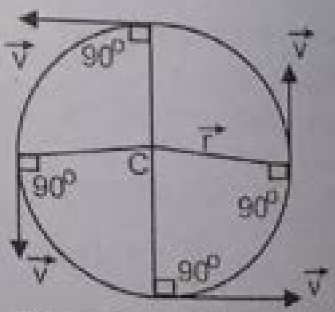
Ans: **Centripetal Force and Centripetal Acceleration**

Centripetal Force

The force which bends the normally straight path of a particle into circular path is called centripetal force.

OR

A force which compels a body to move in a circular path is called centripetal force



At each instant and at each point we see the direction of velocity change

Explanation:

If an object is moving in a circle or along the arc of a circle, it follows that there must be a force acting on it, to change its direction. Moving in a circle means that the direction of motion is constantly changing.

From Fig, we see that, at each instant and at each point, the direction of velocity of body changes.

Formula:

$$F_c = \frac{mv^2}{r}$$

Centripetal Acceleration:

The change in velocity of body produces acceleration directing towards the centre of circle. Such acceleration is known as centripetal acceleration. It is always directed towards the centre of circle.

Mathematically centripetal acceleration is: $a_c = \frac{v^2}{r} \longrightarrow (1)$

force is the same as that of the centripetal force. Hence

$$\text{Centrifugal force} = \frac{mv^2}{r}$$

Assignment 5.2:

An aeroplane dives along a curved path of radius *R* and velocity $\vec{v}$. The centripetal acceleration is $10ms^{-2}$. If both the velocity and the radius are doubled, what will be the new acceleration?

Given Data: $a_c = \frac{v^2}{R} = 10\ m\,s^{-2}$

If velocity and radius of curved path is doubled i.e. $v' = 2v$ and $R' = 2R$, then

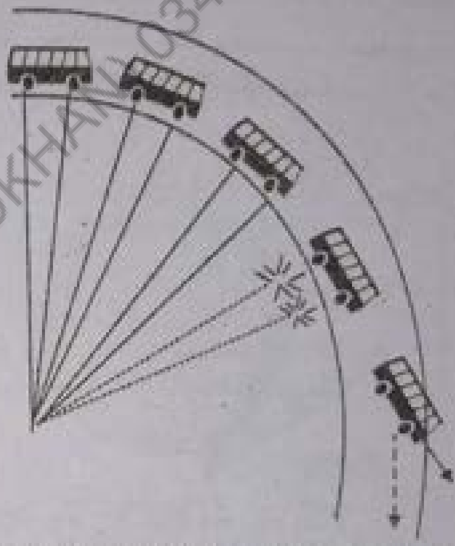
$$a_c{}' = \frac{(2v)^2}{2R} = \frac{4v^2}{2R} = 2\frac{v^2}{R}$$

$$\Rightarrow \quad a_c{}' = 2\,a_c = 2 \times 10 = 20\ m\,s^{-2}$$

Q.10 What is meant by banking of road? Why banked roads are necessary at turns? Derive the formula for necessary centripetal force and angle of banking?

Ans: Banking of road:

"If outer edge of the road is higher than inner edge of the road at turns then it is called banking of road".



If friction 'breaks' the car skids away

Reason for Banking of Road:

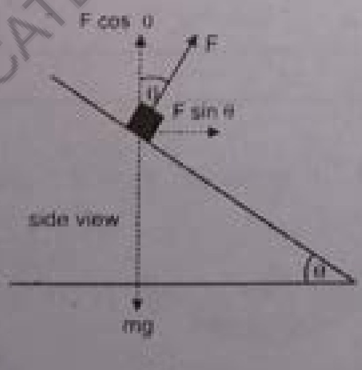
If a car is traveling round a circular path (bend) with uniform speed on horizontal road, the resultant force acting on it must be directed to the centre of its circular path, that is, it must be the centripetal force. This force arises from the interaction of the car with air and the ground.

(i) The direction of the force exerted by the air on the car will be more or less opposite to the instantaneous direction of motion.

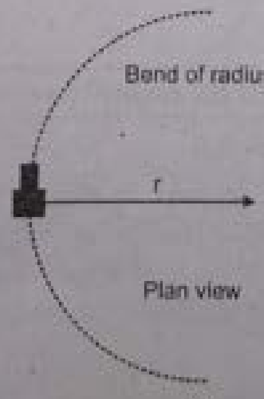
(ii) The other and more important horizontal force is the frictional force inward by ground on the tyres of car, fig (5.10). The resultant of these two forces is the centripetal force.

If friction of air and road is not sufficient to provide necessary centripetal force than vehicle will skid on road. So, road is banked to provide necessary centripetal force without depending on friction.

FIGURE 5.10



(a)



(b)

FIGURE 5.11

Banking of Road

Formula for Necessary Centripetal Force and Angle of Banking:

The problem is to find the angle θ at which a road should be banked so that the centripetal force acting on the car arises entirely from a component of the normal force $\vec{F}$ on the road, Fig. (a). Treating the car as a particle and resolving '$\vec{F}$' vertically and horizontally. Since F Sin θ is direted towards centre of circle

So it provides centripetal force.

$$F \sin\theta = F_c$$

$$\Rightarrow \quad F \sin\theta = \frac{mv^2}{r} \quad \text{----------(1)}$$

Where 'm' and 'v' are the mass and speed respectively of the car and 'r' is the radius of the banked road, Fig. (b).

A long vertical,

Thus

$$F \cos\theta = mg \quad \text{------------(2)}$$

Hence by division equation (1) by (2)

$$\frac{F \sin\theta}{F \cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{gr} \quad \text{------------(3)}$$

The equation shows that for a given radius of banked road, the angle of banking is only correct for one speed. From Eq: (3) we can write

$$v^2 = r\,g\,tan\theta$$

$$\Rightarrow \quad v = \sqrt{r\,g\,tan\theta}$$

This shows that for a given radius and angle, the speed is calculated for the safe turn of vehicle.

Assignment 5.3:

At what speed (in km/h) is a bank angle of 45° required for an aeroplane to turn on a radius of 60m?

Given Data: $\theta = 45^o$

$r = 60\ m$

Since $v = \sqrt{r\,g\,tan\theta}$

$\Rightarrow \quad v = \sqrt{(60)(9.8)tan45}$

$\Rightarrow \quad v = \sqrt{588} = 24.3\ m/s$

$\Rightarrow \quad v = 24.3 \times \frac{3600}{1000}\ km/h$

$\Rightarrow \quad v = 87.34\ km/h$

Q.11 Define Moment of Inertia and give its significance.

Ans: **Moment of Inertia (Rotational Inertia)**

Moment of inertia of a particle is defined as the product of mass of particle and square of its perpendicular distance from (pivot) the axis of rotation.

(OR)

Moment of inertia is the property due to which a body opposes any change in its state of rest or of angular motion.

It is denoted by I and is given by

$$I = mr^2$$

Physical Significance:

Moment of inertia plays the same role in angular motion as inertia (mass) in linear motion.

Example:

Consider the bike wheel as shown, on which we apply the force to rotate it. We make following observations:

i. More massive the wheel, smaller the angular acceleration and vice versa.

ii. If we push on the spike closer to the axle, the angular acceleration will be smaller and vice versa.

Dependence of moment of inertia:

This shows that moment of inertia depends upon

- mass m of body
- square of perpendicular distance from axis of rotation r.
- Practically, it also depends upon distribution of mass from position of axis of rotation.

Q.12 Derive the formula of torque in term of moment of inertia.

Ans: Torque and Moment of Inertia

Consider the bike wheel as shown, on which we apply the force to rotate it. Let force F is applied on the wheel at a point which is at distance r from its axle.

FIGURE 5.13

From Newton's 2nd law $F = ma$

We know that this rotating wheel has angular acceleration α,

Put $a = r\alpha$

⇨ $F = m r \alpha$

Multiplying by r on both sides

$$rF = m r^2 \alpha$$

Since $\tau = rF$

Thus $\tau = (mr^2)\alpha \longrightarrow (1)$

In this equation, during rotatory motion 'm' and 'r' remain constant and we put $mr^2 = I$, which is called moment of inertia or rotational inertia of a body.

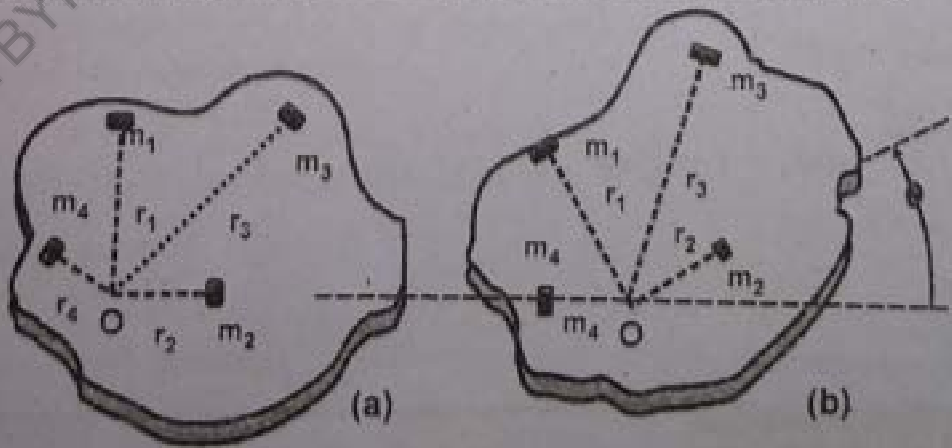
Thus $\tau = I\alpha \longrightarrow (2)$

The product of moment of inertia 'I' and angular acceleration 'α' of body gives the magnitude of the torque acting on it.

Q.13 Derive the formula for Moment of Inertia of Rigid Body.

Ans: Moment of inertia of a rigid body

Mostly the bodies have non-uniform mass distribution. Consider a rigid body made up of 'n' small pieces of masses m_1, m_2.......at distance r_1, r_2 .... from axis of rotation O



In this case, the sum of products of masses of the particles in a body and the squares of their respective perpendicular distance from the axis of rotation is called moment of inertia.

$I = \sum_{i=1} m_i r_i^2$

...h is the moment of inertia of rigid body.

...r Moment of Inertia of Different Objects or Shapes:

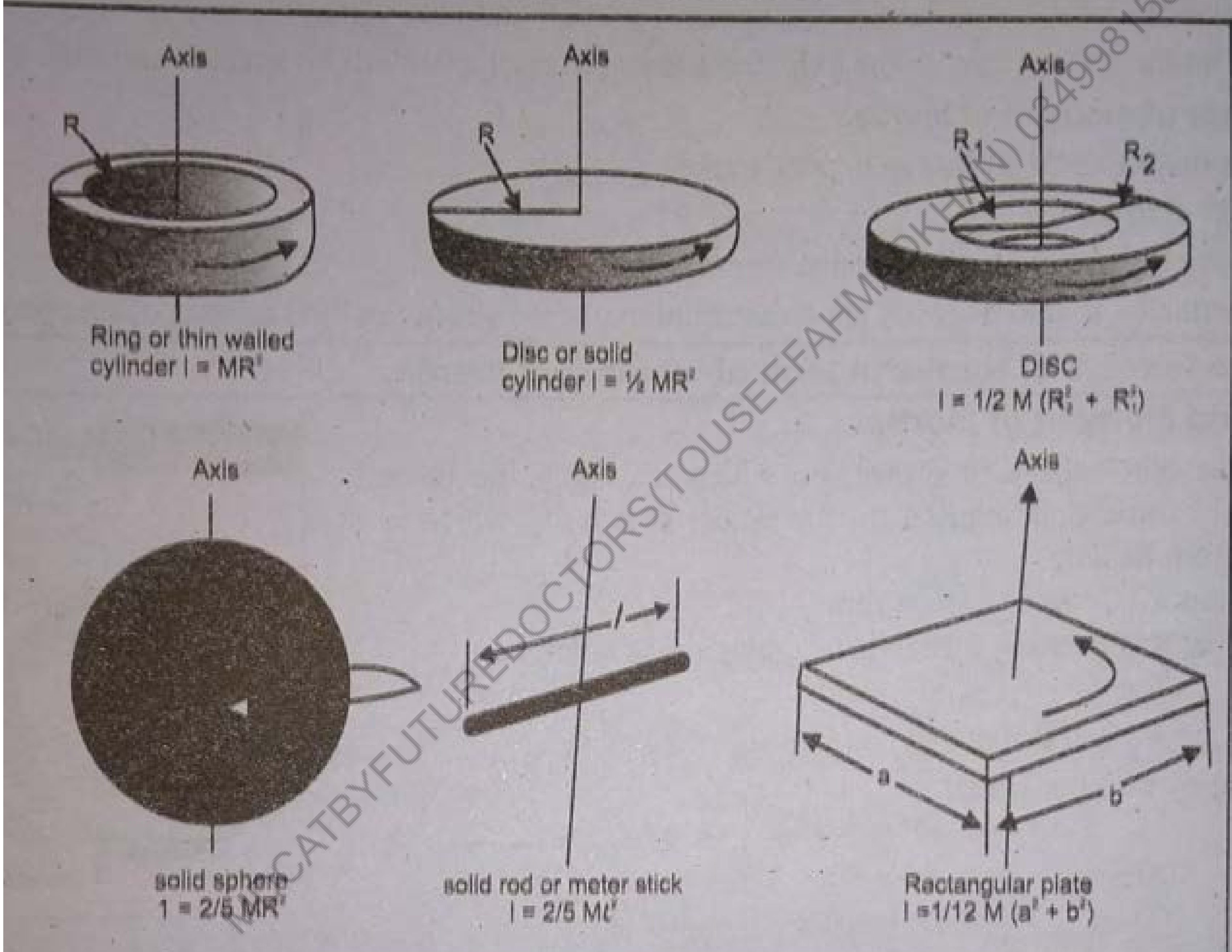


Fig:5.14 Moment of inertia of various bodies about vertical axis.

2. In angular motion the centripetal force is:
(A) m^2r (B) $mr^2\omega$ (C) $mr\omega^2$ (D) $mr^2\omega^2$
3. The angular acceleration is produced by:
(A) Momentum (B) Torque (C) Pressure (D) Power
4. Choose the quantity which plays the same role in angular motion as that of mass in linear motion:
(A) angular acceleration (B) Troque (C) Moment of inertia (D) Angular momentum
5. The ratio of moment of inertia of disc and hoop is:
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{1}{3}$

Answers Key

1. A	2. C	3. B	4. C	5. A

Assignment 5.6:
A belt is wrapped around the edge of a pulley that is 40 cm in diameter. The pulley rotates with a constant angular acceleration of 3.50 rad/s². At t = 0, the rotational speed is 2rad/s. What is the angular displacement and angular velocity of the pulley 2 s later?

Given Data:
$d = 40\ cm$
$\alpha = 3.50\ rad/s^2$
$\omega_i = 2\ rad/s$
$t = 2\ s$
(a) $\omega_f = ?$
(b) $\theta = ?$

Solution: (a) $\omega_f = \omega_i + \alpha t$
$\Rightarrow \omega_f = 2 + (3.50)\,2$
$\Rightarrow \omega_f = 2 + 7 = 9\ rad/s$

(b) $\theta = \omega t + \frac{1}{2}\alpha t^2$
$\Rightarrow \theta = 2(2) + \frac{1}{2}(3.50)(2)^2 = 4 + 7$
$\Rightarrow \theta = 11\ rad$

Q.14 (a) What is angular momentum? What are its formulae? Write its units.
(b) Derive angular momentum of a spinning rigid body?
(c) Show that rate of change of angular momentum of a body is equal to applied torque on it?

Ans: (a) Defination:

Quantity of angular motion in a body is called angular momentum.

OR

If a body changes its angular position w.r.t. its reference axis then it is said that it has angular momentum.

OR

Angular momentum ($\vec{L}$) is the cross product of position vector ($\vec{r}$) and linear momentum $\vec{P}$.

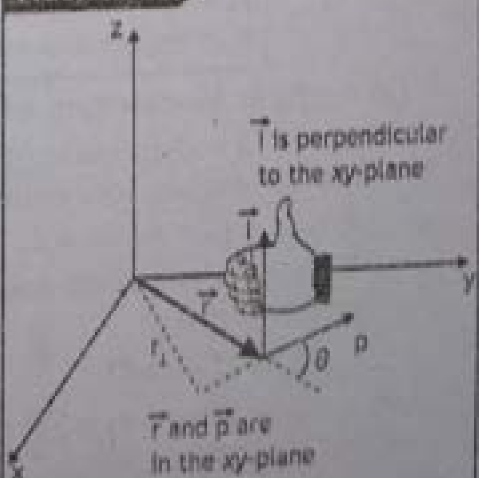
$$\vec{L} = \vec{r} \times \vec{P} \longrightarrow (1)$$

Magnitude of angular momentum is

$$L = r\,\vec{P}\sin\theta \longrightarrow (2)$$

Consider the special case of motion in a circle where $\vec{r}$ is always perpendicular to $\vec{p}$ i.e. $\theta = 90°$.

FIGURE 5.16



In three-dimensional space, the position vector $\vec{r}$ locates a particle in the xy-plane with linear momentum $\vec{p}$. The angular momentum with respect to the origin is $\vec{L} = \vec{r} \times \vec{p}$ which is in the z-direction. The direction of is given by the right-hand rule.

Then the magnitude of angular momentum is

$$L = r\,P\sin 90$$
$$L = r\,P\,(1)$$
$$L = r\,P$$

Putting $P = mv$

$$\boxed{L = mvr} \longrightarrow (3)$$

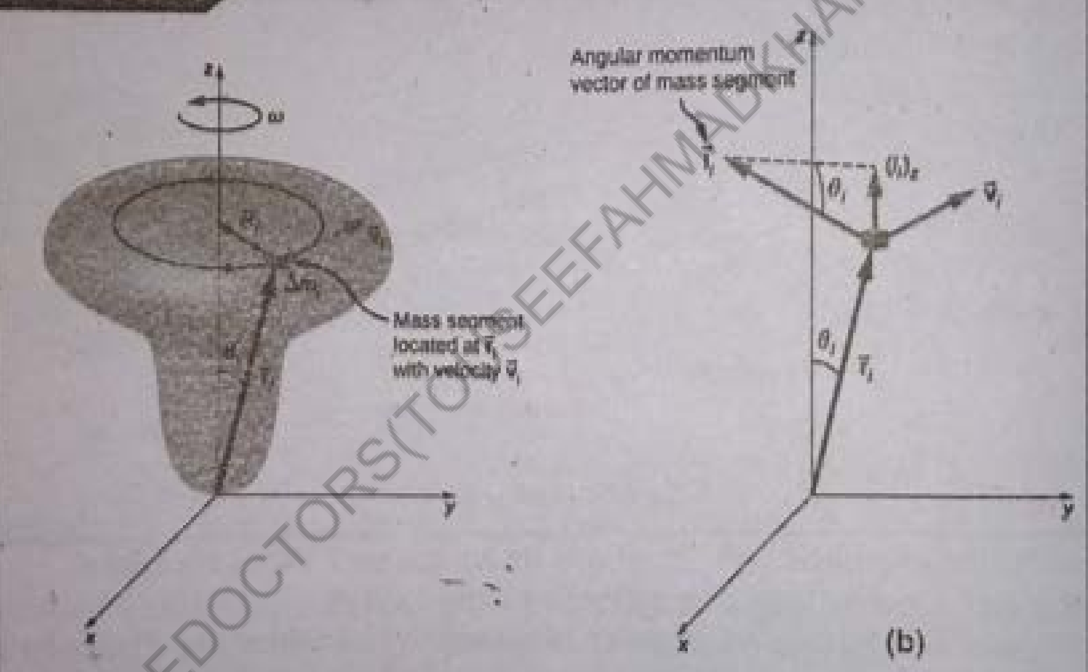
Put $v = r\omega$

⇨ $L = m(r\,\omega)r = mr^2\omega$

⇨ $L = I\,\omega$

- Angular momentum is vector quantity and its direction is along axis of rotations (perpendicular to plane containing vectors $\vec{r}$ and $\vec{P}$)
- S.I unit of angular momentum is kg m^2 s^{-1} or Js.
- Dimensions of angular momentum = $[ML^2T^{-1}]$

FIGURE 5.17



(a) A rigid body is constrained to rotate around the z-axis. The rigid body is symmetrical about the z-axis. (b) A mass segment m_i is located at position r_i, which makes angle θ_i with respect to the z-axis.

(a) Angular Momentum of a Rigid Body:

The rigid body is made of many particles, and the sum of angular momenta of all the particles gives the total angular momentum of the rigid body.

$$L = L_1 + L_2 + \text{-------} + L_n$$

Then, in terms of the masses and velocities of individual particles, we can write the total angular momentum as: $L = mv_1R_1 + mv_2R_2 + \text{-------} + mv_nR_n$

$$L = \sum_1^n m\,v_i R_i \qquad \text{Since } (v_i = R_i\,\omega)$$

⇨ $L = \sum_1^n m\,v_i R_i = \sum_1^n m\,(R_i\,\omega)R_i$

⇨ $L = \sum_1^n (m\,R_i^{\,2})\omega$

⇨ $L = I\,\omega \qquad$ Where $I = \sum_1^n (m\,R_i^{\,2}) = M.O.I.$

(c) Angular Momentum is given by:

$$\vec{L} = \vec{r} \times \vec{P}$$
$$\Delta\vec{L} = \vec{r} \times \Delta\vec{P}$$

Dividing both sides by Δt

$$\frac{\Delta\vec{L}}{\Delta t} = \vec{r}\times\frac{\Delta\vec{P}}{\Delta t}$$

$$\frac{\Delta\vec{L}}{\Delta t} = \vec{r}\times\vec{F} \qquad \because \quad \vec{F} = \frac{\Delta\vec{P}}{\Delta t}$$

$$\frac{\Delta\vec{L}}{\Delta t} = \vec{\tau} \qquad \vec{\tau} = \vec{r}\times\vec{F}$$

$$\boxed{\vec{\tau} = \frac{\Delta\vec{L}}{\Delta t}}$$

"Rate of change in angular motion of the body is equal to applied torque on the body"

Q.15 State and explain law of conservation of angular momentum of the body. Give some examples to explain this law.

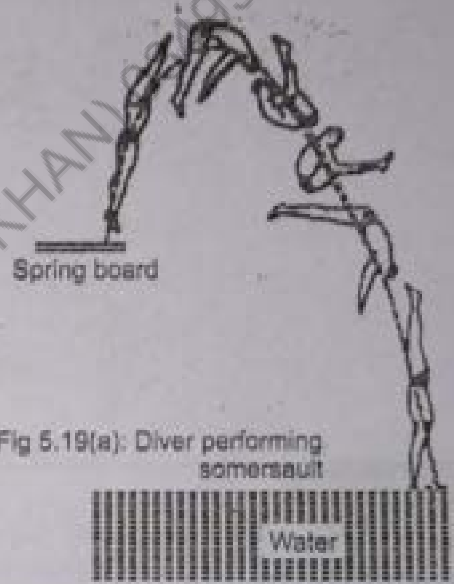


Fig 5.19(a): Diver performing somersault

Ans: **Law of Conservation of Angular Momentum**

Law of conservation of angular momentum which states that:

"In the absence of any external torque, the angular momentum of a system remains constant."

If $\tau = 0$ then

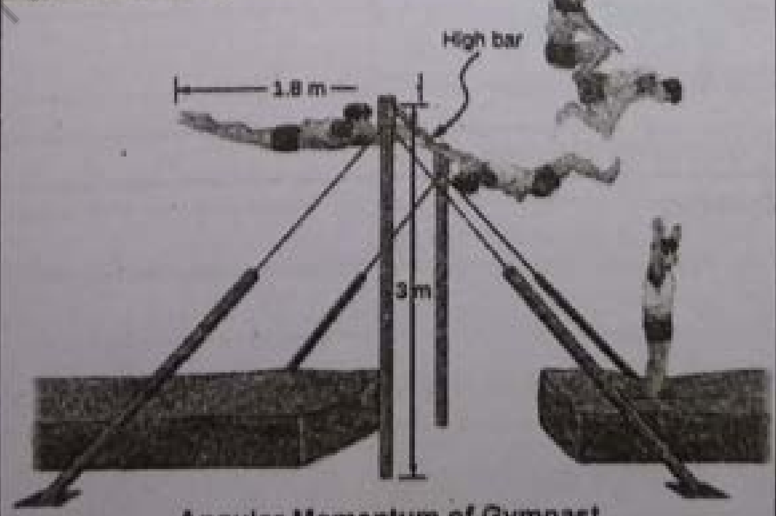
Then $\frac{\Delta L}{\Delta t} = 0 \quad \Rightarrow L = \text{Constant}$

(For isolated system in which no force and no torque acts on system), therefore, angular momentum of an isolated system is conserved.

Applications:

- This law is often used by circus acrobats, divers, ballet dancers, ice skaters and other to perform breath-taking feats.
- In given Fig., a diver leaves the spring board with his arms and legs extended and a small angular speed about a horizontal axis through his centre of gravity. When he pulls his arms and legs in, his moment of inertia becomes smaller. In order to keep his angular momentum '$L = I\omega$' constant, his angular velocity increases. He can thus make one or two extra somersaults.

FIGURE 5.19



Angular Momentum of Gymnast

- A gymnast starts the dismount at full extension, and then by tucking in his knees, he brings his mass closer to the centre of the axis of rotation, thereby decreasing the moment of inertia, his angular velocity increases, in order to keep his angular momentum '$L = I\omega$' constant. His increased angular velocity allows the gymnast to complete the rotation.
- The given fig. shows a man standing on turn table and holding heavy weights in his hands. With arms fully stretched horizontally, he is first set rotating slowly. Upon drawing the hands and weights in toward the chest, the angular velocity is considerable increased.

  He can slow down his spinning speed by stretching his hands again. This fact is due to the conservation of angular momentum.

FIGURE 5.20

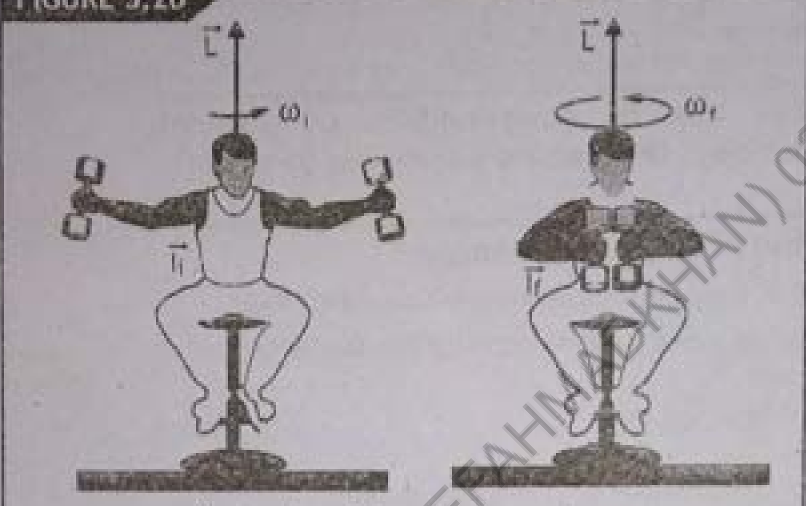


Illustration of Angular Momentum

Assignment:

A DVD disc has a radius of 0.0600 m, and a mass of 0.0200 kg. The moment of inertia of a solid disc is $I = (½)\,MR^2$, where M is the mass of the disc, and R is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of 160.0 rad s^{-1}. What is the angular momentum of this disc?

Given Data: $r = 0.0600\ m$

$M = 0.0200$ kg

$\omega = 160.0$ rad/s

$L = ?$

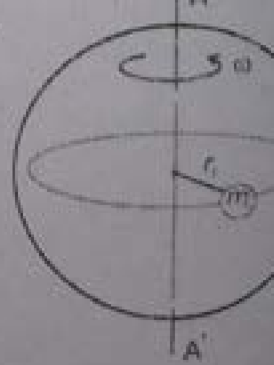
Solution: $L = I\omega$

⇨ $L = \frac{1}{2} m r^2 \omega$ (For Disc $I = \frac{1}{2} m r^2$)

⇨ $L = \frac{1}{2}(0.0200)(0.0600)^2(160.0) = 0.00576\ kg\ m^2/s$

Q.16 Define rotational K.E. Derive its formula

Ans: **Rotational Kinetic Energy**



The energy possessed by the body due to its rotation about an axis is called rotational kinetic energy.

Proof:

In linear motion, *the energy in a body due to its linear motion is called K.E.*

$$K.E = \frac{1}{2} mv^2 \quad \text{\_\_\_\_\_\_ (1)}$$

Where 'm' is mass of body i.e. inertia and $\overrightarrow{v}$ is linear velocity of body.

Similarly, *the energy in a body due to its angular motion, is called rotational kinetic energy* and is given by equation

$$\boxed{K.E_{rot} = \frac{1}{2} I\omega^2}$$

Where 'I' is the moment of inertia of body and 'ω' stands for angular velocity of body.

Roational K.E. of A Irregular Rigid Body

Let we apply some force $\vec{F}$ on a rigid body as shown. We have divided the whole body into a number, of small pieces of masses, $m_1, m_2, \ldots, m_n$ having distances from C.G. point as $r_1, r_2, r_3, \ldots, r_n$ respectively.

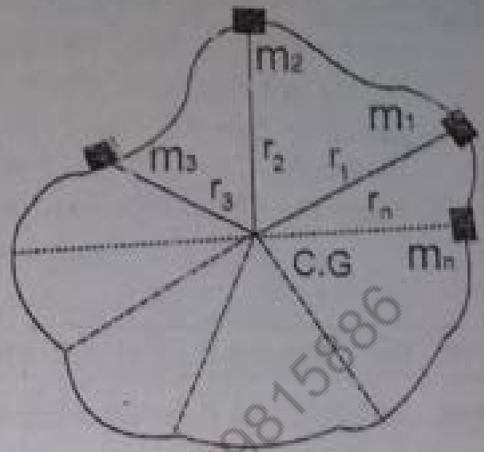


Fig:5.21

The total K.E of body will be: $K.E = K.E_1 + K.E_2 + \ldots K.E_n$

Or $$K.E_T = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots\ldots + \frac{1}{2}m_nv_n^2$$

Using equation $v = r\omega$, then:

$$K.E_{rot} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \ldots\ldots + \frac{1}{2}m_nr_n^2\omega^2$$

Here, each piece of body moves with same angular velocity ω.

Thus $$K.E_{rot} = \frac{1}{2}\omega^2\left(\sum_{i=1}^{i=n} m_ir_i^2\right)$$

Or $$\boxed{K.E_{rot} = \frac{1}{2} I\omega^2} \quad \text{\_\_\_\_\_\_ (2)}$$

Q.17 Compare the equations of motions for linear motion and angular motion.

Ans: Comparing angular and linear motions

Equations for linear motion	Equation for angular motion
(i) $S = vt$	(i) $\theta = \omega t$
(ii) $v_f = v_i + at$	(ii) $\omega_f = \omega_i + \alpha t$
(iii) $v_f^2 - v_i^2 = 2aS$	(iii) $\omega_f^2 - \omega_i^2 = 2\alpha\theta$ g
(iv) $S = v_it + \frac{1}{2}at^2$	(iv) $\theta = \omega_it + \frac{1}{2}\alpha t^2$
(v) Inertia = m	(v) Moment of inertia $I = mr^2$
(vi) Force = ma	(vi) Torque = $\tau = I\alpha$
(vii) Linear momentum = $\vec{p} = m\vec{v}$	(vii) Angular momentum $\vec{L} = \vec{r} \times \vec{p}$ or $\vec{L} = I\vec{\omega}$
(viii) $K.E_{lin} = \frac{1}{2}mv^2$	(viii) $K.E_{rot} = \frac{1}{2}I\omega^2$

Q.18 Discuss the K.E. of the rolling body from an inclined plane.

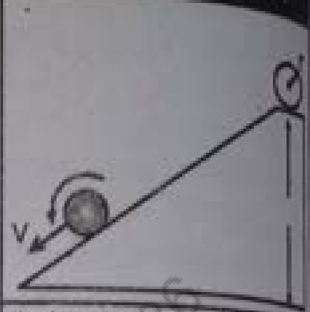
Ans: **K.E. of Rolling Body**

When a body is rolling down an inclined plane, then P.E. of the rolling body converts into two types of kinetic energies i.e. it has linear K.E. due to its linear motion i.e. K.E. = $\frac{1}{2}mv^2$ and it has rotational K.E. due to its rotation i.e. K.E. = $\frac{1}{2}I\omega^2$. So, total K.E. of the body is the sum of its linear and rotational kinetic energies.

$$K.E._{Total} = K.E._{linear} + K.E._{Rotational}$$

Or $$P.E._{Top} = (K.E._{linear} + K.E._{Rotational})_{Bottom}$$

For Your Information



As the sphere rolls to the bottom of the incline, its gravitational potential energy is changed to kinetic energy of rotation and translation.

EXPLANATION:
$E_{Total} = K.E_{trans} + K.E_{rot}$

EXPLANATION
$P.E = K.E_{trans} + K.E_{rot}$

Q.19 Find the rotational K.E. of hoop, disc and sphere?

Ans: **Rotational K.E. of a Hoop:**

As we know

$$(K.E)_{rot} = \frac{1}{2}I\omega^2$$

As moment of inertia of a hoop is

$$I = mr^2$$

So,

$$(K.E)_{rot} = \frac{1}{2}(mr^2)\omega^2$$

$$(K.E)_{rot} = \frac{1}{2}mr^2\omega^2$$

$$K.E_{rot} = \frac{1}{2}m(r\omega)^2 \qquad [v = r\omega]$$

$$\boxed{(K.E)_{rot} = \frac{1}{2}mv^2}$$

Rotational K.E. of a Disk

As we know

$$(K.E)_{rot} = \frac{1}{2}I\omega^2$$

As moment of inertia of a disc is $I = \frac{1}{2}mr^2$

So $$(K.E)_{rot} = \frac{1}{2}(\frac{1}{2}mr^2)\omega^2$$

$$(K.E)_{rot} = \frac{1}{4}m(r^2\omega^2)$$

As we know $v = r\omega$

$$\boxed{(K.E)_{rot} = \frac{1}{4}mv^2}$$

Rotational K.E. of a Sphere

As we know

$$(K.E)_{rot} = \frac{1}{2}I\omega^2$$

As moment of inertia of a sphere is $I = \frac{2}{5}mr^2$

So, $$K.E_{rot} = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$$

$$\Rightarrow \quad K.E_{rot} = \frac{1}{5}m(r^2\omega^2)$$

Putting $r\omega = v$

$$\Rightarrow \quad \boxed{K.E_{rot} = \frac{1}{5}mv^2}$$

Q.20 Derive the relations for the velocities of disc and hoop moving down an inclined plane.

Ans: Hoop and disc are placed at height h on an inclined plane as shown. When these are released, both rolls down the hill, they move forward with translational or linear K.E. and it has rotational K.E. as it also rotates. So, from law of conservation of energy:

$$P.E = K.E_{Tran} + K.E_{rot}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{-------- (1)}$$

Rolling down of Cylinder and Hoop on an inclined plane.

For Hoop

$$I = mr^2$$

Put it in eq (1)

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}m(r\omega)^2 \quad \text{(Since } v = r\omega\text{)}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\Rightarrow mgh = mv^2$$

$$\Rightarrow gh = v^2$$

$$\Rightarrow \sqrt{v^2} = \sqrt{gh}$$

$$\Rightarrow v = \sqrt{gh}$$

$$\Rightarrow \boxed{v_{Hoop} = \sqrt{gh}}$$

For Disc

$$I = \frac{1}{2}mr^2$$

Put it in eq (1)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \quad \text{(Since } v = r\omega\text{)}$$

$$\Rightarrow mgh = \left(\frac{1}{2} + \frac{1}{4}\right)mv^2$$

$$\Rightarrow mgh = \left(\frac{2+1}{4}\right)mv^2$$

$$\Rightarrow gh = \frac{3}{4}v^2$$

$$\Rightarrow \frac{4}{3}gh = v^2$$

$$\Rightarrow \sqrt{v^2} = \sqrt{\frac{4}{3}gh}$$

$$\Rightarrow \boxed{v_{disc} = \sqrt{\frac{4}{3}gh}}$$

$$\Rightarrow \boxed{v_{disc} = \sqrt{1.33\,gh}}$$

From above results, we have

$$V_{Disc} > V_{Hoop}$$

Also, we can write;

$$V_{Disc} = \sqrt{\frac{4}{3}}\sqrt{gh} \Rightarrow V_{Disc} = \frac{2}{\sqrt{3}}V_{Hoop}$$

or

$$V_{Disc} = \sqrt{1.33}\,V_{Hoop}$$

SOME IMPORTANT POINTS RELATED TO ROTATIONAL K.E OF DIFFERENT BODIES

Body	Moment of inertia	Rotational K.E $K.E_R$	Translation K.E $K.E_T$	Total K.E $T.K.E = K.E_R + K.E_T$	Relationship B/W $K.E_R$ and $K.E_T$	$\frac{K.E_T}{T.K.E}$	$\frac{K.E_R}{T.K.E}$	$\frac{K.E_R}{K.E_T}$	$\frac{K.E_T}{K.E_R}$	Speed on reaching bottom
Hoop or hollow cylinder	mr^2	$\frac{1}{2}mv^2$	$\frac{1}{2}mv^2$	mv^2	$K.E_R = K.E_T$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\sqrt{gh}$
Disc or solid cylinder	$\frac{1}{2}mr^2$	$\frac{1}{4}mv^2$	$\frac{1}{2}mv^2$	$\frac{3}{4}mv^2$	$K.E_R = \frac{1}{2}(K.E_T)$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{1}$	$\sqrt{\frac{4}{3}gh}$
Solid sphere	$\frac{2}{5}mr^2$	$\frac{1}{5}mv^2$	$\frac{1}{2}mv^2$	$\frac{7}{10}mv^2$	$K.E_R = \frac{2}{5}(K.E_T)$	$\frac{5}{7}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{5}{2}$	$\sqrt{\frac{10}{7}gh}$
Spherical shell	$\frac{2}{3}mr^2$	$\frac{1}{3}mv^2$	$\frac{1}{2}mv^2$	$\frac{5}{6}mv^2$	$K.E_R = \frac{2}{3}(K.E_T)$	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{3}{2}$	$\sqrt{\frac{6}{5}gh}$

Q.21 What are real and apparent weight? Find the apparent weight in different cases for an object suspended by a string and spring balance in an lift?

Ans: **Real and Apparent Weight**

Real Weight (W)

"It is the gravitational pull of the earth on the object."

$$W = mg$$

Apparent Weight (T)

"Apparent weight is equal and opposite to the force required to stop it from falling in the frame of reference."

OR

The reactional force which prevents the body from falling on ground is called apparent weight.

It is the measurement of the weight of the body by any device.

Apparent weight of an object in lift:

Consider a woman weighs a fish with a spring balance attached to a ceiling of lift.

In this case, the net force on the person is:

$$F_{net} = W - T$$

Where W = Real weight of the person

T = Tension in string (Reaction Force applied by balance to stop the body from falling) i.e. it is Apparent weight of the person

FIGURE 5.23

a When the lift is at rest, the spring scale reads the true weight of the fish.

CASE 1:

When the lift is at rest or moving with uniform velocity i.e. $a = 0$.

When the lift is at rest, Newton's second law tells us that the acceleration of object is zero. So the net force becomes zero.

If 'W' is the gravitational force acting on the object i.e. real weight and T is the Tension in string i.e. Apparent weight then net force on the person is:

Then,

$$F_{net} = W - T$$

$$ma = W - T$$

As $a = 0$

$$0 = W - T$$

or $T = W$

Apparent Weight = Real Weight

Result:

Hence the apparent weight of person is equal to the real weight for observer inside the lift.

CASE II:

When the lift is moving upward with acceleration 'a'.

When the lift is moving upwards with an acceleration a.

So, the upward force i.e. Tension in string is greater than downward force of weight W,

Then the net force acting on the body is

$$T - W = F_{net}$$

$$W - ma = T$$

OR $$T = W + ma$$

Apparent weight = Real weight + ma

Result:

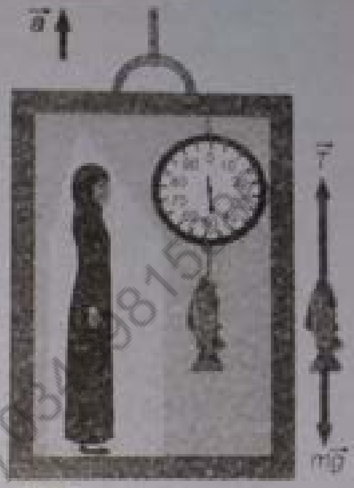
Which shows that the apparent weight of object is increased by an amount of 'ma' than its actual weight.

Above equation can be written as:

$$T = mg + ma$$

$$T = m(g + a)$$

FIGURE 5.23



b When the lift accelerates upward, the spring scale reads a value greater than the weight of the fish.

CASE III:

When the lift is moving downward with acceleration 'a'.

When the lift is moving downwards with an acceleration a. So, the upward force i.e. Tension in string is less than downward force of weight W,

Then the net force acting on the body is

$$W - T = F_{net}$$

$$W - T = ma$$

$$-T = -W + ma$$

Or $$T = W - ma$$

Apparent weight = Real weight – ma

Result:

Which shows that the apparent weight is less than the actual weight by a amount equal to ma.

Above equation can be written as:

$$T = mg - ma$$

$$T = m(g - a)$$

FIGURE 5.23

c When the lift accelerates downward, the spring scale reads a value less than the weight of the fish.

CASE IV:

When Lift is falling freely under gravity i.e. a = g

Now we consider that the lift is falling freely under gravity. Then

$$a = g$$

As $$T = W - ma$$

$$T = mg - ma$$

$$T = mg - mg$$

$$T = 0$$

Apparent weight = 0

Result:

So, the apparent weight of object shown by the balance is zero. The object seems to be weightless. Thus, it is in state of weightlessness.

Do you know?

Your apparent weight differs from your true weight when the velocity of the lift changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

Q.22 Explain the concept of weightlessness in a body.

Ans: **Weightlessness in A Body**

"Weightlessness in a body is the sensation experienced by the body when no external object is exerting a force by touching it."

OR

"Weightless sensation exists when all the contact forces are removed."

Example:

Consider a woman weighs a fish with a spring balance attached to a ceiling of lift. Let suddenly lift falls freely, all the objects in it falls freely, then the spring balance shows zero newton weight of the fish i.e. it appears weightless in this situation because tension in the string of the balance disappears.

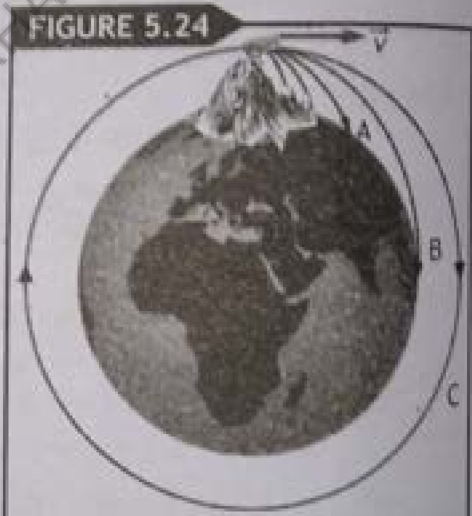
In free fall, the only force acting the body i.e. fish is force of gravity which is non-contact force. Since force of gravity can only be felt by an opposing contact force (e.g. tension in string of balance). During free fall of lift, tension in string disappears, therefore body i.e. fish appears in state of weightlessness.

The feelings of weightlessness are common at amusement parks for riders of roller coasters and other rides in which riders are momentarily airborne and lifted out of their seats.

Q.23 Discuss weightlessness in satellites.
OR Discuss free fall in satellites.

Ans: **Weightlessness in Satellite (Free Fall in Satellite)**

FIGURE 5.24



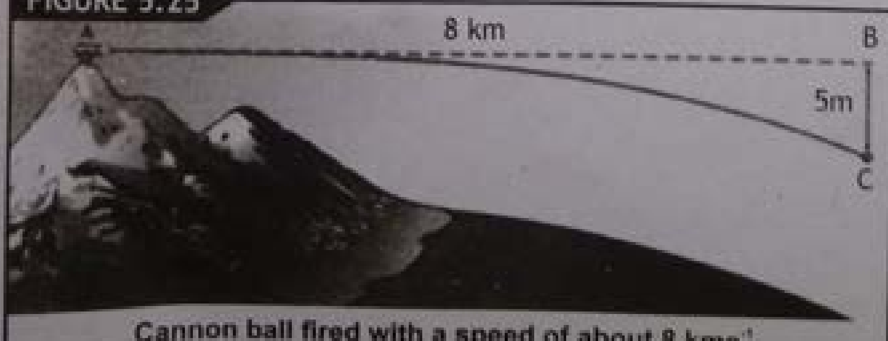
Cannon ball shot parallel to the horizontal surface of the earth

Newton gave law of universal gravitation. Soon after that he started working on the situation in which any object could revolve around earth.

He put idea that place a cannon at the top of mountain and fire cannon balls.

- Consider a cannon ball shot parallel to the horizontal surface of the earth from the top of the mountain ignoring air friction. While moving parallel to the earth, the force of gravity will pull the cannon ball downward and it follows path A. Since the speed of cannon ball was too small, that it eventually falls to earth.
- Similarly, if a cannon ball is fired faster than the earlier it will follow path B and come down further away as illustrated in the Figure.
- *We know about the curvature of the earth that its surface drops a vertical height of 5m for every 8000m tangent to the surface.*
- For a cannon ball to orbit the earth, it should drop a vertical height of 5m for every 8000m distance along the horizon. So the cannon ball fired with a speed of about 8000ms^{-1} will be capable of orbiting the earth in a circular path and follow path C.
- We conclude that if the object is thrown fast enough parallel to the earth, the curvature of its path matches the curvature of the earth, it will circle round the earth. Its radial acceleration is simply 'g', the free fall acceleration.
- When the cannon ball speed exceeds 8000ms it overshoots a circular path and travels an elliptical orbit.

FIGURE 5.25



Cannon ball fired with a speed of about 8 kms^{-1}.

- The orbiting spaceship is accelerating towards the center of the earth at all the times exactly the same way cannon ball is orbiting round the earth as shown in Figure. Its radial acceleration is simply 'g', the free fall acceleration.
- So, the orbiting satellite/ space station is a falling freely object in space, everything within this will appear to be weightless.

Assignment:

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s², what will be the reading of the spring balance?

Given Data: $W = 49\ N$

⇨ $m = \frac{W}{g} = \frac{49}{9.8} = 5\ kg$

and $a = 5\ m/s^2$

Solution: When lift moves downward:

$T = m(g - a)$

⇨ $T = 5(9.8 - 5) = 5(4.8)$

⇨ $T = 24\ N$

Q.24 What are artificial satellites? Find the expression for minimum velocity to put a satellite into the orbit?

Ans: **Artificial Satellite**

Artificial satellites are the man- made objects that orbit around the earth.

Satellite can be launched from earth's surface to circle the earth by mean of rocket. They are kept into their orbit by gravitational attraction of earth. The satellites which are near the earth have the acceleration 9.8 m/s². Other-wise they would fly off in the straight line tangent to earth.

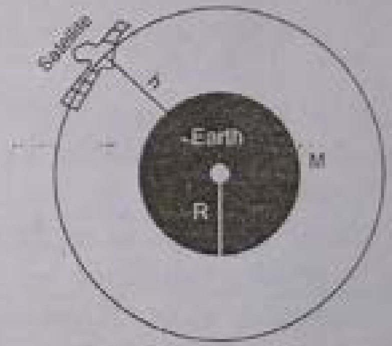


Fig:5.23 Artificial Satellite

Critical orbital velocity

The minimum velocity required to put a satellite into an orbit close to the earth is called critical velocity.

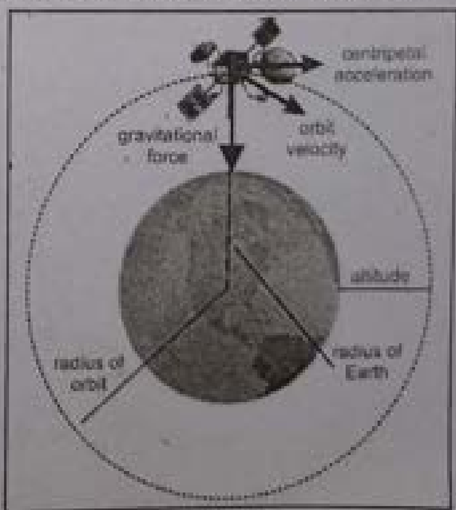
Expression

Consider a satellite of mass 'm' is moving with velocity v close to the earth in a circle of radius R.
The centripetal force acting on the satellite is provided by gravitational force acting on satellite by earth.
So, centripetal acceleration of the orbiting satellite is:

$$a_c = \frac{v^2}{R} \quad \text{______ (1)}$$

This force being provided by it weight. So, its acceleration must be equal to gravitational acceleration.

$$⇨ g = \frac{v^2}{R} \quad \text{______ (2)}$$



(i.e., $R = 6.4 \times 10^6$ m)

Where v is the orbital velocity and R is the radius of earth

Thus from equation (2)

⇨ $v^2 = gR$

⇨ $v = \sqrt{gR}$

As

$g = 9.8 m/sec^2$, $R = 6.4 \times 10^6$ m

Putting values, we get

$v = \sqrt{9.8 \times 6.4 \times 10^6}$

$v = 7900$ m/s

or $v = 7.9$ K m/s ≈ 8 K m/s

*This is the minimum velocity necessary to put the satellite into orbit. This is called **critical velocity**.*

QUIZ

If astronauts cut there hairs in space station,
(a) will it fell to the floor?
(b) if not, what are the reasons?

Ans:
(a) Hairs will not fall in space station.
(b) Orbiting space station is a freely falling object, so it will be in weightlessness, therefore any thing in it will appear weightless. That's why, hair will not fall on the floor of space station.

Do you know?
GPS satellite systems stands for "global positioning system satellite system". It consists of 24 satellites. These are close orbiting satellites. Closest orbiting satellites orbit the Earth at a height of about 400km.

Q.25 What is "Artificial Gravity"? Derive expression for frequency of spaceship required to provide the artificial gravity?

FIGURE 5.26

Astronauts which are not in contact with the floor or walls would be floating within the sp...

Ans: **Artificial Gravity**

In order to have a spaceship in space, we have to provide gravity to the occupants of the spaceship. Such provided gravity is known as artificial gravity, because it does not exist naturally.

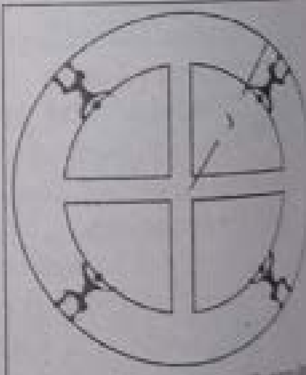
OR

Artificial gravity is the gravity like effect produced in an orbiting satellite by spinning it about its own axis.

Explanation:

There is no force on the space craft or in a gravity free space or orbiting satellites are in state of weightlessness. This weightlessness creates a lot of problems for astronauts present in the spaceship. To overcome this difficulty, an artificial gravity is created in the spaceship.

For this purpose, spaceship spins about its own axis, due to which all objects inside the spaceship or satellite is acted upon by an outward force i.e. centrifugal force which presses the person in the same same way as gravity does on earth. Then we say that artificial gravity is produced.

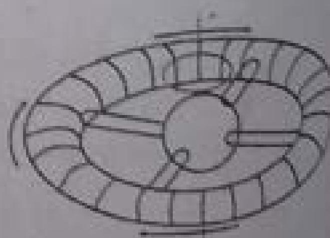


The spaceship rotates with angu... velocity w and frequency ... giving us the artificial gravity like...

Expression for frequency:

Consider a space craft of ring shape, having 'R' as its outer radius. It rotates around its own central axis with angular speed 'ω'. Then its centripetal acceleration a_c is

$$a_c = \frac{v^2}{r} \qquad [\text{As } v = R\omega] \text{ and } a_c = g$$



So $$g = \frac{r^2\omega^2}{r}$$

Or $$g = r\omega^2$$

But $\omega = \frac{2\pi}{T}$, where 'T' is the period of revolution of spaceship.

So, $$g = r\left(\frac{2\pi}{T}\right)^2$$

$$g = r\frac{4\pi^2}{T^2}$$

$$g = 4\pi^2 \times r\left(\frac{1}{T^2}\right) \qquad [\text{As } \frac{1}{T} = f\,]$$

$$g = 4\pi^2 r f^2$$

Or $$f^2 = \frac{g}{4\pi^2 r}$$

Taking square root of both sides

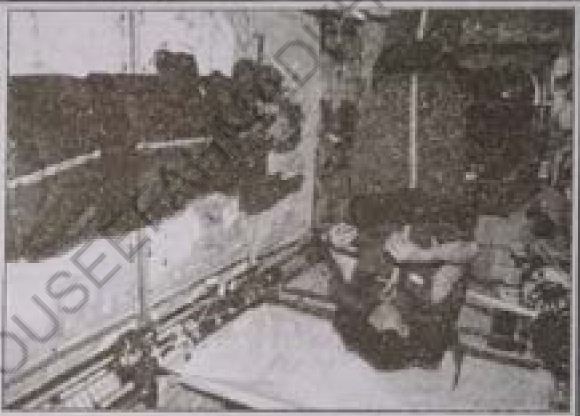
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

FIGURE 5.27

When the spaceship/ satellite rotates with this frequency, then artificial gravity helps the astronauts perform the work easily.

POINT TO PONDER

Astronauts in places with microgravity, like on the ISS, are weightless; they can sleep or rest in any orientation.
However, when it's time for them to sleep, they have to attach themselves so they don't float around and bump into something. ISS astronauts usually sleep in sleeping bags located in small crew cabins.
Each crew cabin is just big enough for one person.
Astronauts also attach themselves to walls or



Q.26 What is orbital velocity of a satellite? Derive its formula.

Ans: **Orbital Velocity**

Orbital velocity is the tangential velocity to put satellite in orbit around the earth.

OR

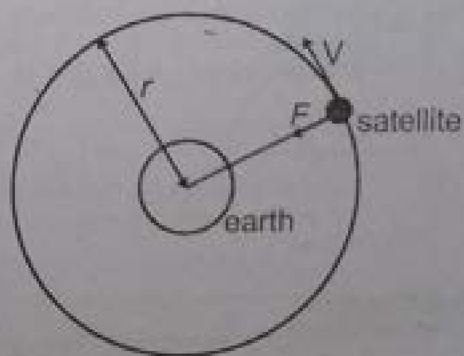
The velocity of satellite with which it revolves round the earth in an orbit is called orbital velocity.

The earth and some other planets revolve round the sun in nearly circular orbits. This type of motion is called orbital motion. Artificial satellites also revolve around the sun.

Expression for Orbital Velocity

Consider a satellite of mass m_s moving with orbital velocity v around the earth of mass M. If r is the radius of the orbit then centripetal force F can be expressed as

$$F_c = \frac{m_s v^2}{r} \qquad \text{\_\_\_\_\_\_\_\_ (1)}$$



This force is provided by gravitational force of attraction between earth and satellite and is given by

$$F = G\frac{Mm_s}{r^2} \qquad \text{\_\_\_\_\_\_\_\_ (2)}$$

Equating equation (1) and (2), we get

$$\frac{m_s v^2}{r} = G\frac{Mm_s}{r^2}$$

Or $v^2 = \frac{GM}{r}$

Or $v = \sqrt{\frac{GM}{r}}$ ________ (4)

Where $r = R + h$

r = Radius of the orbit

R = Radius of the earth

H = Height of the orbit above from the equator

Thus the mass of satellite is not important in describing the satellite orbit. So if the speed of satellite is less than the orbital speed then it will not be able to revolve around the earth and fall back to the earth.

⇒ Orbital speed of satellite is inversely proportional radius of orbit of satellite.

$v \propto \frac{1}{\sqrt{r}}$

i.e satellites closer to earth revolve with fast speed and vice versa.

MCQ's From Past Board Papers

1. The time period of artificial satellite is given by:
(A) $\frac{2\pi r}{v}$ (B) $2\pi rv$ (C) $\frac{2\pi v}{r}$ (D) $\frac{\pi r}{v}$

2. The relation between the escape velocity v_{esc} and the orbital velocity v_o is:
(A) $v_{esc} = 1/2\, v_o$ (B) $v_{esc} = \sqrt{2}\, v_o$ (C) $v_{esc} = v_o$ (D) $v_{esc} = 2v_o$

3. Which of the following is the S.I units of angular momentum?
(A) Js (B) $J s^{-2}$ (C) Js (D) None

4. The direction of angular momentum of body moving in circle is:
(A) Along the tangent (B) Perpendicular to the plane of circle
(C) Radially outward (D) Radially inward

5. If the radius of earth is doubled then the value of critical velocity becomes:
(A) $\frac{1}{\sqrt{2}} v_o$ (B) $\frac{1}{2} v_o$ (C) $\sqrt{2}\, v_o$ (D) $\frac{1}{4} v_o$

6. The value of angular momentum is maximum when θ is:
(A) 90° (B) 60° (C) 45° (D) 0°

7. The angular momentum $\vec{L}$ is given by:
(A) $\vec{r} \times \vec{P}$ (B) $\vec{L} \times \vec{r}$ (C) $\vec{r} \times \vec{F}$ (D) $\vec{F} \times \vec{P}$

8. Which of the following of the dimensions of Angular momentum?
(A) $[MLT^{-2}]$ (B) $[MLT^{-1}]$ (C) $[ML^2T^{-1}]$ (D) $[ML^{-2}T^{-2}]$

9. Orbital velocity near surface of earth is given by:
(A) $\sqrt{2gR}$ (B) $\sqrt{gR}$ (C) $\sqrt{\frac{2g}{R}}$ (D) $\sqrt{\frac{g}{R}}$

10. Close orbiting satellites orbit the earth at a height of about:
(A) 400 km (B) 4000 km (C) 400 m (D) 400 cm

11. The time period of a low flying satellite is:
(A) 1 year (B) 84 minutes (C) 28 hours (D) 1 day

12. The ratio of velocity of disc to velocity of hoop is:
(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{4}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\frac{\sqrt{4}}{3}$

13. A stone of mass 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16N, the maximum velocity of revolution that can be given to the stone without breaking it will be:
(A) $12\ ms^{-1}$ (B) $20\ ms^{-1}$ (C) $16\ ms^{-1}$ (D) $14\ ms^{-1}$

Answers Key

1. A	2. B	3. C	4. B	5. C	6. A	7. A	8. C	9. B	10. A	11. B	12. D
13. A											

Q.27 What are geo-stationary satellites and geo-stationary orbits? Find the orbital radius of geo-stationary satellites?

Ans: **Geostationary Orbit**

The orbit in which the time period of revolution of satellite is exactly equal to the time period of rotation of earth about its axis, is called geo-stationary orbit.

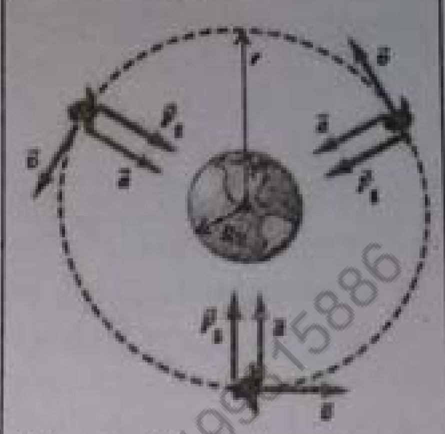
Geostationary Satellites

The satellite which completes its one revolution around earth in 24 hours is called geo-stationary satellite.

OR *A satellite whose orbital motion is synchronized with the rotation of the earth about its own axis is called geo-stationary satellite.*

This type of satellite is the one whose orbital motion becomes equal to the period of rotation of earth. So the satellite remains always over the same point on the equator as the earth spins on its axis i.e. these are satellites which remain stationary w.r.t. earth.

FIGURE 5.31



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Applications

Such satellites are used in communication system, weather observation and other military uses.

Fig. 5.31

Expression of orbital radius of geo-stationary satellite:

As the orbital speed necessary for circular orbit is given by

$$v_o = \sqrt{\frac{GM_e}{r_o}} \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

But this speed must be equal to average speed of satellite in one day. So,

$$v_o = \frac{S}{t}$$

Or $$v_o = \frac{2\pi r_o}{T} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Where 'T' is period of revolution of satellite that is equal to one day. So the satellite also complete one rotation in exactly one day.

For Your Information

A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationary. It is used now for international communications.

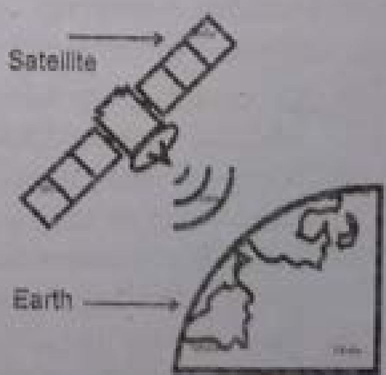
Equating equations (1) and (2), we get

$$\frac{2\pi r_o}{T} = \sqrt{\frac{GM_e}{r_o}}$$

Squaring, both sides, we get

$$\frac{4\pi^2 r_o^2}{T^2} = \frac{GM_e}{r_o}$$

$$r_o^3 = \frac{GM_e T^2}{4\pi^2}$$



Taking cube root of both sides.

Or $$\boxed{r_o = \left[\frac{GM_e T^2}{4\pi^2}\right]^{\frac{1}{3}}}$$

This equation gives the orbital radius of the geostationary satellite.

Substituting the values,

$$G = 6.67 \times 10^{-11}\ Nm^2/kg^2, \quad M = 6 \times 10^{24}\ Kg$$

$$T = 1\ \text{day} = 24\ \text{hours} = 24 \times 60 \times 60\ \text{s} = 86400\ \text{s}$$

So
$$r = \left[\frac{6.67\times10^{-11}\times6\times10^{24}\times(86400)^2}{4\times(3.14)^2}\right]^{\frac{1}{3}}$$

$$= 0.423 \times 10^8\ \text{m}$$

$$= 4.23 \times 10^7\ \text{m}$$

$$\boxed{r = 4.23 \times 10^4\ \text{km}}$$

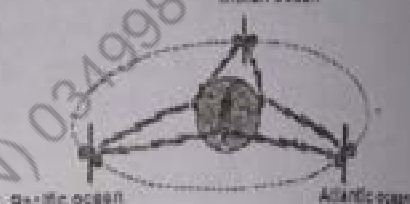
Which is the orbital radius measured from center of the earth for geo-stationary satellite.

⇨ A satellite at this height will always stay directly above a particular point on the surface of earth.

⇨ This type of orbit is ideal for many communications and weather satellites.

⇨ A geostationary orbit has an altitude of 22,240 miles (35,790 km).

⇨ Which results in an orbital speed of 6,880 mph^{-1} (11,070 km h^{-1}).



The whole Earth can be covered by just three geo-stationary satellites.

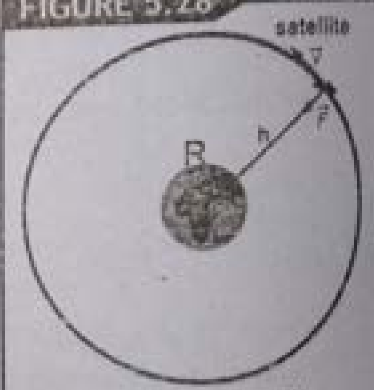
Q.28 Describe different types of artificial satellites

Ans: THE ARTIFICIAL SATELLITES

A satellite is anything that orbits around another object. Moons are natural satellites that orbit around planets, whereas artificial satellites are human-built objects orbiting the Earth and other planets in the Solar System and launched into orbit using rockets.

- There are currently over a thousand active satellites orbiting the Earth.
- The size, altitude and design of a satellite depend on its purpose.
- Artificial satellites are spacecraft that stay in orbit around the Earth.

FIGURE 5.28



Artificial Satellite

Types of Satellites

Navigation satellites:

The GPS (global positioning system) is made up of 24 satellites that orbit at an altitude of 20,000 km above the surface of the Earth.

The difference in time for signals received from four satellites is used to calculate the exact location of a GPS receiver on Earth.

Communication satellites

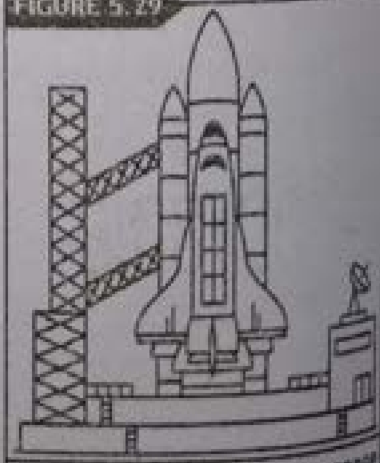
These satellites are used for television, phone or internet transmissions.

For example, the **Optus D1 satellite** is in a geostationary orbit above the equator and has a coverage footprint to provide signals to all of Australia and New Zealand.

- Communications satellites are often in geostationary orbit at the high orbital altitude of 35,800 kilometers.
- A geostationary satellite orbits the Earth in the same amount of time it takes the Earth to revolve once.
- From Earth, therefore, the satellite appears to be stationary, always above the same area of the Earth.
- Such a satellite covers **120°** of longitude. So the whole of the populated earth's surface can be covered by **three** correctly positioned satellites.

  The area to which it can transmit is called a satellite's footprint.

FIGURE 5.29



A space shuttle launches into space from NASA's Kennedy Space Centre in Florida.

Weather satellites

These satellites are used to image clouds and measure temperature and rainfall.

- Both geostationary and low Earth orbits are used depending on the type of weather satellite.
- Weathe satellites are used to help with more accurate weather forecasting.

MCQ's From Past Board Papers

1. Weight of the object at the centre of the earth is:
(A) Maximum (B) Minimum (C) Zero (D) Infinite

2. Weight of 60 kg man in moving lift (downward) with constant acceleration of $\frac{1}{2}$ g (where g = 10m/sec^2):
(A) Zero (B) 300 N (C) 600 N (D) 200 N

3. The apparent weight of a man in a lift moving down with an acceleration of 9.8 m/s^2 is:
(A) 0 (B) 9.8 N (C) Infinite (D) 19.6 N

4. If a body of mass 1kg is lifted upward with an acceleration of 9.8 ms^{-2}. its apparent weight be observed as:
(A) Zero Newton (B) 9.8 newton (C) 19.6 newton (D) 100 newton

5. The weight of a man in an lift descending with an acceleration of 4.9ms^{-2} will become:
(A) twice (B) Half (C) Zero (D) Unchanged

6. A man of weight W is standing on the elevator which is ascending with an acceleration of 'a' the apparent weight of the man is:
(A) W + ma (B) W – ma (C) W (D) ma

7. Rotational inertia of two equal masses of cylinders, but one has larger diameter will be:
(A) Lesser (B) Larger (C) Same (D) None of these

8. Which is unimportant in describing the satellite's orbit?
(A) Distance of satellite from earth's center (B) Gravitational constant G
(C) Mass of satellite (D) Mass of earth

9. The ratio of escape velocity to the critical orbital velocity is:
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

10. The speed of disc at the bottom of an inclined plane is:
(A) $v = \sqrt{gh}$ (B) $v = \sqrt{2gh}$ (C) $V = \sqrt{\frac{4gh}{3}}$ (D) None of these

11. A body of mass 2 kg is suspended from the ceiling of an elevator moving up with an acceleration g, its apparent weight in the elevator will be:
(A) 0 (B) 9.8 N (C) 19.6 N (D) 39.2 N

Answers Key

1. C	2. B	3. A	4. C	5. B	6. A	7. B	8. C	9. D	10. C	11. D

FORMULAE

1	Relation between S , r and θ	$S = r\theta$	$\theta = \frac{S}{r}$
2	Average angular velocity	$\vec{\omega}_{av} = \frac{\Delta\vec{\theta}}{\Delta t}$	
3	Instantaneous angular velocity	$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t}$	
4	Average angular acceleration	$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$	$\vec{\alpha}_{av} = \frac{\Delta\vec{\omega}}{\Delta t}$

5	Instantaneous angular acceleration	$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t}$		
6	Relation between v , r and ω	$v = r\omega$	$\vec{v} = \vec{\omega} \times \vec{r}$	
7	Relation between a_t , r and α	$a_t = r\alpha$	$\vec{a}_t = \vec{\alpha} \times \vec{r}$	
8	Equations of angular motion	$\omega_f = \omega_i + \alpha t$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$
9	Centripetal acceleration	$a_c = \frac{v^2}{r}$		
10	Centripetal force	$F_c = \frac{mv^2}{r}$		
11	Centripetal force (in angular measures)	$F_c = mr\omega^2$		
12	Moment of inertia of a particle	$I = mr^2$		
13	Torque on a particle rotating about a fixed point	$\tau = mr^2\alpha$		
14	Moment of inertia of a thin rod of length L	$I = \frac{1}{12}mL^2$		
15	Moment of inertia of a hoop	$I = mr^2$		
16	Moment of inertia of a disc	$I = \frac{1}{2}mr^2$		
17	Moment of inertia of a sphere	$I = \frac{2}{5}mr^2$		
18	2nd law of motion for rotational motion	$\tau = I\alpha$		
19	Angular momentum	$\vec{L} = \vec{r} \times \vec{P}$	$L = rp\sin\theta$	$L = mvr\sin\theta$
20	Angular momentum (in angular measures)	$L = mr^2\omega$	$L = I\omega$	
21	Rotational K.E.	$K.E._{rot} = \frac{1}{2}I\omega^2$	$K.E._{rot} = \frac{1}{2}mr^2\omega^2$	
22	Rotational K.E. of disc	$K.E._{rot} = \frac{1}{4}mr^2\omega^2$	$K.E._{rot} = \frac{1}{4}mv^2$	
23	Rotational K.E. of hoop	$K.E._{rot} = \frac{1}{2}mr^2\omega^2$	$K.E._{rot} = \frac{1}{2}mv^2$	
24	Velocity of hoop falling from an inclined plane of height h	$v = \sqrt{gh}$		
25	Velocity of disc falling from an inclined plane of height h	$v = \sqrt{\frac{4}{3}gh}$		

26	Velocity of sphere falling from an inclined plane of height h	$v=\sqrt{\frac{10}{7}gh}$	
27	Critical orbital velocity	$v=\sqrt{gR}$	$v=\sqrt{\frac{GM}{R}}$
28	Time period of close orbiting satellite	$T=\frac{2\pi R}{v}$	
29	Apparent weight of an object at rest or moving up with uniform velocity	$T=mg$	
30	Apparent weight of an object moving up with uniform acceleration a	$T=mg+ma$	
31	Apparent weight of an object moving down with uniform acceleration a	$T=mg-ma$	
32	Apparent weight of an object falling freely	$T=0$	
33	Orbital velocity of a satellite	$v=\sqrt{\frac{GM}{r}}$	
34	Spinning frequency of a satellite about its own axis	$f=\frac{1}{2\pi}\sqrt{\frac{g}{R}}$	
35	Orbital radius of a geo-stationary satellite	$r=\left(\frac{GmT^2}{4\pi^2}\right)^{\frac{1}{3}}$	

Key Points

- Angular displacement is the angle forms at the centre of a circle when a body moves in circle. It is related to linear displacement by equation
  $S=r\theta$ or $\vec{S}=\vec{\theta}\times\vec{r}$
- Angular acceleration of a body is the change in angular velocity of a body in particular time. It is related to linear acceleration by equation: $=\vec{\alpha}\times\vec{r}$
- The force which attracts a body towards the centre of circle, when a body moves in circle, is called centripetal force. It formula is: $F_c=\frac{mv^2}{r}$.
- The torque in terms of moment of inertia is: $\tau=I\alpha$
- Relation between torque and angular momentum is: $\tau=\frac{\Delta L}{\Delta t}$.
- The orbital speed of a satellite related to its orbit is given by: $v_o=\sqrt{\frac{GM_e}{r_e}}$.
- Geo-stationary satellite is one whose angular velocity is synchronized with angular velocity of earth.
- The gravity provided to the inhabitants of a spaceship is called artificial gravity.

- Apparent weight of a body is the force needed to prevent the body falling in the gravitational field of the earth.
- The whole surface of earth can be covered by using three geo-stationary satellites.
- The state of a body in which it becomes weightless, is called weightlessness.
- The torque produced in body of mass 'm' when force $\vec{F}$ is applied on it at position vector $\vec{r}$, is given by: $\tau = |\vec{\tau}| = rF\sin\theta$.

Solved Examples

Example 5.1: **An electric motor turns at 400 rpm. What is the angular velocity? What is the angular displacement after 4 s?**

Given Data:

Frequency of rotation= 400rpm time t=4s

Required:

Angular velocity ω =?

Angular displacement θ =?

Solution:

$$\omega = 400\,\text{rpm} = 400 \times \frac{2\pi}{60}\,\text{rad/s}^{-1} = 41.9\,\text{rad/s}$$

$$\theta = \omega t = 41.9\text{rad/s} \times 4\text{s} = 167.6\text{ rad}$$

41.9rad/s, 167.6 rad

Example 5.2: **In a carnival ride, the passenger travel in a circle of radius 5.0m, making one complete circle in 4.0s. What is its acceleration?**

Given Data:

Radius of circle r = 5.0m, time T = 4.0s

Required:

Acceleration a = ?

Solution:

The speed is the circumference of the circle divided by the period T

$$V = \frac{S}{t} = \frac{2\pi R}{t} = \frac{2(3.14)(5)}{4} = 7.85\text{ ms}^{-1}$$

$$a = \frac{v^2}{R} = \frac{(7.85\text{ms}^{-1})^2}{5.0\text{m}} = 12.3\text{ms}^{-2}$$

Example 5.3: **The curved roadway is designed in such a way that a car will not have to rely on friction to round the curve even when the road is covered with ice. Suppose the designated speed for the road is to be 12 m/s and the radius of the curve is 36.0 m. At what angle should the curve be banked?**

Given Data:

Circle of radius r = 36.0m, speed v = 12 m/s

Required:

Angle of bank θ =?

Solution:

$$\tan\theta = v^2 / rg$$

Angle of bank $\theta = \tan^{-1}((12\ m/s)^2 / (36m \times 9.8)) = 22^o$

22^o

Example 5.4: **A 2-kg mass swings in a circle of radius 50-cm at the end of a light rod. What resultant torque is required to give an angular acceleration of 2.5 rad/s²?**

Given Data:

Radius r = 50cm = 0.5 m, mass, m = 2kg

Angular acceleration $\alpha = 2.5\ rad\ s^{-2}$

Required:

Torque, t = ?

Solution:

$I = mR^2$

$= (2\ kg)(0.5\ m)^2 = 0.5\ kg\ m^2$

$\tau = I\alpha = (0.5\ kg\ m^2)(2.5\ rad/s^2)$

$= 1.25\ N\ m$

1.25 N m

Example 5.5: **Find Earth's angular momentum using Earth-Sun distance and mass of Earth,**
Earth - Sun distance 149.6×10^9 m
Mass of the Earth 5.9742×10^{24} kg

Given Data:

Mean distance from Earth to Sun = 149.6×10^9 km

Mass of the Earth M_e 5.9742×10^{24} kg

Required:

Angular momentum L = ?

Solution:

For a circular orbit, angular momentum is

$L = r \times M_e\ \vec{v}$

The average angular momentum is $M_e vr$, treating the Earth as if it were a point mass.

Earth takes 365 days to go one complete circle around Sun.

$$V = \frac{s}{t} = \frac{2\pi r}{t}$$

$$= \frac{2 \times 3.14 \times 149.6 \times 10^9}{365 \times 24 \times 3600} = 2.98 \times 10^4 ms^{-1}$$

Average angular momentum

$$L = M_e v r = 5.9742 \times 10^{24}\ kg \times 2.98 \times 10^4 ms^{-1} \times 149.6 \times 10^9\ m$$

$$= 2.663 \times 10^{40} kgm^2 s^{-1}$$

Earth angular momentum is $2.663 \times 10^{40}\ kgm^2\ s^{-1}$

Example 5.6: **A body of moment of inertia 0.80kgm² about a fixed axis, rotates with constant angular velocity of 100 rad s⁻¹. Calculate:**
i. Its angular momentum
ii. Torque to sustain this position

Given Data:

Moment of inertia = I = $0.80\ kg\ m^2$

Angular velocity = w = 100 rads ^{Q}

Required:

i. Torque = $\vec{\tau}$ =?

ii. angular momentum = L =?

Solution:

i. For L: We use the equation: $L = I\omega$

$L = 0.80 \times 100 = 80\ kg\ m^2 s^{-1}$

ii. As angular velocity is uniform, so there is no change in angular velocity (ω = constant) and as a result angular acceleration is zero. So

$\vec{t} = \vec{a} = PNXP \times P = \vec{P}$

$$\alpha = \frac{\Delta\omega}{t} = \frac{0}{t} = 0 \quad (\text{as } \Delta\omega = 0)$$

$\tau = I\alpha = I(o) = 0$

$\boxed{0}$

Example 5.7: **A child of mass 25 kg stands at the edge of a rotating platform of mass 150 kg and radius 4.0 m. The platform with the child on it rotates with an angular speed of 6.2 rad/s. The child jumps off in a radial direction. What happens to the angular speed of the platform? Treat the platform as a uniform disk.**

Given Data:

Mass of child m = 25kg

mass of plat form, m_2 = 150kg, radius r = 4.0m

initial angular speed, ω_i = 6.2 rad s^{-1}

Required:

Final angular speed ω_f = ?

Solution:

$\boxed{I_{platform} = \frac{1}{2}mr^2 = \frac{1}{2}\ 150kg\ (4m)^2 = 1200\ kgm^2}$

From Conservation of angular momentum

The angular momentum of two interacting objects is constant.

$(I_{platform} + I_{child})\omega_i = I_{platform}\omega_f$

$(1200 + 400)(6.2) = (1200)\omega_f$

$$\frac{(1600)(6.2)}{1200} = \omega_f$$

$\Rightarrow \omega_f = 8.27$ rad s^{-1}

$\boxed{I_{child} = mr^2 = 25kg\ (4m)^2 = 400\ kgm^2}$

Example 5.8: **A 70 kg man is standing on a scale in an elevator which is accelerating, as it heads for the top floor of a building at 4 m/s². What apparent weight will show on the scale?**

Given Data:

Mass = m = 70kg

Acceleration $\vec{a}$ = 4 ms^{-2}

Required: Apparent weight, $\vec{T}$ = ?

Solution:

$T = mg + ma$

$T = 70(9.8ms^{-2} + 4ms^{-2})$

$T = 966\ N$

Example 5.12: **Determine the orbital speed for the International Space Station (ISS). If its orbit 4.0 × 10² km above the earth surface.**

Given Data:

The radius at which it orbits

$r = R_E + 4.00 \times 10^2\ km = 6.36 \times 10^6 m + 4.00 \times 10^2\ km$

$G = 6.673 \times 10^{MI}\ Nm^2\ kg^{-2}$

$M_e = 6 \times 10^{24}\ kg$

Required:

Orbital speed $v_{orbit} = ?$

Solution:

$$v_o = \sqrt{\frac{GM_e}{r_o}} = \sqrt{\frac{6.67 \times 10^{-QI}\ Nm^2 kg^{-2} (6 \times 10^{24}\ kg)}{(6.36 \times 10^6 m + 4.00 \times 10^2 km}}$$

$$\boxed{v_o = 7.67 \times 10^3 m/s}$$

Example 5.13: **What should be the orbital speed to launch a satellite in a circular orbit 900 km above the surface of the earth?**

Given Data:

Height above the surface of earth is: h 900 km = 9×10^U m

Radius of earth = R_e 64×10^U m

So radius of orbit will be: $r_o = R_e$ h 73×10^U m

Here G $6.673 \times 10^{MQQ}\ Nm^R\ kg^{MR}$

And mass of earth = $M_e = 6 \times 10^{RT}$ kg.

Required:

Orbital speed v_o =?

Solution:

Using the formula: $v_o = \sqrt{\frac{GM_1}{r_o}}$

We get: $v_o = \sqrt{\frac{6.673 \times 10^{MI} \times 6 \times 10^{24}}{73 \times 10^5}}$

Or $v_o = 7.4 \times 10^3 ms^Q$

$$\boxed{v_o = 7.4 \times 10^3 ms^Q}$$

Text Book Exercises

Q.1 Select the correct answer of the following questions.

1. **The angular speed in radians/hours for daily rotation of our earth is?**
   (a) 2π (b) 4π (c) $\pi/6$ (d) $\pi/12$
2. **Linear acceleration**
   (a) 0° (b) 180° (c) 360° (d) 90°
3. **What is moment of inertia of a sphere**
   (a) MR^2 (b) $\frac{1}{2}MR^2$ (c) $\frac{2}{5}MR^2$ (d) $\frac{1}{2}M^2R$
4. **Aparticle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of the particle takes place in a horizontal plane. It follows**
   (a) Linear momentum is constant (b) Velocity is constant
   (c) It moves in a circular path (d) particle move in straight line
5. **Abody moving in a circular path with constant speed has**
   (a) Constant acceleration (b) Constant retardation
   (c) Variable acceleration (d) Variable speed and constant velocity

6. **Astronauts appear weightless in space because**
(a) there is no gravity in space
(b) there is no floor pushing upwards on the
(c) satellite is freely falling
(d) there is no air in space

7. **Which one is constant for a satellite in orbit?**
(a) Velocity (b) K.E (c) Angular Momentum (d) Potential Energy

8. **If the earth suddenly stops rotating the value of 'g' at equator would:**
(a) Decrease (b) Remain unchanged (c) Increase (d) Become Zero

9. **If solid sphere and solid cylinder of same mass and density rotate about their own axis, the moment of inertia will be greater for**
(a) Solid sphere
(b) Solid cylinder
(c) The one that has the largest mass arrives first.
(d) The one that has the largest radius arrives first.

10. **The gravitational force exerted on an astronaut on Earth's surface is 650N down. When she is in the international Space Station, the gravitational force on her is**
(a) larger,
(b) exactly the same
(c) smaller
(d) nearly but not exactly zero, or (e) exactly zero?

11. **A solid cylinder of mass M and radius R rolls down an incline without slipping. Its moment of inertia about an axis through its center of mass is $MR^2/2$. At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy?**
(a) ½ (b) ¼ (c) 1/3 (d) 2/4

No.	Option	ANSWER	EXPLANATION
1.	(d)	$\frac{\pi}{12}$	$\omega = \frac{\theta}{t} = \frac{2\pi_{rad}}{24_{hour}} = \frac{\pi}{12}$ rad h^{-1}
2.	(d)	90°	$\vec{a} = \vec{\alpha} \times \vec{r} = \alpha r \sin\theta \hat{n}$ $\Rightarrow a = \alpha r \sin 90° = \alpha r (1) = \alpha r = r\alpha$
3.	(c)	$\frac{2}{5}MR^2$	For sphere $I = \frac{2}{5}MR^2$
4.	(c)	It moves in a circular path	Centripetal force is always perpendicular to motion i.e. velocity, and it moves body in circle.
5.	(a)	Constant acceleration	When body moves in circle, it has centripetal acceleration which has constant magnitude ($a_c = \frac{v^2}{r}$) and it is directed always towards centre of circle.
6.	(c)	satellite is freely falling	
7.	(c)	Angular Momentum	As no external torque acts on satellite, so L = constant, from law of conservation of angular momentum.
8.	(c)	Increase	Rotation of earth causes centrifugal force acting on body, when earth stops centrifugal force becomes zero, so net attractive force i.e., gravity increases.
9.	(b)	Solid cylinder	Moment of inertia of sphere $= \frac{2}{5}mr^2$ Moment of inertia of cylinder $= \frac{1}{2}mr^2$ From above equations moment of inertia of cylinder is greater.
10.	(c)	Smaller	Gravitational force is inversely proportional to the distance from centre of earth. i.e. $F \propto (\frac{1}{r})$

11.	(c)	1/3	$\frac{K.E._{rot}}{K.E._{total}} = \frac{K.E._{rot}}{K.E._{translational} + K.E._{rotational}}$ *For solid cylinder or dics* $\frac{K.E._{rot}}{K.E._{total}} = \frac{\frac{1}{4}mv^2}{\frac{1}{2}mv^2 + \frac{1}{4}mv^2} = \frac{\frac{1}{4}mv^2}{\frac{1}{4}mv^2(2+1)} = \frac{1}{3}$

SHORT ANSWERS OF THE EXERCISE

Q.2 Write short answers of the following questions.

1. Why is the fly wheel of an engine made heavy in the rim?

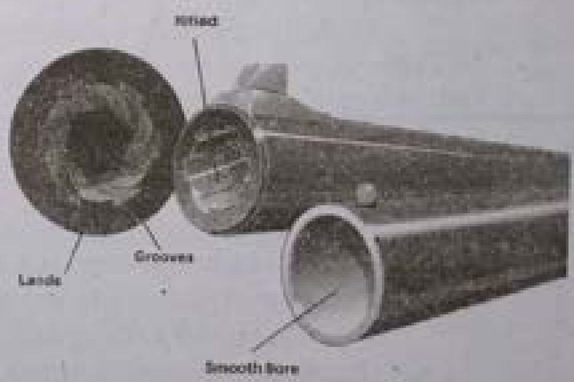
Ans: The rim of fly wheel of an engine is made heavy to concentrate more and more mass at greater perpendicular distance from axis of rotation i.e. at edges. Due to which moment of inertia increases.

It has large moment of inertia, due to which it rotates with uniform rotational speed.

Reason: As we know that moment of inertia of hoop is greater as compared to disc, its reason is that it is hollow from middle and all its mass is concentrated at periphery (rim). It increases its moment of inertia, therefore its opposition to any change in uniform rotatory motion is large. Due to which it maintains its uniform rotational motion.

2. Why is a rifle barrel 'rifled'?

Ans: Rifling of a rifle barrel means cutting spiral like groove in the interior of the barrel. The barrel of a rifle is rifled to keep the bullet along the required direction.



Explanation:

When a rifle is fired the bullet moves through the barrel and has translational motion as well as rotational motion about its own axis. This rotational motion helps the bullet to keep it along the required direction with aerodynamic shape. The total kinetic energy of the bullet is the sum of translational K-E and rotational K-E.

$$K.E = (K.E)_{trans} + (K.E)_{rot}$$

$$K.E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

This much K.E of the bullet gives the directional stability to the bullet so that it could preserve its direction for larger distances and hits the target with less chance of missing the target.

3. Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on a table? Explain.

Ans: Hard-boiled egg will spin faster than raw egg when same torque is applied on both.

Explanation:

The raw egg contains liquid in it and when you rotate it the centrifugal force will act on the liquid and push it towards outer shell. Therefore moment of inertia of raw egg is greater and angular velocity is smaller. Hard-boiled egg act as a rigid body, while rotating. The moment of inertia of hard-boiled egg is smaller and its angular velocity is greater. Hence hard-boiled egg will spin faster than raw egg when same torque is applied in both the cases.

4. Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?

Ans: When a body is moving uniformly in a circle then magnitude of its velocity (speed) is constant and the direction of the velocity is changing at each point due to which centripetal acceleration produces. In order to move in a circle the centripetal force acts on it, which is always directed towards centre of the circle. According to Newton's 2[nd] law of motion the force produces acceleration in its direction. Therefore the direction of centripetal acceleration is also towards centre of the circle and along the direction of centripetal force.

$$F_c = \frac{mv^2}{r} = m\left(\frac{v^2}{r}\right)$$

$$\vec{F}_c = m\vec{a}_c$$

5. A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?

Ans: When the mass is in equilibrium, the tension in the string balances the weight i.e.

$T = W = mg$.

But when the mass oscillates, the tension in the string increases due to action of centrifugal force.

Now Tension > weight

Due to this increase in tension, the string breaks.

6. An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.

Ans: Insect will fly off the wheel tangent to the surface of the wheel i.e. in the direction of velocity, when it rotates at higher speed.

Reason:

When wheel rotates then centrifugal force acts on the insect which tries to move away from the axis of rotation, but friction keeps it in contact with the wheel. But the insect will fly away, as rotational speed of the wheel increases due to which centrifugal force becomes greater than friction between insect and wheel.

7. Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.

Ans: Minimum *three correctly positioned geo-stationary satellites* are required for the global coverage of T.V transmission.

Reason:

As each satellite in geo-stationary orbit may covers *120° of longitude* so for whole populated surface of the earth, there must be minimum three correctly positioned geo-stationary satellites to provide the coverage of 360° of longitude.

8. Explain the significance of moment of inertia in rotatory motion.

Ans: **Moment of Inertia:**

- Moment of inertia is the property of body to maintain its state of rest or state of uniform angular motion. $I = mr^2$ is called moment of inertia or rotational inertia of a body.
- It plays the same role in angular motion which mass 'm' plays in linear motion i.e. that of inertia, that is why, it is named as rotational inertia.
- Greater the moment of inertia, greater will be tendency of the body to maintain its state either of rest or of uniform rotational motion.

9. Why does the coasting rotating system slow down as water drops into the beaker?

Ans: **EXPLANATION:**

When water drops into the beaker, the mass of the contents in the beaker increases which increases the moment of inertia as $I = mr^2$.

Angular momentum is given by $L = I\omega \rightarrow \omega = \frac{L}{I}$

$$\omega \propto \frac{1}{I}$$

According to the law of conservation of momentum the angular momentum remains constant. Therefore when moment of inertia increases then angular velocity decreases.

Point to Ponder

Why does the coasting rotating system slow down as water drops into the beaker?

10. A body will be weightless when the elevator falls down just like a free falling body. Explain.

Ans: When Elevator is falling freely under gravity i.e. $a = g$:

Now we consider that the elevator is falling freely under gravity. Then $a = g$

As $T = W - ma$

$T = mg - ma$

$T = mg - mg$

$T = 0$

Result:

So, the apparent weight of object shown by the spring balance is zero. The object seems to be weightless. Thus it is in state of weightlessness.

11. When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel, why? Explain.

Ans: Large tyre has large moment of inertia so its angular velocity is small that's why heavier tyre rotates slower than its lighter tyre.

Reason: The moment of inertia is: $I = mr^2$.

Where m = mass of the body

And r = Distance from the axis of the rotation.

The mass and size (r) of the heavier wheel is large value, due to which it has large rotational inertia. So when a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel. Lighter wheel have smaller moment of inertia and their speed of rotation is greater.

Comprehensive Questions

1. Q3. Give a short response to the following questions.

1. What are centripetal acceleration and centripetal force? Derive their equations.

Ans: Q. No. 8

2. Show that angular momentum in magnitude is given by $|\vec{L}| = |\vec{r} \times \vec{p}| = mr^2\omega = mvr$

Ans: Q. No. 14(a)

3. Show that role playing by mass in linear motion is playing by moment of inertial in rotatory motion.

Ans: Q. No. 11

4. What do you mean by 'INTELSAT'. At what frequency it operates. For how many T.V. stations this system is used?

Ans: **Satellite Organization (INTELSAT):**

The largest satellite system, managed by 126 countries is called INTELSAT VI "INTELSAT" means international telecommunication, satellite organization.

It works at the microwave frequencies of 4, 6, 11 and 14 GHz and has the capacity of 30,000 two-way telephone circuit plus three T.V channels.

5. Show that in angular form, centripetal acceleration is: $\vec{a}_c = -\omega^2\vec{r}$.

Ans: Q. No. 8

6. Show that centripetal force is also shown by $\vec{F}_c = \frac{-mv^2}{r}\hat{r} = -m\omega^2\vec{r} = \frac{-mv^2}{r^2}\vec{r}$

Ans: Q. No. 8

7. Show that a satellite near the earth will have greater velocity.

Ans: The orbital velocity of the satellite is:

$$v = \sqrt{\frac{GM}{r}}$$

Where $r = R + h$

r = Radius of the orbit

R = Radius of the earth

h = Height of the orbit above from the equator

→ Speed of satellite is inversely proportional to the distance from the centre of the earth i.e.

$$v \propto \frac{1}{\sqrt{r}}$$

Or it is inversely proportional to the square of its altitude from earth surface.

$$v \propto \frac{1}{\sqrt{h}}$$

i.e. Satellites closer to earth revolve with fast speed and vice versa.

8. What do you mean by weight of a body? Use examples to distinguish between real weight and the apparent weight of a body.

Ans: Q. No. 21

9. Explain, how gravity is provided to the occupants of the space ship.

Ans: Q. No. 25

10. Give different three examples to illustrate the phenomenon of conservation of angular momentum.

Ans: Q. No. 15

11. Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles?

Ans: Mud guard are used to stop mud to fly off and travel in a straight line.

Reason: Mud sticks to the surface of tyre. When tyre rotates then centrifugal force acts on it. When centrifugal force becomes greater than the adhesive (attractive) force between mud and tyre then it will fly off to the tangent of the surface of tyre in the direction of velocity. So the mud may strike the vehicles coming from behind. Mud guards are used to stop mud from hitting the vehicles.

Numerical Problems

1. If plate of microwave oven has a radius of 0.15 m and rotates at 6.0 rev/min, calculate the total distance travelled by the fly (plate) during a 2.0 minutes cooking period?

Solution:

Given Data: Radius = r = 0.15m

Angular velocity = ω = 6rev/min = $6 \times \frac{2\pi}{60}$ rad/sec = 0.628rad/sec

Time period = T = 2 min = 2×60sec = 120sec

To Find:

Distance travelled = S = ?

Calculation:

We know that, $S = r\theta$ ______ (1)

We also know that, $\theta = \omega T$ ______ (2)

Putting equation (2) in equation (1), we get,

$S = r\,\omega T$ ______ (3)

Putting the values in equation (3), we get,

$S = [0.15 \times 0.628 \times 120]m \Rightarrow S = 11.3m$

Or **$S = 11m$** approximately

Distance travelled by fly = S = 11m

2. A Circular drum of radius 40 cm is initially rotating at 400 revolutions/min. It is brought to stop after making 50 revolutions. What is angular acceleration and stopping time?

Solution:

Given Data: Radius of drum = $r = 40$ cm

Initial angular velocity = $\omega_i = 400\text{rev/min} = \frac{400 \times 2\pi}{60}$ rad/sec

$\Rightarrow$ $\omega_i = 41.866$ rad/sec

Angular displacement = $\theta = 50$ revolution $= 50 \times 2\pi$ rad $= 314$ rad

$\Rightarrow$ Final angular velocity = $\omega_f = 0$

To Find:

(a) Angular acceleration = α =?

(b) Stopping time = t =?

Drum comes to rest, so put $\omega_f = 0$

Calculation:

(a) We know that,

$2\alpha\theta = \omega_f^2 - \omega_i^2$ ______ (2)

Putting the values in equation (1), we get,

$2 \times \alpha \times 314 = 0^2 - (41.866)^2 \Rightarrow 628\alpha = -1752.76$

$\Rightarrow$ $\alpha = \left[\frac{-1752.76}{628}\right] \text{rad/sec}^2 = -2.79 \text{rad/sec}^2$

$\alpha = -2.79 \text{rad/sec}^2$

(b) We know that,

$\omega_f = \omega_i + \alpha t$ ______ (1)

Putting the values in equation (1), we get,

$0 = 41.866 - 2.79t$

$\Rightarrow$ $-2.79t = -41.866$

$\Rightarrow$ $t = \left(\frac{41.866}{2.79}\right)$ sec

$\Rightarrow$ **$t = 15.0$ sec**

Stopping time = 15.0 sec

3. A string 1m long is used to whirl a 100g stone in a horizontal circle at a speed of 2m s^{-1}. Find the tension in the string.

Solution:

Given Data: Length of string = $l = r = 1$ m,

Mass of stone = $m = 100$ gm = 0.1 kg.

To Find: Tension in the string = T = ?

Here the tension in the string must be equal to the centripetal force is

Calculation: $T = F_c = \frac{mv^2}{r}$

$\Rightarrow$ $T = \frac{0.1 \times 2^2}{1}$ $\Rightarrow$ **$T = 0.4$ N**

4. The moon revolves around the earth in almost a circle of radius 382400 km in 27.3 days. What is the centripetal acceleration?

Solution:

Given Data: Radius of circle = r = 382400 km

⇒ $r = 382400 \times 10^3$ m

Time period = t = 27.3 day = 27.3 × 24 × 3600 = 2358720 s

To Find: Centripetal acceleration = a_c = ?

$V = r\omega$
$T = 2\pi/\omega$
⇒ $\omega = 2\pi/T$

We know that,

Calculation: $a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r}$

⇒ $a_c = r\omega^2$

⇒ $a_c = r\left(\frac{2\pi}{T}\right)^2 = 4\pi^2\left(\frac{r}{T^2}\right)$

⇒ $a_c = \frac{4 \times 3.14^2 \times 382400 \times 10^3}{(2358720)^2}$

⇒ $\boxed{a_c = 0.00271\ m/s^2}$

5. A modern F1 car can accelerate from 0 to 62 mile/h (100 km/h) in 2.50 s. What is the angular acceleration of its 170 mm radius wheel?

Solution:

Given Data: Initial velocity = v_i = 0m/sec

Final velocity = v_f = 100km/h = $100 \times \frac{10^3\ m}{3600\ sec}$

⇒ v_f = 27.77m/sec = 27.8m/sec

Time taken = t = 2.5 sec

Radius of wheel = r = 170mm = 170×10^{-3}m = 0.17m

Given Data:

Angular acceleration = α = ?

Calculation:

We know that,

$\omega_f = \omega_i + \alpha t$

⇒ $\alpha t = \omega_f - \omega_i$ (1)

Also $v = r\omega$ ⇒ $v_i = r\omega_i$ ⇒ $\omega_i = \frac{v_i}{r}$ ______ (2)

Similarly, $\omega_f = \frac{v_f}{r}$ ______ (3)

Putting equation (2) and equation (3) in equation(1), we get

$\alpha t = \frac{v_f}{r} - \frac{v_i}{r} = \frac{1}{r}(v_f - v_i)$

⇒ $\alpha = \frac{1}{rt}(v_f - v_i)$ ______ (4)

Putting the value in equation (4), we get,

$\alpha = \frac{1}{(0.170 \times 2.5)}[27.8 - 0] = \left(\frac{27.8}{0.425}\right) rad/sec^2$

⇒ $\boxed{\alpha = 65.4\ rad/sec^2}$

6. An electric motor is running at 1800 rev min⁻¹. It comes to rest in 20s. If the angular acceleration is uniform find the number of revolutions it made before stopping.

Solution:

Given Data: Initial angular speed = ω_i = 1800 rev/min

⇒ $\omega_i = 1800 \times 2\pi/60$ rad/s = 188.4 rad/s

Final angular speed = $\omega_f = 0$

Time taken = t = 20 s

To Find: Number of revolutions = N = ?

We have,

$$\text{No. of revolution} = \frac{\text{total angular displacement}}{2\pi}$$

$$\Rightarrow \quad N = \frac{\theta}{2\pi} \longrightarrow (1)$$

Now we have, $\left[\alpha = \frac{\omega_f - \omega_i}{t}\right]$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \quad \theta = \omega_i t + \frac{1}{2} \frac{(\omega_f - \omega_i)t^2}{t}$$

$$\Rightarrow \quad \theta = \omega_i t + \frac{1}{2}(\omega_f - \omega_i)t$$

$$\Rightarrow \quad \theta = 188.4 \times 20 + \frac{1}{2}(0 - 188.4) \times 20$$

$$\Rightarrow \quad \theta = 188.4 \times 20 + (-188.4) \times 10 = 3768 - 1884$$

$$\Rightarrow \quad \theta = 1884 \text{ rad}$$

Putting θ = 1884 rad in eq (1), we get,

$$N = 1884/2\pi \quad \Rightarrow \quad \boxed{N = 300 \text{ rev}}$$

7. **What is the moment of inertia of a 100 kg sphere whose radius is 50 cm.**

Solution:

Given Data: Mass of sphere = m = 100 kg,

Radius of sphere = R = 50 cm = 0.5 m.

To Find: Moment of inertia = I = ?

We know that the moment of inertia of a solid sphere is given by,

$$I = \frac{2}{5} MR^2$$

$$\Rightarrow \quad I = \frac{2}{5} \times 100 \times (0.5)^2 \Rightarrow \boxed{I = 10 \text{ kg.m}^2}$$

8. **A rope is wrapped several times around a cylinder of radius 0.2 m and mass 30 kg. What is the angular acceleration of the cylinder if the tension in the rope is 40 N and it turns without friction**

Solution: Radius of cylinder = 0.2 m

Mass of cylinder = 30 kg

Required: Angular acceleration = α =?

Calculation:

We know that,

$$\tau = I\alpha \quad \_\_\_\_\_\_ (1)$$

We also know that,

$$\tau = rF \quad \_\_\_\_\_\_ (2)$$

Comparing equation (1) and equation (2), we get,

$$I\alpha = rF \quad \Rightarrow \quad \alpha = \frac{rF}{I} \quad \_\_\_\_\_\_ (3)$$

The moment of inertia of cylinder is given by,

$$I = \frac{1}{2} mr^2 \_\_\_\_\_\_\_\_ (4)$$

Putting equation (4) in equation (3), we get,

$$\alpha = \frac{rF}{\left(\frac{mr^2}{2}\right)} = \frac{2rF}{mr^2} = \frac{2F}{mr} \quad \Rightarrow \quad \alpha = \frac{2F}{mr} \_\_\_\_\_\_\_\_ (5)$$

Putting the values in equation (5), we get,

$$\alpha = \left(\frac{2 \times 40}{30 \times 0.2}\right) rad/sec^2 \quad \Rightarrow \quad \boxed{\alpha = 13.3 rad/sec^2}$$

9. What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15m if it rolls across a level surface with a speed of 2 ms^{-1}?

Solution:

Given Data: Mass of solid ball = M = 5 kg,
Diameter of ball = D = 15 cm = 0.15 m
Radius of solid ball = R = D/2 = 0.15/2 = 0.075 m
Speed of ball = V = 2m/s,

To Find: K.E of rolling ball = K.E = ?

We know that a rolling body posses both linear K.E and rotational K.E, so its total K.E is given by,

$$(K.E)_{Total} = (K.E)_{Linear} + (K.E)_{rot}$$

$$\Rightarrow \quad K.E = \frac{1}{2} mv^2 + \frac{1}{5} mv^2$$

$$\Rightarrow \quad K.E = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{5} \times 5 \times 2^2$$

$$\Rightarrow \quad K.E = (10 + 4)J \quad \Rightarrow \quad \boxed{K.E = 14 \text{ joules}}$$

10. A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction.

Solution:

Given Data: Diameter of cylinder = D = 50 cm = 0.5 m
Radius of solid cylinder = R = D/2 = 0.5/2 = 0.25 m
Height = h = 29.4 cm = 0.294 m

To Find:
(a) linear speed = v = ?
(b) Angular speed = ω = ?

Solution:

(a) $$V = \sqrt{\frac{4}{3} gh}$$

$$\Rightarrow \quad V = \sqrt{4 \times 9.8 \times 0.294/3} \quad \Rightarrow \quad \boxed{V = 1.96 \text{ m/s}}$$

(b) As $V = R\omega$

$$\Rightarrow \quad \omega = V/R = 1.96/0.25$$

$$\Rightarrow \quad \boxed{\omega = 7.84 \text{ rad/s}}$$

11. A disc without slipping rolls down a hill of vertical height 1000cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

Solution:

Given Data: Height of hill = h = 1000 cm = $\frac{1000}{100}$m = 10m

$$\Rightarrow \quad h = 10 \text{ m}$$

To Find: Speed at the bottom = v = ?

$\Rightarrow \quad v = \sqrt{\frac{4}{3}gh}$

$\Rightarrow \quad v = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$

$\Rightarrow \quad \boxed{v = 11.4\ m/s}$

12. A motor car is travelling at a speed of 30 ms^{-1}. If the wheel has a diameter of 1.5 m, find its angular speed in rad s^{-1} and rev s^{-1}

Solution:

Given Data: $v = 30\ ms^{-1}$

$d = 1.5\ m$

$\Rightarrow \quad r = \frac{d}{2} = \frac{1.5}{2} = 0.75\ m$

To Find: $\omega = ?$

We know that $v = r\omega$

$\Rightarrow \quad \omega = \frac{v}{r} = \frac{30}{0.75} = 40\ rad\ s^{-1}$

$\Rightarrow \quad \omega = 40\ rad\ s^{-1} = 40 \times \frac{1}{2\pi}\ rev\ s^{-1} = 6.37\ rev\ s^{-1}$

Additional Conceptual Short Questions With Answers

1. Is there any work done by centripetal force?

Ans: When a body is moving in circular path then centripetal force is perpendicular to displacement $\theta = 90°$.

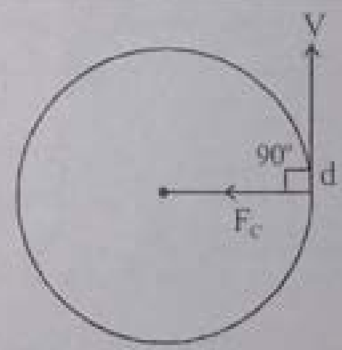
Work done by centripetal force is

$W = \vec{F_c} \cdot \vec{d}$

$W = F_c\ d \cos 90°$

$\Rightarrow \quad W = F\ d\ (0)$

$W = 0$



No work is done by centripetal force. It only changes the direction of the body to move it in circular path.

2. The ratio of rotational K.E. of two bodies of moment of inertia 9 kgm^2 and 1 kgm^2 are same. Find ratio of their angular momentum?

Ans: For linear motion, we can write:

Linear K.E $= \frac{P^2}{2m}$

Similarly, for angular motion,

Rotational K.E $= \frac{L^2}{2I} \quad \Rightarrow \quad 2I\,(K.E) = L^2$

$\Rightarrow \quad L = \sqrt{2I\,(K.E.)}$

$\Rightarrow \quad L = \text{Constt.}\sqrt{I}$

$\Rightarrow \quad \frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$

$\Rightarrow \quad \frac{L_1}{L_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$

$\boxed{L_1 : L_2 = 3 : 1}$

3. Find the rotational K.E. and speed of sphere at the bottom of inclined plane without slipping?

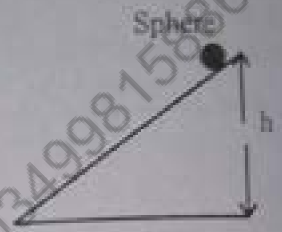
Ans: **Rotational K.E.**

$$\text{Rotational K.E} = \frac{1}{2} I\omega^2 \qquad \left[\because I = \frac{2}{5} mr^2\right]$$

$$= \frac{1}{2}\left(\frac{2}{5} mr^2\right)\omega^2$$

$$= \frac{1}{5} m (r\omega)^2$$

$$\text{Rotational K.E} = \frac{1}{5} mv^2$$



For sphere rolling on inclined plane

Loss of P-E = gain of translational K-E + gain of rotational K-E

$$P.E = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{5} mv^2$$

$$mgh = \left(\frac{1}{2} + \frac{1}{5}\right) mv^2$$

$$mgh = \frac{5+2}{10} mv^2$$

$$mgh = \frac{7}{10} mv^2$$

$$gh = \frac{7}{10} v^2$$

$$\Rightarrow v^2 = \frac{10}{7} gh$$

$$\Rightarrow \boxed{v = \sqrt{\frac{10}{7} gh}}$$

4. Show that rotational K.E. of disc, hoop and sphere have following relation:

(i) $(K.E)_{hoop} = 2\,(K.E)_{disc}$ **(ii)** $(K.E)_{sphere} = \frac{2}{5}(K.E)_{hoop}$

Ans: (i)

$$(K.E)_{disc} = \frac{1}{4} mv^2$$

$$(K.E)_{hoop} = \frac{1}{2} mv^2$$

$$(K.E)_{sphere} = \frac{1}{5} mv^2$$

$$\Rightarrow (K.E)_{disc} = \frac{1}{4} mv^2$$

$$(K.E)_{disc} = \frac{1}{2}\left(\frac{1}{2} mv^2\right)$$

$$(K.E)_{disc} = \frac{1}{2}(K.E)_{hoop}$$

$$\Rightarrow 2(K.E)_{disc} = (K.E)_{hoop}$$

$$\Rightarrow \boxed{(K.E)_{hoop} = 2(K.E)_{disc}}$$

(ii) $(K.E)_{sphere} = \frac{1}{5}mv^2$

$(K.E)_{sphere} = \frac{2}{5}\left(\frac{1}{2}mv^2\right)$

$$\boxed{(K.E)_{sphere} = \frac{2}{5}(K.E)_{hoop}}$$

5. What is relation between $(K.E)_{sphere}$ and $(K.E)_{disc}$?

Ans: $(K.E)_{disc} = \frac{1}{4}mv^2$

$(K.E)_{sphere} = \frac{2}{5}mv^2$

$(K.E)_{sphere} = \frac{2}{5} \times 4\left(\frac{1}{4}mv^2\right)$

$(K.E)_{sphere} = \frac{8}{5}(K.E)_{disc}$

6. The speed of a body moving in uniform circular motion is doubled and the radius of the circular path is halved. What happens to the centripetal force?

Ans: Centripetal force $F_c = \frac{mv^2}{r}$

When υ becomes 2υ and r becomes r/2, then

Centripetal force $= F_c' = \frac{m(2v)^2}{r/2} = \frac{mv^2}{r} \times 8$

$F_c' = 8F_c$

The centripetal force, increases 8 times than its original value.

7. Find angular momentum of the earth about its own axis?

Ans: Earth is like sphere, so its moment of inertia is:

$I = \frac{2}{5}MR^2$

$I = \frac{2}{5} \times 6 \times 10^{24} \times (6.4 \times 10^6)^2$

$I = 9.83 \times 10^{37}$ kg m^2

And angular velocity is: [For one complete rotation in one day $\theta = 2\pi$ and $T = 86400$ s]

$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{86400}$

$\omega = 7.27 \times 10^{-5}$ rad/s

So its angular momentum is:

$L = I\omega$

$L = (9.83 \times 10^{37})(7.27 \times 10^{-5})$

$L = 7.1 \times 10^{33}$ J s

What is relation between escape velocity and orbit velocity?

Ans: We know that

$V_{esc} = \sqrt{2gR}$

Or $V_{orb} = \sqrt{gR}$

So $V_{esc} = \sqrt{2gR}$

$V_{esc} = \sqrt{2}(\sqrt{gR})$ Putting $\sqrt{gR} = V_{orb}$

$$\boxed{V_{esc} = \sqrt{2}\,V_{orb}}$$

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

(FBISE 20

1. **What is moment of inertia of a sphere:**
   A. MR^2 B. $\frac{1}{2}MR^2$ C. $\frac{2}{5}MR^2$ D. $\frac{1}{2}M^2R$

(FBISE 20

2. **If the earth suddenly stops rotating the value of 'g' at equator would:**
   A. Decrease B. Remain unchanged C. Increase D. Become Zero

3. **Time Period of circular motion is given by:** (FBISE 2018 supply)
   A. $T = rV$ B. $T = \omega V$ C. $T = 2\pi\omega$ D. $T = \frac{2\pi}{\omega}$

4. **SI unit of Angular momentum is:**
   A. Nm B. Radian C. Ns D. Js

5. **Time period of pendulum in lift moving upward with constant velocity:**
   A. increases B. decrease C. remains constant D. None

6. **The relation between the orbital speed v_0 of a planet and its orbital radius r_0 is:**
   A. $v_o \propto 1/\sqrt{r_o}$ B. $v_o \propto 1/r_o$ C. $v_o \propto r_o$ D. $v_o \propto \sqrt{r_o}$

7. **A body of mass 2kg is suspended in an lift by means of a spring balance. The balance read i weight when the lift moves up with an acceleration of $5ms^{-2}$ as**
   A. 30.5 N B. 29.6 N C. 26.5 N D. 9.8 N

8. **The rotational kinetic energy of a hoop of mass m moving down an inclined plane with veloci v will be**
   A. $\frac{1}{4}mv^2$ B. $\frac{1}{2}mv^2$ C. $\frac{4}{3}mv^2$ D. $\frac{3}{4}mv^2$

9. **Apparent Weight of freely falling body**
   A. increases B. decrease C. zero D. remains same

10. **Which of following is not axial vector**
    A. linear momentum B. angular velocity C. angular acceleration D. Centripetal force

11. **Point moves along arc of length "l" and radius "r" in time "t", its angular velocity is given by**
    A. $\frac{l}{rt}$ B. $\frac{r}{lt}$ C. $\frac{2\pi}{lt}$ D. $\frac{2\pi r}{t}$

12. **Minute hand of large clock is 2m long, What is its angular speed approximately.**
    A. $1 \times 10^{-3}\ rad/s^{-1}$ B. $1.7 \times 10^{-3}\ rad/s$ C. $3 \times 10^{-3}\ rad/s$ D. $3.5 \times 10^{-3}\ rad/s$

13. **How many satellites form GPS in geostationary orbit _____:**
    A. 3 B. 9 C. 12 D. 24

14. **Height of geostationary satellites above equator is:**
    A. $6.4 \times 10^6\ m$ B. $3.6 \times 10^7\ m$ C. $3.6 \times 10^{-7}\ m$ D. $4.2 \times 10^7\ m$

15. **Expression for orbital speed**
    A. $\sqrt{\frac{GM}{r}}$ B. $\frac{\sqrt{GM}}{r}$ C. $\frac{GM}{r}$ D. $\sqrt{\frac{GM}{2r}}$

16. **Moment of Inertia of solid disc is:**
    A. mr^2 B. $\frac{1}{2}mr^2$ C. $\frac{3}{2}mr^2$ D. $\frac{2}{5}mr^2$

17. **100 kg motorcycle moving around a curved path of radius 100m, with velocity 144km Centripetal force:**
    A. 1600N B. 14000N C. 260N D. 377N

18. **SI Units of angular Momentum is:**

A. $kgm^{-3}s^{-1}$ B. kgm^{-1} C. $kgm^{2}s^{-1}$ D. None

19. **Displacement covered by body during two revolutions on a circle of radius r is:**

A. πr B. $2\pi r$ C. $4\pi r$ D. Zero

Answers Key

1.	C	2.	C	3.	D	4.	D	5.	C
6.	A	7.	B	8.	B	9.	C	10.	D
11.	A	12.	B	13.	A	14.	B	15.	A
16.	B	17.	A	18.	C	19.	D		

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 **Choose the correct answer from the given options.** (1 x 6 = 6)

SECTION – A

1. **A wheel of radius 50 cm having the angular speed of 5 rad/s will have linear speed in m/s**
   (A) 1.5 (B) 2.5 (C) 3.5 (D) 4.5
2. **The ratio of moment of inertia of disc and hoop is:**
   (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{1}{3}$
3. **2 radian = ________**
   (A) 114.6° (B) 57.3° (C) 75.3° (D) 37.5°
4. **A stone of mass 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16N, the maximum velocity of revolution that can be given to the stone without breaking it, will be;**
   (A) 12 ms^{-1} (B) 20 ms^{-1} (C) 16 ms^{-1} (D) 14 ms^{-1}
5. **Which one is constant for a satellite in orbit?**
   (A) Velocity (B) K.E (C) Angular Momentum (D) Potential Energy
6. **If the earth suddenly stops rotating the value of 'g' at equator would:**
   (A) Decrease (B) Remain unchanged (C) Increase (D) Become Zero

Question.No.2 **Give short answers of followings:** (3 x 7 = 21)

SECTION – B

(i) Why is the fly wheel of an engine made heavy in the rim?
(ii) A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?
(iii) Why does the coasting rotating system slow down as water drops into the beaker?
(iv) Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles?
(v) Explain, why is there weightlessness in satellites?
(vi) At what speed (in km/h) is a bank angle of 45° required for aero-plane to turn on a radius of 60 m?
(vii) How artificial gravity is produced in spaceships? Explain

Question.No.3 **Extensive Questions.** (13)

SECTION – C

(a) Define Centripetal force. Derive its formula. (06)
(b) Derive the formula for radius of geostationary orbit of geostationary satellites. (04)
(c) An electric motor is running at 1800 rev min^{-1}. It comes to rest in 20s. If the angular acceleration is uniform. Find the number of revolutions it made before stopping. (03)

The End

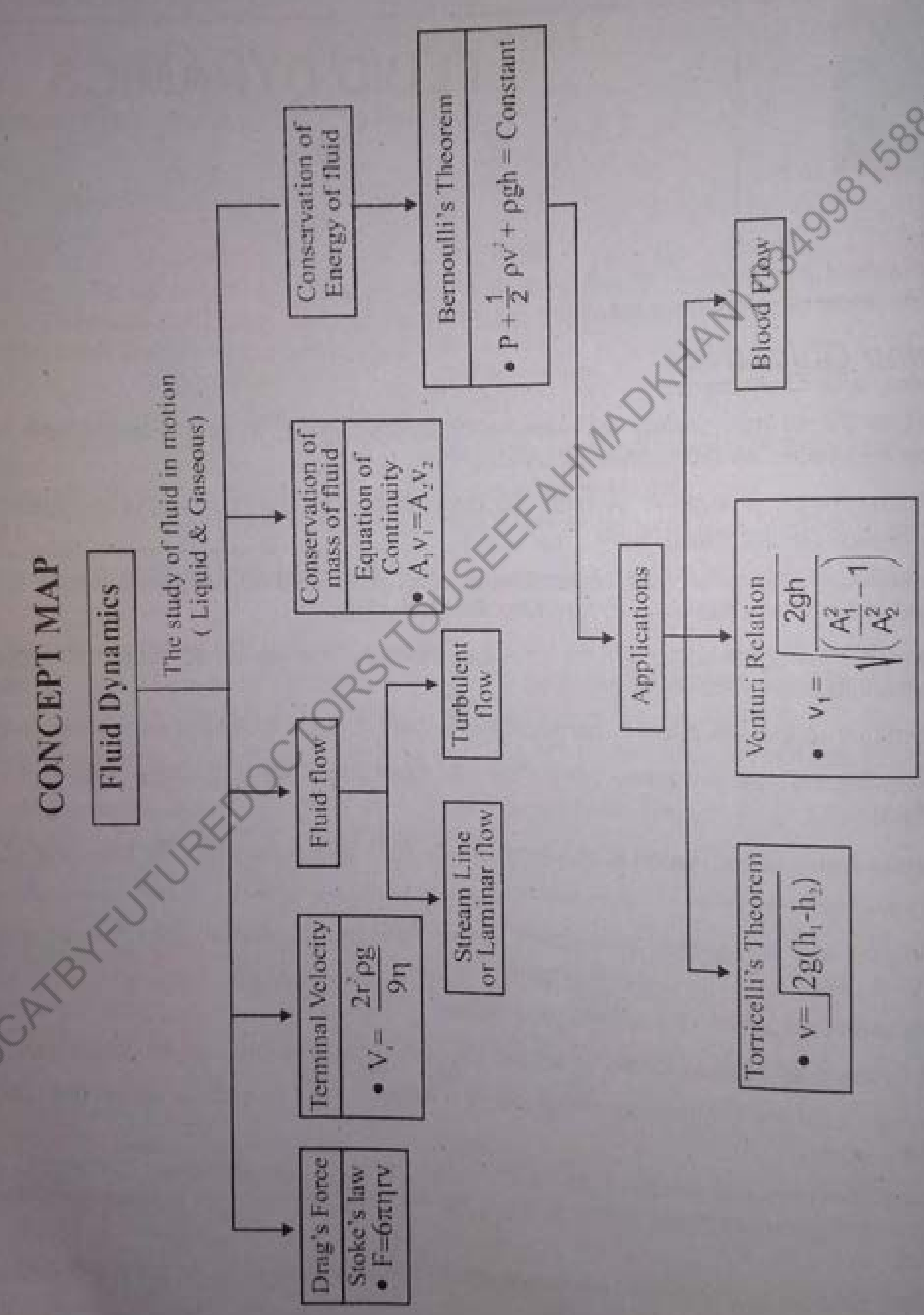
CHAPTER 6

FLUID DYNAMICS

Learning Objectives

- Define the terms: steady (streamline or laminar) flow, incompressible flow and non viscous flow as applied to the motion of an ideal fluid.
- Explain that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
- Describe that the majority of practical examples of fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions.
- Describe equation of continuity Av = Constant, for the flow of an ideal and incompressible fluid and solve problems using it.
- Identify that the equation of continuity is a form of the principle of conservation of mass.
- Describe that the pressure difference can arise from different rates of flow of a fluid (Bernoulli effect).
- Derive Bernoulli equation in the form $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant for the case of horizontal tube of flow.
- Interpret and apply Bernoulli Effect in the: filter pump, Venturi meter, in atomizers, flow of air over an aero foil and in blood physics.
- Describe that real fluids are viscous fluids.
- Describe that viscous forces in a fluid cause a retarding force on an object moving through it.
- Explain how the magnitude of the viscous force in fluid flow depends on the shape and velocity of the object.
- Apply dimensional analysis to confirm the form of the equation $F = A\eta rv$ where 'A' is a dimension-less constant (Stokes' Law) for the drag force under laminar conditions in a viscous fluid.
- Apply Stokes' law to derive an expression for terminal velocity of spherical body falling through a viscous fluid.

Chapter No. 6


CONCEPT MAP
Fluid Dynamics
The study of fluid in motion (Liquid & Gaseous)
Drag's Force
Stoke's law
• F=6πηrv
Terminal Velocity
• $V_t = \frac{2r^2\rho g}{9\eta}$
Fluid flow
Stream Line or Laminar flow
Turbulent flow
Conservation of mass of fluid
Equation of Continuity
• $A_1v_1=A_2v_2$
Conservation of Energy of fluid
Bernoulli's Theorem
• $P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant}$
Applications
Torricelli's Theorem
• $v = \sqrt{2g(h_1-h_2)}$
Venturi Relation
• $v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2}-1\right)}}$
Blood Flow

Fluid

The state of matter which can ***flow*** from one place to another is called a fluid.

Examples

Liquids and ***gases*** are classified as fluids.

Fluid dynamics

The branch of physics which deals with the study of ***fluids in motion*** is called fluid dynamics.

Viscous Fluids

The ***frictional effect*** between different layers of a flowing fluid is called viscosity.

- Viscosity of a fluid is the measure of its ***resistance to flow***.
- It measures that ***how much force*** is required to slide one layer of the liquid over another layer.
- Viscosity is the resistance to flow of a fluid. Honey has a high viscosity at room temperature, and freely flowing gasoline has a low viscosity.
- In general, the stronger the intermolecular forces of attraction, the more viscous is the liquid.
- Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy.

This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

- The numeric value of resistance to flow of fluid (viscosity) is called coefficient of viscosity 'η'.
- Substances like ***honey*** ,**Syrup** , **engine oil** and thick ***tar*** have large coefficient of viscosity so they can not flow easily.
- Substance like ***water and air*** has small coefficient of viscosity η. So they can flow easily.

Viscosity depends upon

- **Nature of fluid**
- **Cohesive force**
- **Temperature of fluid**

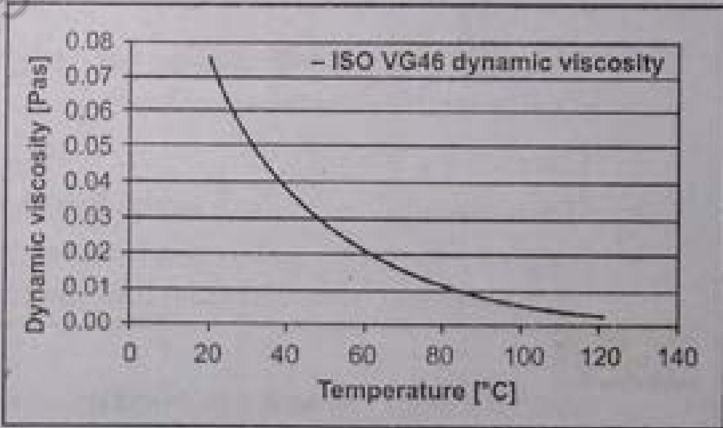
DO YOU KNOW?

Drag force acts opposite to the direction of motion of object, through fluid.

Viscosity of liquids and gases

Liquids and gases have ***non-zero*** viscosity.

- Viscosity of gases ***increases*** with increase in temperature (due to random motion of molecules)
- The viscosity of liquid deceases with rise in temperature (Cohesive force decreases with rise in temperature of liquid)



Unit: The SI unit of coefficient of viscosity is ***$kgm^{-1}s^{-1}$ or $Nm^{-2}s$*** or Pa s

Dimensions of coefficient of viscosity (η) = $[ML^{-1}T^{-1}]$.

- The most common unit of viscosity is the dyne second per square centimeter (dyne second/cm^2), which is given the name poise (P) after the French physiologist Jean Louis Poiseuille (1799-1869).

1 poise = 1dy(1s)/1cm^2

1 pascal second = 10 poise

1 centipoise = 1 millipascal second

Table: COEFFICIENT OF VISCOSITY OF VARIOUS SUBSTANCES

Material	Viscosity (Pa s)	Material	Viscosity (Pa s)
Air	1.8×10^{-5}	Ethanol	1.00×10^{-3}
Acetone	2.9×10^{-4}	Blood	1.6×10^{-3}
Methanol	5.1×10^{-4}	Honey	1.42
Benzene	1.00×10^{-3}	Blood (at body temperature)	4.0×10^{-3}
Water	8.91×10^{-4}		

Q.1 What is meant by the fluid friction and Stokes law? Explain.

Ans: **FLUID FRICTION AND STOKES LAW**

Drag Force

An object moving through a fluid experiences a retarding force called drag force.

OR

The force of friction acting on a body while moving through the fluid is called drag force or fluid friction.

For Your Information

Viscosities of Liquids and Gases at 30°C

Material	Viscosity 10[illegible] (Nsm[illegible])
Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.000
Plasma	1.5
Glycerin	6.29

For example

When we take out our hand out of the window of a fast-moving car, we feel a force opposite to our motion.

- The moving body exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force.

The drag force depends upon the

- Size, shape and orientation of the object
- Properties of the fluid (viscosity and density)
- Speed of the object relative to the fluid

The drag force F acting on a small sphere of radius r moving slowly with velocity v in a fluid of viscosity η is given by

$F_D \propto \eta r v$

or $F_D = A \eta r v$

Where A is the constant of proportionality and its experimentally determined value is 6π i.e A = 6π

Therefore $\boxed{F_D = 6\pi\eta r v}$ .... (1)

POINT TO PONDER

Skydivers and swimmers change their effective size and orientation by bending, twisting and starching their body parts. This allow them to manipulate drag and thereby allowing them to control speed and direction of motion.

- This equation first set forth by the British scientist Sir George G. Stokes in 1851 is termed as stokes law.
- Stokes's law finds application in several areas, particularly with regard to the settling of sediment in fresh water and in measurements of the viscosity of fluids.

MCQ's

1. Which one of the following has the maximum viscosity?
   (A) Air (B) Water (C) Acetone (D) Glycerin
2. Stokes law holds for:
   (A) Motion through free space (B) Motion through viscous medium (C) Bodies of all shapes (D) All mediums
3. As the speed of the object moving through a fluid increases, the upward drag force experienced by it;
   (A) Decreases (B) Increases (C) Remains same (D) Becomes zero
4. In the relation F = 6πηr v. Which one of the following are the dimensions of co-efficient of viscosity η?
   (A) $[ML^{-1}T^{-1}]$ (B) $[MLT^{-1}]$ (C) $[ML^{-2}T^{-1}]$ (D) [MLT]
5. Pascal is the unit of:
   (A) Pressure (B) Force (C) Tension (D) Weight
6. Which of the following is the base unit of coefficient of viscosity?
   (A) $Kg^{-1}\ m\ s^{-2}$ (B) $Kg\ m^{-1}s^{-1}$ (C) $Kg^{-1}\ m^{-2}s$ (D) $Kg\ m\ s^{-1}$
7. The frictional effect between different layers of a flowing fluid is called:
   (A) Fluidity (B) Density (C) Viscosity (D) Flow rate

Answers Key

1. D	2. B	3. B	4. A	5. A	6. B	7. C

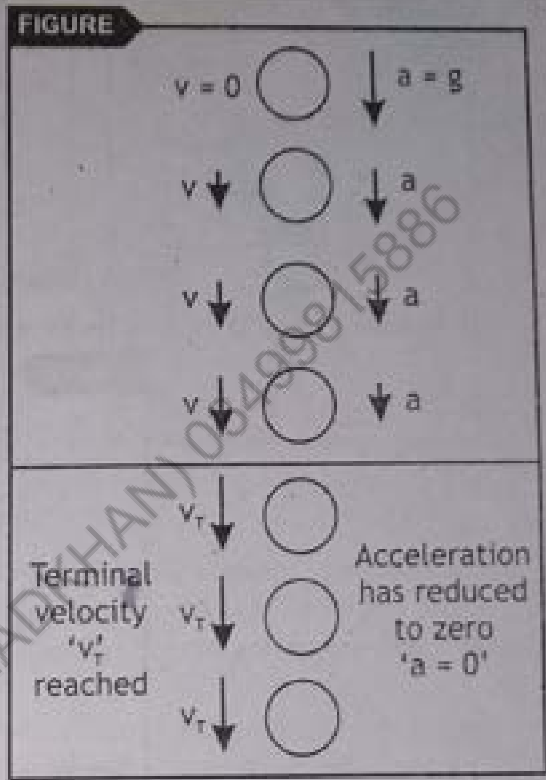
Q.2 What is terminal velocity? Show that terminal velocity of fog droplet is directly proportional to the square its radius?

Ans: **TERMINAL VELOCITY**

The maximum constant velocity of a freely falling body for which drag force becomes equal to its weight is called terminal velocity.

- The constant maximum velocity attained and maintained by an object while falling through a resistive medium is called terminal velocity 'v_t'.
- We will be concerned the terminal velocity for the simplest case, that is the uniform density spherical object falling though a consistent medium as shown in Figure.
- Newton's laws apply for all objects, whether freely falling or falling in the presence of resistive forces. The accelerations, however, are quite different, due to difference in net force.
- In a vacuum the net force is the weight because it is the only force.
- However, in the presence of air resistance, the net force is less than the weight, it is the weight minus drag force.

FIGURE



Net force = Weight – Drag force

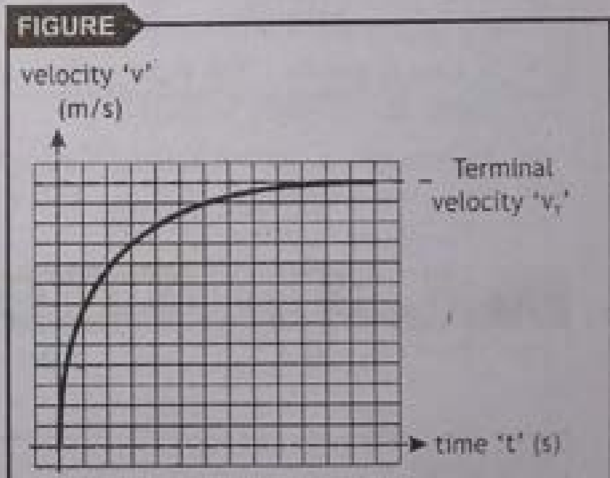
$F_{net} = mg - 6\pi\eta r\, v$

$ma = mg - 6\pi\eta r\, v$ ______ (1)

where $(F_G = W = mg)$

- When drag force becomes equal to the weight of droplet then it will start moving with uniform velocity, called terminal velocity (v_t).
- Net force is zero i-e $F_{net} = 0$
- So its acceleration becomes zero. (i.e. a = 0)

FIGURE



Graphical Representation of Terminal velocity

Putting a = 0 in equation (1)

$m(0) = mg - 6\pi\eta r v_t$

or $0 = mg - 6\pi\eta r v_T$

therefore $6\pi\eta r v_t = mg$

or $V_T = \dfrac{mg}{6\pi\eta r}$ ______ (2)

Equation (2) represents terminal velocity of a spherical object of mass 'm' and radius 'r', falling with acceleration due to gravity 'g' in a medium of co-efficient of viscosity 'η'.

Since density = $\dfrac{\text{mass}}{\text{volume}}$ (OR) $\rho = \dfrac{m}{V}$

OR

$m = \rho V$

$m = \left(\dfrac{4}{3}\pi r^3\right)\rho$ ______ (3) where $[\because V = \dfrac{4}{3}\pi r^3$ for sphere]

Putting value of m from equation (3) in equation (1), we get

$$V_t = \frac{\frac{4}{3}\pi r^3 \rho g}{6\pi\eta r}$$

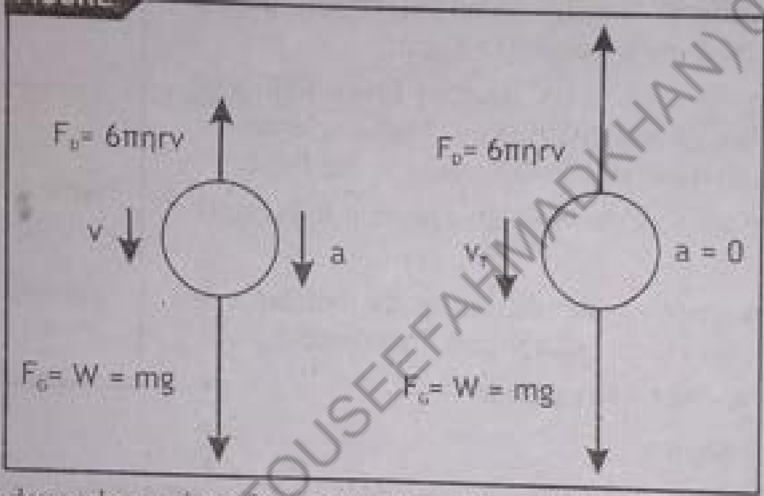
By rearranging we get

$$\boxed{v_t = \frac{2gr^2\rho}{9\eta}} \qquad \text{———} \quad (4)$$

$v_t \propto r^2$ (When density is constant)

Thus terminal velocity is directly proportional to the ***square*** of the radius of droplet for constant density.

FIGURE



- As terminal velocity depends on size, shape and orientation of the object and density of object.
- It also depends upon the coefficient of viscosity of the medium and speed, therefore there is no single speed for terminal velocity.
- In general, a person falling through the air on Earth reaches terminal velocity after about 12 seconds, covering a distance of about 450 meters. Table shows the terminal velocities of various objects falling through air.

Table: TERMINAL SPEEDS FOR VARIOUS OBJECT FALLING THROUGH AIR

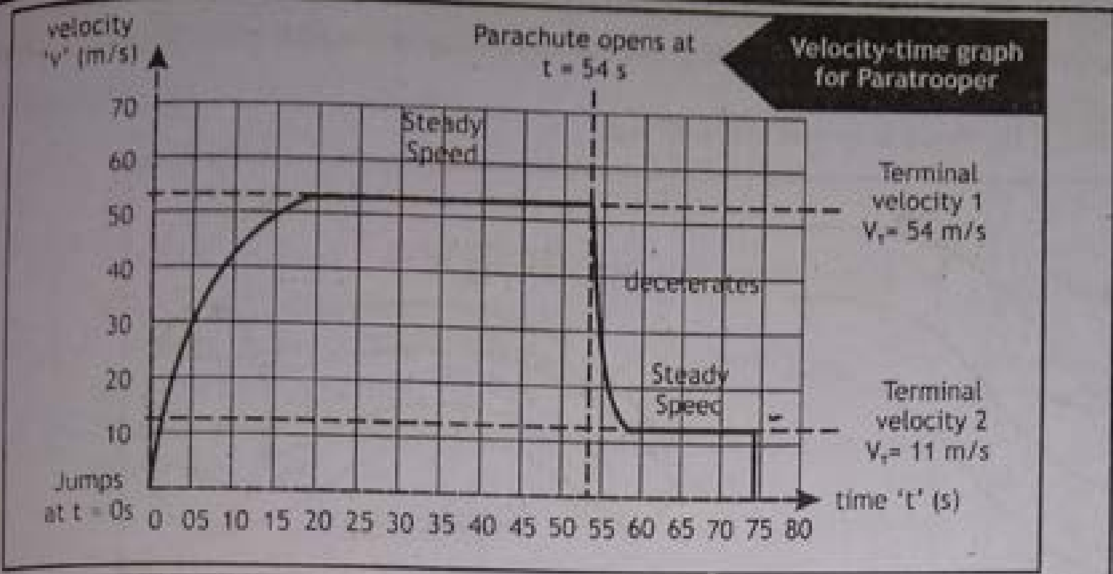
Object	mass (kg)	Cross- sectional Area (m^2)	Terminal Speed (m/s)
Sky Diver	70	0.70	54
Base Ball (radius 3.7 cm)	0.145	4.2 × 10-3	43
Golf Ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hail stone (radius 0.5 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Rain Drop (radius 0.2 cm	3.4×10^{-5}	1.3×10^{-5}	09

DO YOU KNOW?

In free fall the paratrooper attains his terminal velocity twice, once before opening his chute and the other after opening the chute. Without opening the chute paratrooper offers lower radius to air and therefor has a high terminal speed. Whereas after opening the chute he has large radius thereby having sufficiently low terminal speed to allow him to fall safely on the ground. For example, consider the graph below which explain the motion of paratrooper.

Do you know?

The largest ever hailstone weighed over 1kg and fell in Bangladesh in 1986.



Stage 1 at t = 0 s – after just jumping from the plane the skydiver is not moving very fast – weight is a bigger force than air resistance, so he accelerate downwards.

Stage 2 at t = 19 s – eventually the force of the air resistance has increased so much that it is the same size as the skydiver's weight – the forces are balanced and the speed remains constant (this is terminal velocity 1)

Stage 3 at t = 54 s – when the parachute opens air resistance increases dramatically: the air resistance force is much greater than the weight force, so the skydiver slows down.

Stage 4 at t = 58s – as the skydiver slows, the air resistance force from the parachute is reduced, until it is the same size as the weight force – the forces are balanced and the speed remains constant (this is terminal velocity 2)

Assignment 6.1:

A certain globular protein particle has a density of 1246 kg m^{-3}. It falls through water (having coefficient of viscosity 8.91×10^{-4} Pa s) with a terminal speed of 8.33×10^{-6} ms^{-1}. Find the radius of the particle.

Given Data: Density of protein particle = ρ = 1246kgm^{-3}

Co – efficient of viscosity = η = $8.91\text{x}10^{-4}$ Pa s

Terminal velocity = $8.33\text{x}10^{-6}$m/s

To Find: Radius of particle = r = ?

Calculations: As $$v_t = \frac{2gr^2\rho}{9\eta}$$

Or

$$r^2 = \frac{9\eta v_t}{2\rho g}$$ Putting values, we get

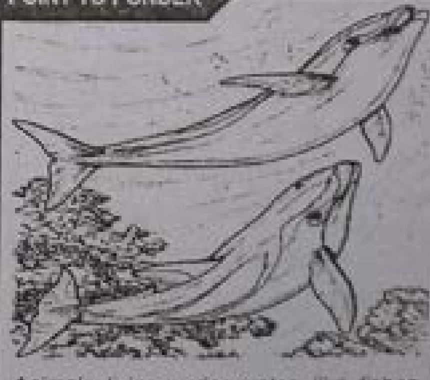
$$r^2 = \frac{9\times8.91\times10^{-4}\times8.33\times10^{-6}}{2\times1246\times9.8}$$

$$r^2 = 2.735\times10^{-12}$$

Taking square root

$$r = 1.65\times10^{-6}\text{ m}$$

POINT TO PONDER



Animals living under water, like fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined to reduce air drag and migratory species that fly large distances often have particular features such as long necks.

Q.3 What is the difference between steady and turbulent flow?

Ans:
Fluid Flow

Let us consider the flow of the fluid through the pipes. It may be either streamline or turbulent.

Laminar Flow

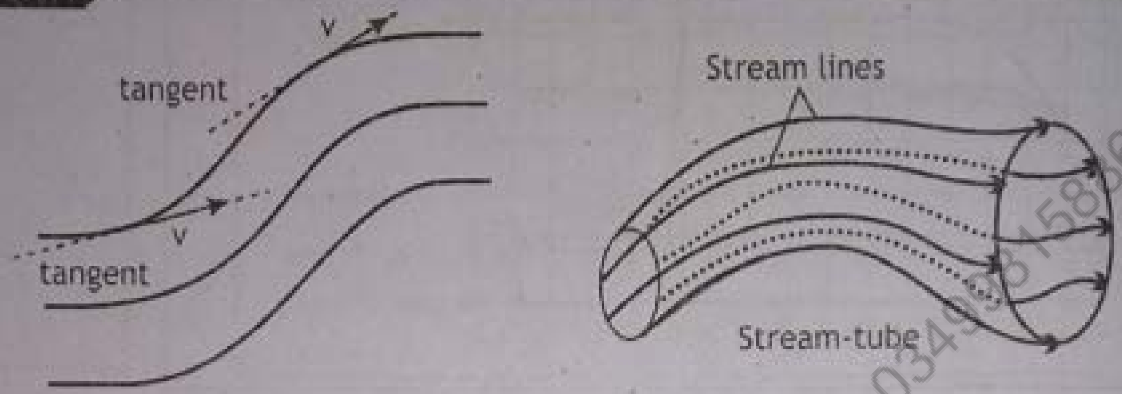
The flow is said to be streamline, steady, or laminar, if every particle of a fluid that passes through a particular point, moves along exactly the same path, as followed by particles that have passed that point earlier.

(OR)

The flow of a fluid in which every particle of the fluid moves along a smooth path is called laminar flow.

- **The smooth path followed by fluid is called streamline.**

FIGURE



A streamline is a line everywhere tangent to the velocity vector.

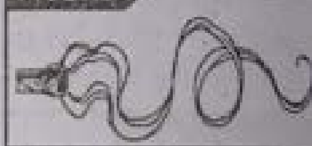
A stream-tube consists of a bundle of streamlines

Turbulent Flow

The irregular or unsteady flow of the fluid is called turbulent flow.

- When the speed of flowing fluid exceeds a certain ***critical*** value, the flow becomes extremely irregular and complex and it changes continuously with time.
- For example consider the figure 6.6, close to the incense, the smoke's flow is very smooth, or laminar.
- As the smoke rises higher and higher, it speeds up due to the lower density of warm air compared to the surrounding air (natural convection). Since it is speeding up as it rises, it will eventually reach a speed at which its flow becomes chaotic, or turbulent.
- In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. Turbulent flow tends to occur at higher velocities and low viscosity; therefore, most kinds of fluid flow are turbulent.
- The examples of turbulent flow include blood flow in arteries, oil transport in pipelines, lava flow, atmosphere and ocean currents, the flow through pumps and turbines, and the flow in boat wakes and around aircraft-wing tips.

FIGURE



Smoke rising from incense shows laminar flow near the bottom and turbulent flow farther up.

POINT TO PONDER

Extreme turbulent flow, can be seen in the form of a tornado. Tornadoes are violently rotating columns of air that extend from a thunderstorm to the ground. Tornadoes can destroy buildings, flip cars, and create deadly flying debris.

Difference between Laminar Flow and Turbulent Flow

- In laminar flow, each particle of fluid moves along a ***smooth path*** which does not change with time.
- In turbulent flow the ***flow pattern is not smooth*** but continuously changes with time.
- In laminar flow, stream lines ***do not*** cross each other while in turbulent flow it does not happen.
- In turbulent flow, the velocity of fluid changes ***abruptly.***
- For steady flow, different streamlines can ***never*** intersect each other. This condition is called steady flow condition.

Note (Steady flow condition)

...characteristics of an ideal fluid?

The discussion of fluid flow can be simplified by considering the fluid flow as ideal flow. In our model of an ideal flow, we make the following assumptions:

The fluid is non-viscous: In a non-viscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.

The flow is steady: In steady (laminar) flow, the velocity of the fluid at each point remains constant.

The fluid is incompressible: The density of an incompressible fluid is constant.

The flow is irrotational: In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

The temperature does not vary: Phenomena such as the convection of fluids in which a liquid in the bottom of a vessel is heated, rises, cools, and falls in a circulating pattern will not be considered.

For Your Information

Formula one racing car... a streamlined design.

Dolphins have streamlined bodies to assist their movement in water

State and explain equation of continuity.

EQUATION OF CONTINUITY

"For an ideal fluid flowing through a pipe, the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe remains constant".

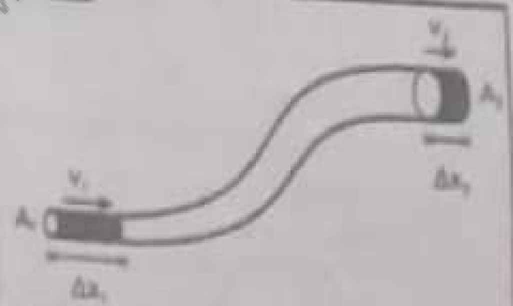
(OR)

"For an ideal fluid, the volume flow per second of the fluid (or simply flow rate) always remains constant" at every point in a pipe.

$$A_1v_1 = A_2v_2$$

$$A\,v = \text{constant}$$

(Where v is the velocity and A is the area of cross-section)



A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval t must equal the volume flowing through area A_2 in the same time interval.

- Consider an ideal fluid flowing through a pipe of non-uniform size, as shown in Figure.
- The particles in the fluid move along streamlines in steady flow.
- It obeys the law of conservation of mass.
- As there is no source or sink in the pipe so the mass of liquid that flows into the bottom of the pipe through A_1 in time Δt must be equal to mass of the liquid that flows out through A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2 = \Delta m \quad \text{..........(1)}$$

$$\rho = \frac{\Delta m}{\Delta V}$$

$$\Delta m = \rho\Delta V \quad \text{..........(2)}$$

$$\text{Volume} = \Delta V = A\Delta x \quad \text{..........(3)}$$

...ing $\Delta X = v\Delta t$ in equation (3)

$$\Delta V = Av\,\Delta t \quad \text{..........(4)}$$

...ing value from equation (4) in (2)

$$\Delta m = \rho\,Av\,\Delta t \quad \text{..........(5)}$$

- The fluid that moves through the lower end of the pipe in the time Δt has a mass given by equation (5) as

$$\Delta m_1 = \rho A_1 v_1 \Delta t \quad \text{..........(6)}$$

- The fluid that moves through the upper end of the pipe in the time Δt has a mass given by as

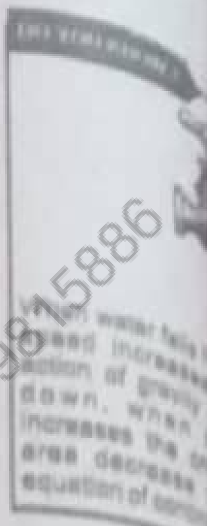
$$\Delta m_2 = \rho A_2 v_2 \Delta t \quad \ldots\ldots\ldots (7)$$

Putting values from equation (6) and equation (7) in equation (1) i-e $\Delta m_1 = \Delta m_2$, we get

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 \quad \ldots\ldots\ldots (8)$$

Therefore Av = Constant



Flow rate: The volume of liquid flowing per unit time through the pipe is called <u>flow rate.</u>

- The flow rate <u>remains constant</u> at every position in pipe.

Thus $$Av = \frac{\Delta V}{\Delta t} = \text{constant}$$

- The S – I unit of flow rate is m^3/s
- The dimensions of flow rate = $[L^3T^{-1}]$.

From equation (8) we have

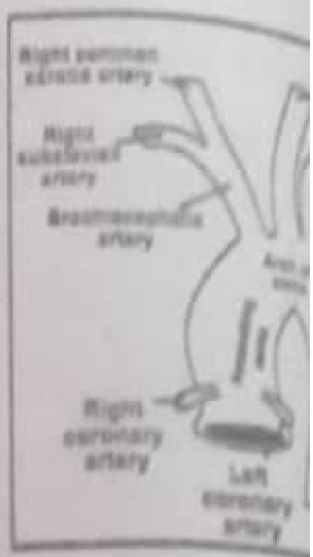
$$\frac{A_1}{A_2} = \frac{v_2}{v_1} \quad \ldots\ldots\ldots (9)$$

- Thus speed of fluid through any pipe is inversely proportional to the cross-sectional area of pipe

$$v \propto \frac{1}{A}$$

Assignment 6.2

The heart pumps blood into the aorta, which has an inner radius of 1.0 cm. The aorta feeds 32 major arteries (each have an inner radius of 0.21 cm). If blood in the aorta travels at a speed of 25 cm/s, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid.



Given Data: Radius of aorta = r_1 = 1 cm = 0.01m

Radius of each arteries = r_2 = 0.21 cm = 0.21×10^{-2} m

Number of arteries = n = 32

Speed of blood through aorta = v_1 = 25 cm/s = 0.25 m/s

Speed of blood through arteries = v_2 = ?

Solution: Area of aorta = $A_1 = \pi r_1^2 = 3.14\,(0.01)^2 = 3.14 \times 10^{-4}\ m^2$

Area of one artery $A = \pi r_2^2 = 3.14\,(0.21 \times 10^{-2})^2 = 1.3847 \times 10^{-5}\ m^2$

Area of 32 arteries = $A_2 = nA = 32 \times 1.3847 \times 10^{-5}\ m^2 = 4.43 \times 10^{-4}\ m^2$

$$A_1 v_1 = A_2 v_2$$

or $$v_2 = \frac{A_1 v_1}{A_2}$$

$$v_2 = \frac{3.14 \times 10^{-4} \times 0.25}{4.43 \times 10^{-4}}$$

$$v_2 = 0.177\ m/s$$

State and explain Bernoulli's Equation.

Bernoulli's Equation

Bernoulli's equation that relates the pressure, flow speed, and height for flow of an ideal

Statement

It states that the sum of pressure, K.E. per unit volume and P.E. per unit volume of an ideal fluid flowing in steady state remains constant at each point along a stream line path in a pipe.

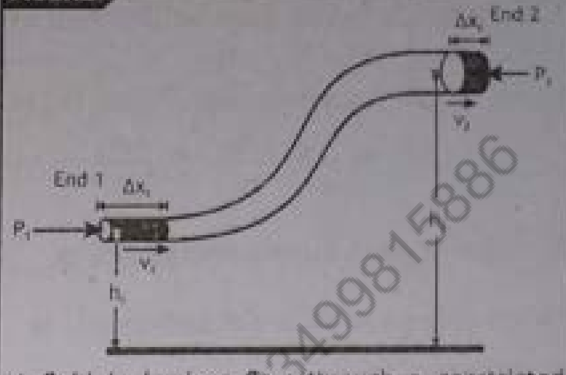
Mathematically

$$P_1+\frac{1}{2}\rho v_1^2+\rho g h_1=P_2+\frac{1}{2}\rho v_2^2+\rho g h_2$$

$$P+\frac{1}{2}\rho v^2+\rho g h=\text{constant}$$

- Bernoulli's equation is simply **law of conservation of energy** applied to fluids in motion.
- Consider an ideal flow through a pipe of non-uniform size, as illustrated in Figure.
- The work 'W' is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element.

FIGURE



A fluid in laminar flow through a constricted pipe. The volume of the shaded section on the left is equal to the volume of the shaded section on the right.

$$W=\Delta E$$

(or) $$W=\Delta K+\Delta U$$

For end (1) By definition of work

$$W_1=\vec{F}_1\cdot\Delta\vec{x}_1$$

$$W_1=F_1\Delta X_1\cos\theta$$

Here $\theta=0^0$ and $\cos 0^0=1$

Therefore $W_1=F_1\Delta x_1$ ------------- (1)

By definition of pressure $P=\frac{F}{A}$

$$F=PA$$

Putting $F_1=P_1A_1$ in equation (1)

$W_1=P_1A_1\Delta x_1$ ------------- (2)

For end (2) By definition of work

$$W_2=\vec{F}_2\cdot\Delta\vec{x}_2$$

$$W_2=F_2\Delta x_2\cos\theta$$

Here $\theta=180°$ and $\cos 180°=-1$

Therefore $W_2=-F_2\Delta x_2$

Putting $F_2=P_2A_2$ in above equation

$W_2=-P_2A_2\Delta x_2$ ------------- (3)

Since volume $\Delta V=A\Delta x$ ------------- (4)

By definition of density $\rho=\frac{\Delta m}{\Delta V}$

$\Delta V=\frac{\Delta m}{\rho}$ ------------- (5)

Comparing equation (4) and (5)

$\frac{\Delta m}{\rho}=A\Delta x$ ------------- (6)

Putting $A_1\Delta x_1=\frac{\Delta m_1}{\rho}$ in equation (2)

For end 1 $W_1=P_1\frac{\Delta m_1}{\rho}$ ... ------------- (7)

DO YOU KNOW ?

Daniel Bernoulli (1700–1782) Daniel Bernoulli, a Swiss physicist and mathematician, made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics. Bernoulli's most famous work, Hydrodynamica, was published in 1738; it is both a theoretical and a practical study of equilibrium, pressure, and speed in fluids. He showed that as the speed of a fluid increases, its pressure decreases. In Hydrodynamica Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases.

Putting $A_2 \Delta x_2$ in equation (3)

For end 2 $W_2 = -P_2 \dfrac{\Delta m_2}{\rho}$ ------------- (8)

- The total work done will be sum of all the individual work done.

$$W = W_1 + W_2 \quad \text{------------- (9)}$$

Putting values from equation (7) and (8) in equation (9)

$$W = P_1 \frac{\Delta m_1}{\rho} - P_2 \frac{\Delta m_2}{\rho} \quad \text{------------- (10)}$$

The net change in kinetic energy ΔK is $\Delta k = \frac{1}{2}\Delta m_2 v_2^2 - \frac{1}{2}\Delta m_1 v_1^2$ ------------- (11)

The net change in potential energy ΔU is $\Delta U = \Delta m_2 g h_2 - \Delta m_1 g h_1$ ------------- (12)

As we know that $W = \Delta K + \Delta U$ ------------- (13)

Putting values from equation (10), equation (11) and (12) in equation (13)

$$P_1 \frac{\Delta m_1}{\rho} - P_2 \frac{\Delta m_2}{\rho} = \frac{1}{2}\Delta m_2 v_2^2 - \frac{1}{2}\Delta m_1 v_1^2 + \Delta m_2 g h_2 - \Delta m_1 g h_1 \quad \text{------------- (13)}$$

Since for ideal fluid equal mass should flow across both ends, therefore

Putting $\Delta m_1 = \Delta m_2 = \Delta m$ in equation (13) we get

$$P_1 \frac{\Delta m}{\rho} - P_2 \frac{\Delta m}{\rho} = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 + \Delta m g h_2 - \Delta m g h_1$$

Taking Δm as common $\frac{\Delta m}{\rho}(P_1 - P_2) = \Delta m\left[\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + gh_2 - gh_1\right]$

Multiplying both sides by ρ, we get $P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$

therefore $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ -----------------(14)

or $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$ -----------------(15)

For understanding statement of Bernoulli's equation

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constant}$

Putting $\rho = \frac{m}{\Delta V}$

$P + \frac{1}{2}\left(\frac{m}{\Delta V}\right)v^2 + \left(\frac{m}{\Delta V}\right)gh = \text{constant}$

$P + \frac{\frac{1}{2}mv^2}{\Delta V} + \frac{mgh}{\Delta V} = \text{constant}$

$\text{Pressure} + \frac{K-E}{Volume} + \frac{P-E}{Volume} = \text{constant}$

Equations (14) and (15) are termed as Bernoulli's equations. Bernoulli's equations is based on conservation of energy such that for an incompressible and non-viscous fluid, the total mechanical energy of the fluid is constant.

Assignment 6.3:

Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1}, while at another point 3 m higher, the speed is 4ms^{-1}. At lower point the pressure is 80 kPa. Find the pressure at the upper point. **(47.1 kPa)**

Given Data:

Speed of water at one point = $v_1 = 3\ ms^{-1}$

Speed of water at second point = $v_2 = 4\ ms^{-1}$

Difference of height between two points $h_2 - h_1 = 3$ m

Pressure at lower point = $P_1 = 80$ kPa = 80,000 Pa

To Find:

Pressure at upper point = P_2 = ?

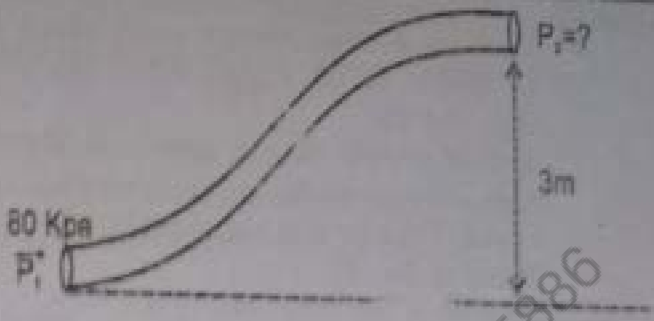
Calculation:

According to Bernoulli's theorem

$$P_1+\frac{1}{2}\rho v_1^2+\rho gh_1=P_2+\frac{1}{2}\rho v_2^2+\rho gh_2$$

Or $$P_2=P_1+\frac{1}{2}\rho v_1^2-\frac{1}{2}\rho v_2^2+\rho gh_1-\rho gh_2$$

$$P_2=P_1+\frac{1}{2}\rho(v_1^2-v_2^2)-\rho g(h_2-h_1)$$



Putting values, we get

$$P_2=80000+\frac{1}{2}\times1000(3^2-4^2)-1000\times9.8\times3$$

$$=80000+500(9-16)-29400$$

$$=80000+4500-8000-29400=84500-37400$$

$$=47100$$

$$=47.1\times10^3\,Pa$$

$$\boxed{P_2=47.1\ KPa}$$

Q.8 Explain application of Bernoulli's equation in filter pump, Atomizer and engine carburetor.

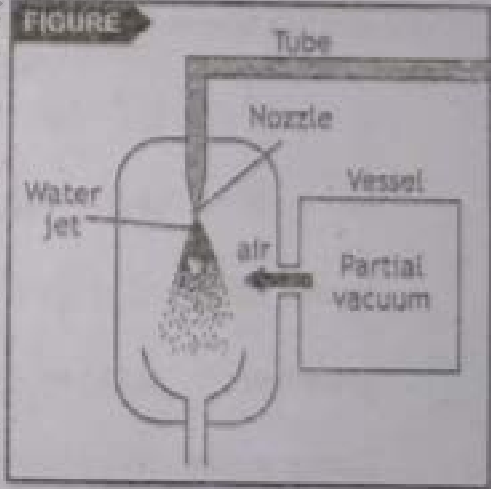
Ans: APPLICATIONS OF BERNOULLI'S EQUATION

A number of devices operate by means of the pressure difference that result from changes in a fluid's speed.

(A) Filter Pumps:

- A filter pump consists of a tube with jet attached to it, in which water flows from the tube toward the jet as shown in Figure 6.9.
- When water flows through the jet section its speed increases, as a result the pressure drops near it. This drop in pressure allows air to flow in from the side tube to which the vessel is connected.
- The air and water are forced together at the bottom of the filter pump.

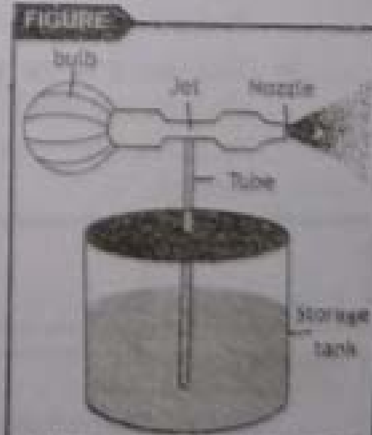
FIGURE



(B) Atomizers:

- Automizer is a device for emitting water, perfume, or other liquids as a fine spray.
- For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure 6.10.
- This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets.
- Such atomizers can be seen in perfume bottles, engine carburetor, water filter pumps and paint sprayers.

FIGURE



A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

Q.9 State and explain Torricelli's Theorem.

Ans: Application of Bernoulli's Equation

(C) Torricelli's theorem (Speed of efflux)

Statement

"Torricelli's theorem states that 'the speed of efflux is equal to the speed gained by fluid while falling through height h under the action of gravity".

Proof

- Consider a large storage tank, which develop a leak at the bottom as shown in the Figure 6.11. The pressure at both ends is same ($P_1 = P_2 = P$).

Area of cross-section of upper end of tank = A_1
Area of cross-section of lower small hole = A_2
Speed of the fluid at upper surface of tank = v_1
Speed of the efflux = v_2
Pressure at the upper end of tank = P_1
Pressure at the lower small hole = P_2
Height of fluid level from bottom = h_1
Height of hole from bottom = h_2
Height of water level above orifice = $h = h_1 - h_2$
Density of fluid = ρ
Velocity at the top is considered as zero ($v = 0$).
While the bottom velocity is to be determined ($v_2 = v$).

By Bernoulli's equation $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Substituting appropriate values and rearranging

$$P + \frac{1}{2}\rho (0)^2 + \rho g h_1 = P + \frac{1}{2}\rho v^2 + \rho g h_2$$

or $\quad P - P + \rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v^2$

hence $\rho g (h_1 - h_2) = \frac{1}{2}\rho v^2$ as $h_1 - h_2 = h$

therefore $\rho g (h) = \frac{1}{2}\rho v^2$ or $2gh = v^2$

Taking square root on both sides $\sqrt{v^2} = \sqrt{2gh}$

Therefore $v = \sqrt{2gh}$ .... (1)

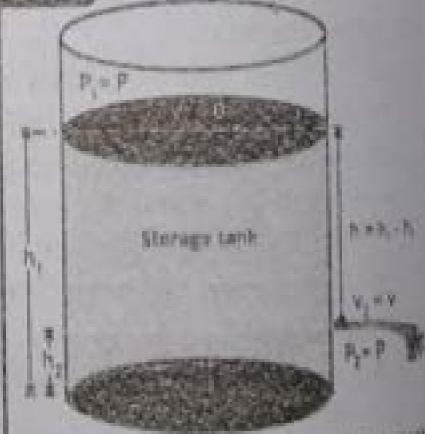
- The speed is the same as the vertical velocity which a body gain after falling freely through a height 'h'. The equation (1) is termed as Torricelli's equation for the speed of fluid emerging from water storage.

DO YOU KNOW ?

Engine Carburetor

An Engine Carburetor is a device that blends air and fuel for an internal combustion engine. Part of the carburetor is a tube with a constriction, as shown in the diagram. The pressure on the petrol in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be changed. Newer cars tend to have Electronic Fuel Injectors (EFI) rather than

FIGURE



A leak at the bottom of a large storage tank

EXTENSION EXERCISE

If the opening from spigot points upward, how high does the resulting 'fountain' go?

Assignment 6.4:

A tank full of water has a (small) hole near its bottom at a depth of 2.0 m from the top surface which is open to air. What is the speed of the stream of water emerging from the hole? (6.3 m/s)

Data: $h = 2m$

Speed of efflux = v = ?

$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 2} = \sqrt{39.2}$

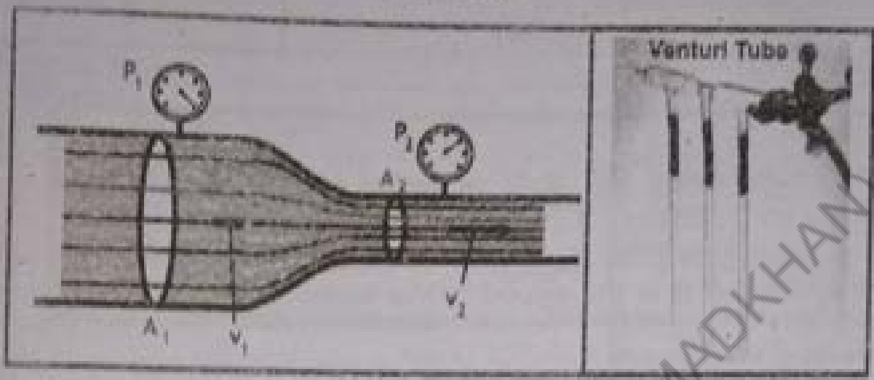
$V = 6.26$ m/s

Q.9 What is flow meter or venturi meter? Explain.

Ans: **Venturi Meter (Flow meter):**

Venturi meter is a device used to measure the flow speed or flow rate through a piping system.

- It works on the principle of pressure difference between restricted and unrestricted flow regions.
- Consider the steady flow of the incompressible fluid and it has negligible internal friction.
- Hence we can use Bernoulli's equation.
- Consider the Figure 6.12, let P_1 and P_2 be the pressure and v_1 and v_2 be the velocities of wide (end1) and narrow (end 2) sections of the tube respectively.



Since $h_1 = h_2 = h$

The Bernoulli's equation can be written as

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh$$

or $$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \quad \text{——1}$$

Now by equation of continuity

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1 v_1}{A_2} \quad \text{——2}$$

Putting value of v_2 from equation (2) in equation (1)

$$P_1 - P_2 = \frac{1}{2}\rho\left[\frac{A_1 v_1}{A_2}\right]^2 - \frac{1}{2}\rho v_1^2$$

or $$P_1 - P_2 = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]v_1^2 \quad \text{rearranging} \quad v_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(A_1/A_2\right)^2 - 1\right]}$$

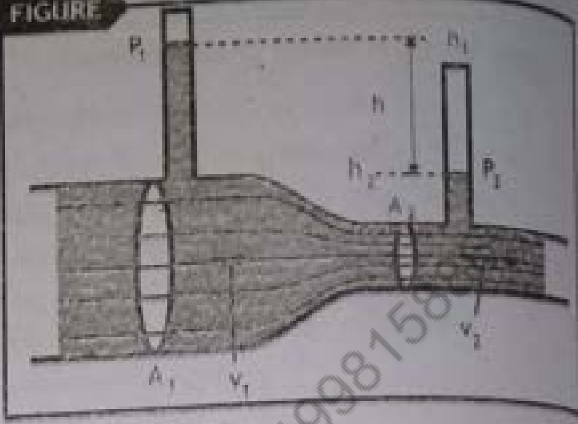
Taking square root of both sides $$\sqrt{v_1^2} = \sqrt{\frac{2(P_1 - P_2)}{\rho\left[A_1^2/A_2^2 - 1\right]}}$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left[\frac{A_1^2 - A_2^2}{A_2^2}\right]}} \quad \text{(or)} \quad v_1 = \sqrt{A_2^2} \times \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

Therefore $$v_1 = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}} \quad \text{------(3)}$$

- This is expression for speed of liquid in a pipe by Venturi's meter when any barometer (device used to measure pressure) is used.
- However, when no barometer is used and height of the fluid in smaller then

FIGURE



Putting $(P_1 - P_2) = \rho g h$ in equation (3) we get

$$v_1 = A_2\sqrt{\frac{2(\rho gh)}{\rho(A_1^2 - A_2^2)}}$$

Therefore $$v_1 = A_2\sqrt{\frac{2(gh)}{(A_1^2 - A_2^2)}} \quad \text{..............(4)}$$

Assignment 6.5:

A venturi meter is measuring the flow of water in a pipe having cross-sectional area of 0.0038m², a throat with cross-sectional area of 0.00031 m_ is connected to it. If the pressure difference is measured to be 2.4 kPa, what is the speed of the water in the pipe?

CLASSROOM DEMONSTRATION

Cut a long piece of paper, blowing over top of paper, make the paper rise. As we blow the speed of the air at the top is greater than the speed at the bottom. In the high speed region (that is at top) the pressure is reduced to fill

Data:

Area of cross-section of pipe = $A_1 = 0.0038\ m^2$

Area of cross-section of throat = $A_2 = 0.00031\ m^2$

Pressure difference = $\Delta P = P_1 - P_2 = 2.4$ K Pa $= 2.4 \times 10^3$ Pa

Density of water = $\rho = 1000\ kg/m^3$

Speed of water in pipe = v_1 = ?

Solution:

$$v_1 = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$v_1 = 0.00031\sqrt{\frac{2(2.4\times10^3)}{1000[(0.0038)^2 - (0.00031)^2]}}$$

$$v_1 = 0.00031\sqrt{\frac{2(2.4\times10^3)}{1000[(0.0038)^2 - (0.00031)^2]}}$$

$$v_1 = 0.00031\sqrt{\frac{4.8\times10^3}{0.0143439}}$$

$$v_1 = 0.00031 \times 578.5 = 0.2\ m/s$$

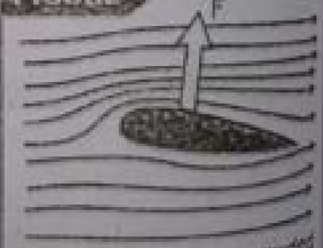
Q.10 Explain aero-foil and lift on an air-plane wing ?

Ans: **(D) Aero-foil**

The devices which are shaped so that the relative motion between it and the fluid produces a force perpendicular to the flow are called aero-foils.

- The shape of aero-foil is made such that the fluid speed at the top surface is greater than the bottom (closer stream lines). as shown in figure.
- An airfoil-shaped body moved through a fluid produces an aerodynamic force.
- The component of this force perpendicular to the direction of motion is called lift.
- The component parallel to the direction of motion is called drag.

FIGURE



Stream lines are crowded together above the aero-foil so flow speed is higher and pressure is low. Because of this decreased pressure a lift is exerted.

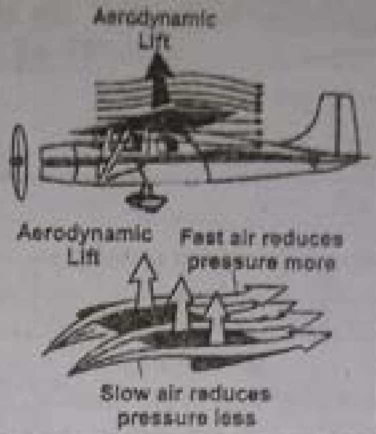
USES:

Aero-foils are found in aero-plane wings, helicopters, sailboats, propellers, fans, compressors and turbines.

TID BITS

Lift on an Airplane Wing

One of the most spectacular examples of how fluid flow affects pressure is the dynamic lift on airplane wings. Figure shows an airplane with its wing moving to the left, relative to it the air flow to the right. Due to the wing's shape, the flow lines crowd together above the wing, it causes the air to travel faster over the curved top surface and more slowly over the flatter bottom. Thus, the pressure above the wing is reduced relative to the pressure under the wing as a result the wing is lifted upward.



DO YOU KNOW ?

Spoilers: All cars are designed to avoid lift and stick to the ground at all times. Because when cars suddenly rise up due to high velocity, the driver loses its control. So scientists designed cars completely opposite to that of an airplane.

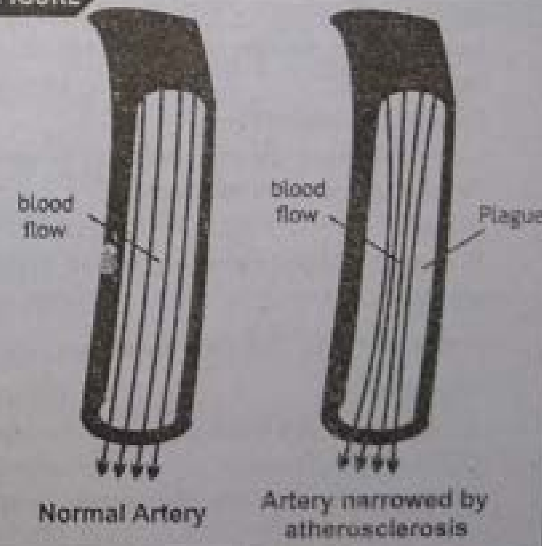
A race car employs Bernoulli's principle to keep its wheels on the ground while traveling at high speeds. A race car's spoiler—shaped like an upside-down wing, with the curved surface at the bottom—produces negative lift (or downforce) to push them down against the track surface so they can take turns quickly without sliding out into the track wall.

Q.11 Explain the blood flow?

Ans: **Blood Flow:**

- Bernoulli's equation ignores viscosity (fluid friction).
- A non-viscous fluid could flow through a level tube or pipe without a force being applied.
- Viscosity acts like a sort of friction (between fluid layers moving at slightly different speeds).
- The volume flow rate $\Delta V/\Delta t$ for laminar flow of a viscous fluid through a horizontal, cylindrical pipe depends on its radius.
- Similar is the case of human arteries i-e 'blood flow in the human body' depends upon the radius of its arteries as shown in Figure.
- If the radius of arteries is reduced as a result of arteriosclerosis (thickening and hardening of artery walls) and by cholesterol buildup, the pressure must be increased to maintain the same flow rate.
- If the radius is reduced by half, the heart would have to increase the pressure by a factor of about $2^4=16$ in order to maintain the same blood –flow rate.
- The heart must work much harder under these conditions, but usually cannot maintain the original flow rate.
- The pressure is lower where the fluid (blood) is flowing faster.
- The pressure difference can dislodge the plaque.
- The plaque can then lodge in and block a smaller artery which can cause heart attack.

FIGURE



Systolic Pressure

- It represents the ***maximum*** pressure exerted when the heart contracts. The value of high blood pressure (systolic pressure) is ***120 torr***

Diastolic Pressure

- It represents the minimum pressure in the arteries when the heart is relaxed. The value of low blood pressure (diastolic pressure) is about ***75 – 80 torr***.

Relation between torr and pascal

- 1 torr = 133.3Pa = 133.3 N/m^2

MCQ's

1. **Pressure will be low where the speed of fluid is:**
(A) Zero (B) High (C) Low (D) Medium
2. **One torr is equal to:**
(A) 13.33Nm^{-2} (B) 1.333Nm^{-2} (C) 133.3 Nm^{-2} (D) 1333 Nm^{-2}
3. **The law of conservation of energy is the basis of:**
(A) Streamline flow (B) Equation of continuity (C) Bernoullis equation (D) Venturi relation
4. **Blood pressure is measured by:**
(A) Hydrometer (B) Barometer (C) Sphygmomanometer meter (D) Galvanometer
5. **A 6.0 m high tank is full of water, a hole appears at its middle. What is the speed of efflux:**
(A) 7.66 m/s (B) 5.66 m/s (C) 6.66 m/s (D) 8.66 m/s
6. **The law of conservation of mass is related to:**
(A) Bernoulli's equation (B) Equation of continuity (C) Stokes law (D) Viscosity
7. **If the Stream lines of fluid are forced closer together then:**
(A) Speed of the Fluid increases (B) speed of the fluid decrease
(C) Pressure of the fluid increases (D) Speed of the fluid remains same
8. **Venturi meter is a device used to measure:**
(A) Density of fluid (B) Speed of fluid (C) Pressure of fluid (D) Viscosity of fluid
9. **Which of the following is S.I unit of pressure?**
(A) pascal (B) $N.m^{-1}$ (C) torr (D) Bar
10. **The working of carburetor of car uses:**
(A) Equation of continuity (B) Gravitational law (C) Bernoulli's theorem (D) Stokes law
11. **A horizontal pipe narrows from a diameter of 10cm to 5cm. For a fluid flowing from larger diameter to smaller diameter:**
(A) The velocity and pressure both increases (B) The velocity increases and pressure decreases
(C) The velocity decreases and pressure increase (D) Velocity and pressure both decreases
12. **torr is the unit of:**
(A) Power (B) Energy (C) Blood pressure (D) Work
13. **Normally the human blood density is nearly equal to:**
(A) 111 kg m^{-3} (B) 133 kg m^{-3} (C) (80-120) kgm^{-3} (D) Water density
14. **Swing is produced to:**
(A) Increase the speed of ball (B) Decrease speed of ball (C) Deceive the player (D) Apply the force on ball
15. **The S.I unit of flow rate are:**
(A) m^2s^{-1} (B) m^3s^{-2} (C) m^3s^{-1} (D) m^2s^{-2}
16. **Pressure will be low where speed of fluid is:**
(A) zero (B) high (C) low (D) medium
17. **One torr in Nm^{-2} is expressed as:**
(A) 130.6Nm^{-2} (B) 133.3Nm^{-2} (C) 140.2 Nm^{-2} (D) 135.3 Nm^{-2}
18. **When water falls from tap, its cross-sectional area decreases due to:**
(A) Decrease of speed (B) Increase of speed (C) Air pressure (D) Gravity increase
19. **The systolic pressure of a normal healthy person is:**
(A) 120 torr (B) 125 torr (C) 115 torr (D) 130 torr
20. **The ratio of the velocities of water in a pipe lying horizontally at two ends is 1 : 4. The ratio of diameters of pipe at these two ends is:**
(A) 1 : 2 (B) 2 : 1 (C) 1 : 4 (D) 4 : 1
21. **High concentration of red blood cells increases viscosity of blood from:**
(A) 2 – 3 time that of water (B) 3 – 4 times that of water (C) 3 – 5 time that of water (D) 4 – 5 times that of water
22. **Bunsen burner works on the principle of:**
(A) Venturi effect (B) Terricilli's effect (C) Bernoulli's effect (D) None of these

23. The instrument which detects the instant at which the external pressure becomes equal to the systolic pressure is called:
(A) Monometer (B) Sphygmomanometer (C) Barometer (D) Stethoscope

24. Venture relation is given as:
(A) $p = \frac{1}{2}\rho v^2$ (B) $P_1 - P_2 = \frac{1}{2}\rho v^2$ (C) $v = 2g\sqrt{(h_1 - h_2)}$ (D) $p_1 - p_2 = \sqrt{\rho gh}$

25. On the average for normal healthy person diastolic pressure is:
(A) 120 torr (B) 110 torr (C) 100 torr (D) 75-80 torr

Answers Key

1. B	2. C	3. C	4. C	5. A	6. B	7. A	8. B	9. A	10. C	11. B	12. C
13. D	14. C	15. C	16. B	17. B	18. B	19. A	20. B	21. C	22. C	23. D	24. B
25. D											

FORMULAE

1	Drag force	$F = 6\pi r\eta v$	
2	Terminal velocity of fog droplet	$v_t = \frac{mg}{6\pi r\eta}$	$v_t = \frac{2g\rho}{9\eta}r^2$
3	Equation of continuity	$\frac{V}{t}$ = constant Av = constant	$A_1v_1 = A_2v_2$
4	Bernoulli's equation	$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$	$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$
5	Torricelli's Theorem (speed of efflux)	$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2gh}$	
6	Venturi's relation	$v_1 = \sqrt{\frac{2gh}{(\frac{A_1^2}{A_2^2} - 1)}}$	

Key Points

- **Fluid Flow:** The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.
- **Viscosity:** the resistance to flow of a fluid.
- **Drag force:** retarding force experienced by an object moving through a fluid.
- **Terminal velocity:** the maximum velocity attained and maintained by an object while falling through a fluid.
- **Streamline, steady, or laminar flow:** every particle of a fluid moving along exactly the same path, as followed by particles that have passed that point earlier.
- **Turbulent flow:** irregular flow characterized by small whirlpool-like regions.
- **Equation of Continuity:** The volume of an incompressible fluid passing any point in a pipe of non uniform cross-section is the same in the steady flow.
  v A = constant (v is the velocity and A is the area of cross-section)
  The equation is due to mass conservation for ideal flow.
- **Bernoulli's Equation:** Bernoulli's principle states that as we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume 2 (ρv /2) and the potential energy per unit volume (ρgh) remains a constant.
  P + ρv2/2 + ρgh = constant (ρ is the density and g is acceleration due to garvity)
  The equation is due to energy conservation for ideal flow.

Solved Examples

Example 6.1: **The radius of small fog droplet in air is found to be 5.1×10^{-6} m. the coefficient of viscosity of air is $1.9\times10^{-5}\ kgm^{-1}s^{-1}$. Find out the settling speed of the droplet in air.**

Given: Radius 'r' = 5.1×10^{-6} m

Coefficient of viscosity 'η' = $1.9 \times 10^{-5}\ kg\ m^{-1}s^{-1}$

Density 'ρ' 1000 kg/m^3

Acceleration due to gravity 'g' = 9.8 ms^{-2}

Terminal velocity 'v_t'=?

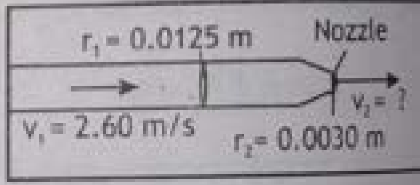
Solution: The terminal velocity is $v_T = \frac{2}{9}\times\frac{\rho r^2 g}{\eta}$

Putting values $$v_T = \frac{2}{9}\times\frac{(1000kgm^{-3})\times(5.1\times10^{-6}m)^2\times(9.8ms^{-2})}{1.9\times10^{-5}kgm^{-1}s^{-1}}$$

Hence $\boxed{v_T = 2.98\times10^{-3}ms^{-1}}$ **Answer**

The Fog droplet will settle with speed of 0.00298 m/s in air.

Example 6.2: **A garden hose of inner radius 1.25 cm carries water at 2.60 m/s. The nozzle at the end has radius 0.30 cm. How fast does the water emerge out through the nozzle?**



Given: Radius of garden hose 'r_1' = 1.25 cm = 0.0125 m

Radius of the nozzle 'r_2' = 0.30 cm = 0.0030 m

Speed though garden hose 'v_1' = 2.60 m/s

Requited: Speed out of nozzle 'v2'=?

Solution: The Equation of continuity is $A_1v_1 = A_2 v_2$

The area of circle is $A = \pi r^2$

therefore, Equation of continuity can also be written as $\pi r_1^2 v_1 = \pi r_2^2 v_2$

or $r_1^2 v_1 = r_2^2 v_2$ and $v_2 = \frac{r_1^2 v_1}{r_2^2}$

putting values $$v_2 = \frac{(0.0125m)^2\times2.60ms^{-1}}{(0.0030m)^2}$$

hence $\boxed{v_2 = 45.14ms^{-1}}$ **Answer**

The speed of the water from the nozzle is 45.14 m/s.

Example 6.3: **Water is flowing smoothly through a pipe. At one point the pressure is 33.2kPa and the speed of water is 2 m/s. While at another point 2.3 m higher the pressure is 3.7kPa, at what speed is the water flowing through this point?**

Given: Pressure 'P_1'=33.2 kPa =$33.2\times10^3\ Nm^{-2}$

Pressure 'P_2'=3.7 kPa =$3.7\times10^3\ Nm^{-2}$

Speed of water 'v_1' =2 ms^{-1}

Height 'h_1' = 0 m , Height 'h_2'= 2.3 m

Density of water 'ρ' =1000 kgm^{-3}

Required:

Speed of water 'v_2'=?

Solution:

The Bernoulli's equation is

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Or

$$\frac{1}{2}\rho v_2^2 = P_1 - P_2 + \frac{1}{2}\rho v_1^2 + \rho g h_1 - \rho g h_2$$

Multiplying both sides by $2/\rho$ we get $v_2^2 = \frac{2}{\rho}(P_1 - P_2) + v_1^2 + 2g(h_1 - h_2)$

Taking square root on both sides $v_2 = \sqrt{\frac{2}{\rho}(P_1 + P_2) + v_1^2 + 2g(h_1 - h_2)}$

or

$$v_2 = \sqrt{\frac{2}{1000kgm^{-3}}(33,200kgmm^{-1}s^{-2} - 3700kgm^{-1}s^{-2}) + (2ms^{-1})^2 + 2\times 9.8ms^{-2}(0m\ 2.3m)}$$

hence $v_2 = 4ms^{-1}$ **Answer**

The water will flow at 4 m/s in the upper part of the pipe.

Example 6.4: **A cylindrical water storage tank has a horizontal spigot near the bottom, at a depth of 1.2 m beneath the water surface. (a) When the spigot opened, how fast does the water come out? (b) If the radius of spigot is 6.0×10^{-3} m, what will be the volume flow rate?**

Given:
Height of water in tank 'h' =1.2 m
Radius of spigot 'A'=6.0×10^{-3} m
Acceleration due to gravity 'g'= 9.0 ms^{-2}

Required:
Speed of water 'v'=?
Volume flow rate '$\Delta V / \Delta t$' =?

Sulution: (a) By Torricelli's theorem $v = \sqrt{2gh}$

putting value $v = \sqrt{2\times 9.8ms^{-2}\times 1.2m}$

hence $v = 4.85ms^{-1}$ —— Answer

(b) From equation of continuity the volume flow rate is $\frac{\Delta V}{\Delta t} = Av$

Since $A = \pi r^2$ therefore $\frac{\Delta}{\Delta t} = \pi r_2 \times v$

Putting value $\frac{\Delta V}{\Delta t} = 3.14\times(6.0\times 10^{-3}m)^2 \times 4.85ms^{-1}$

Hence $\frac{\Delta V}{\Delta t} = 5.48\times 10^{-4}m^3s^{-1}$ —— Answer

The water of volume 0.00548 m^3 will emerge out of spigot each second.

Example 6.5: **What is the aero-foils lift (in newtons) on a wing of area 88m^2 if the air passes at speed over its top surface at 280m/s and bottom surface at 150m/s?**

Given:
Surface area 'A' = 88 m^2
Speed at top of wing 'v_2' = 280 m/s
Speed at bottom of wing 'v_1' = 150 m/s
density of the air 'ρ' = 1.28 kg/m^3

Required: Force 'F' = ?

Solution: Pressure is defined as $(P_1 - P_2) = \frac{F_{lift}}{A}$

or $F_{lift} = (P_1 - P_2) \times A$ ....(1)

By Bernoulli's equation $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g h_2 - \rho g h_1$

Since $h_1 \approx h_2$ therefore $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$ ....(2)

Putting value from equation (2) in equation (1), we get

$$F_{lift} = \frac{1}{2}\rho(v_2^2 - v_1^2) \times A$$

putting values

$$F_{lift} = \frac{1}{2} \times 1.28\,kgm^{-3}\left\{(280\,ms^{-1})^2 - (150\,ms^{-1})^2\right\} \times 88\,m^2$$

$$F_{lift} = 3.2 \times 10^6\,N \quad \textbf{Answer}$$

There will be an upward force of 3.2×10^6 N on air craft wing.

Text Book Exercises

Q.1 Select the correct answer of the following questions.

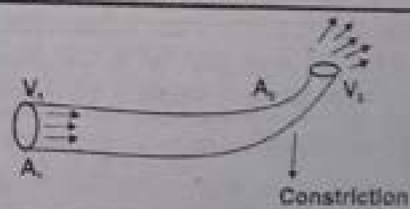
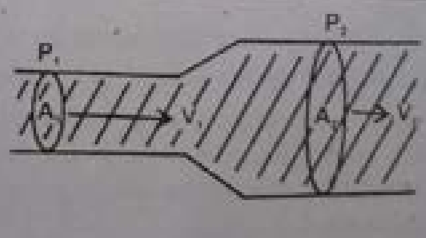
1. **For substances that do not flow easily (like honey) have the ______ value for the coefficient of viscosity**
   A. low B. high C. zero D. negative
2. **A unit for viscosity, the centipoise, is equal to which of the following?**
   A. 10^{-3} N s/m^2 B. 10^{-2} N s/m^2 C. 10^{2} N s/m^2 D. 10^{3} N s/m^2
3. **Stokes law is applicable only if a body is moving through a liquid with slow speed and has ______ shape.**
   A. a cubical B. a spherical C. a rough D. any
4. **The net force that acts on a 10-N falling object, when it encounters 4 N of air resistance is**
   A. 0 N B. 4 N C. 6 N D. 10 N
5. **A skydiver jumps from a high-flying helicopter. Before reaching terminal velocity, her acceleration:**
   A. increase B. decrease C. remain the same D. is zero
6. **At terminal velocity the acceleration of a falling object is:**
   A. 0 m s B. 1 m s C. - 9.8 m s D. + 9.8 m s
7. **According to equation of continuity Av = constant. This constant is equal to**
   A. volume of fluid B. mass of fluid
   C. density of fluid D. volume flow rate
8. **At the constriction in the cross-section for ideal flow, from equation of continuity it follows that, the speed of fluid is**
   A. greater B. less C. same D. zero
9. **As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, the pressure against the wall:**
   A. increases B. decreases C. remains the same D. is zero
10. **A 4 m high tank filled with water is drilled with four identical small holes at 1 m, 1.5 m, 2 m and 2.5 m from the bottom of tank, the speed of efflux will be greatest from the hole at:**
    A. 1 m B. 1.5 m C. 2 m D. 2.5 m
11. **Venturi meter is a device used to measure the:**
    A. mass of fluid B. viscosity of fluid
    C. speed of fluid D. density of fluid

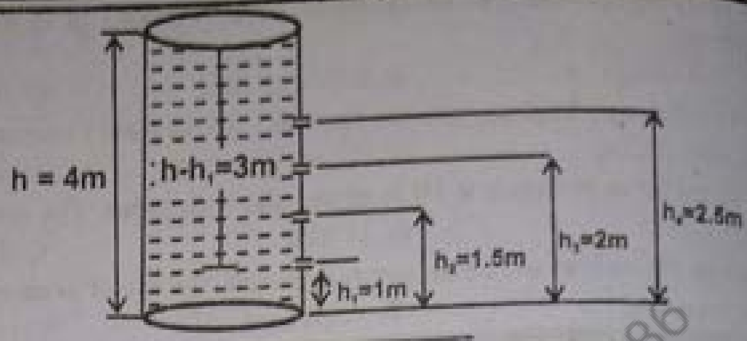
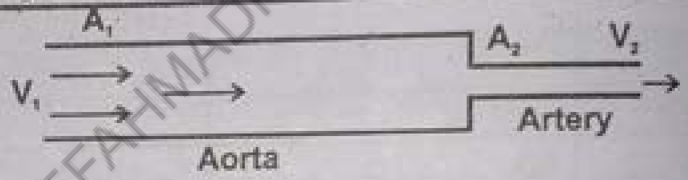
12. **A certain pipe has a cross-sectional area of 0.0001 m in which water is flowing at 10 m/s. The volume flow rate is**
A. 0.00001 m^3/s B. 0.001 m^3/s C. 1 m^3/s D. 10.0001 m^3/s

13. **At sufficiently high speeds the flow of viscous fluid becomes:**
A. unexpected B. stream line C. non-viscous D. turbulent

14. **The water in the tank is 10 m above the leak point. The speed with which the water emerge from the leak is**
A. 10 m/s B. 14 m/s C. 194 m/s D. 0.1 m/s

15. **When the radius of the artery is reduced, the blood pressure**
A. increased B. decreased C. remains the same D. is zero

No.	Option	ANSWER	EXPLANATION
1.	B	High	
2.	A	10^{-3} N s/m^2	$1\text{ poise} = \frac{1dy(1s)}{1cm^2}$ But 1N = 10^5 dy putting 1dy = 10^{-5} N $1\text{poise} = \frac{10^{-5}N(1s)}{(1\times10^{-2}m)^2} = \frac{10^{-5}N(1s)}{1\times10^{-4}m^2}$ 1poise = 10^{-1} Ns/m^2 $1\text{ centipoise} = \frac{10^{-1}Ns/m^2}{100}$ 1 centipoise = 10^{-3} Ns/m^2
3.	B	spherical	
4.	C	6 N	$F_{net} = w - F_D$ $F_{net} = 10 - 4 = 6N$
5.	B	Decreases	$F_{net} = w - F_D$ $ma = mg - F_D$ $a = g - \frac{F_D}{m}$ when drag force increases then net force decreases and acceleration decreases
6.	A	Zero	$F_{net} = w - F_D$ $F_{net} = 0$ and $a = 0$
7.	D	volume flow rate	Av = constant m^2 (m/s) = m^3/s it is unit of volume flow per unit time
8.	A	greater	$A_1v_1 = A_2v_2$ $A_2 < A_1$ $v_2 > v_1$ 
9.	A	increases	$A_1v_1 = A_2v_2$ $v_2 = \frac{A_1v_1}{A_2}$ $A_2 > A_1$ $v_2 < v_1$  When area of cross-section of pipe increases then speed of liquid decreases. Where speed of liquid is low the pressure will be high

10.	A	1 m	 $V=\sqrt{2g(h-h_1)}=\sqrt{2\times 9.8(4-1)}$ =7.7 m/s The height of water level is maximum from the hole which is 1m above from the base therefore speed of liquid is maximum through it.
11.	C	speed of fluid	
12.	B	0.001 m^3/s	Volume flow rate= Av= 0.0001x10= 0.001 m^3/s
13.	D	Turbulent	
14.	B	14 m/s	$V=\sqrt{2gh}=\sqrt{2\times 9.8\times 10}=\sqrt{196}$ = 14 m/s
15.	B	Decreases	 $A_1v_1=A_2v_2$ $\pi r_1^2v_1=\pi r_2^2v_2$ $v_2=\frac{r_1^2v_1}{r_2^2}$ When r_2 decreases then v_2 increases Where speed of fluid is high the pressure will be low.

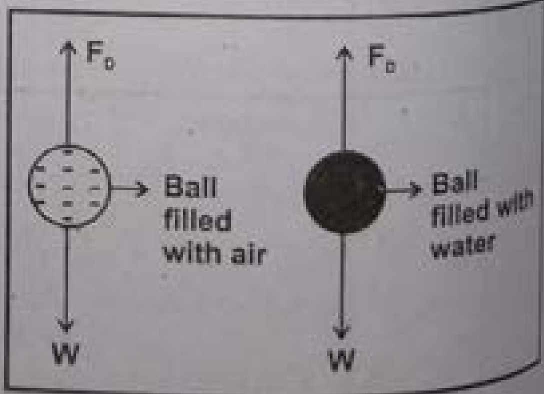
Short Answers of the Exercise

Q.2 Write short answers of the following questions.

Q.1 From the top of a tall building, you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?

Ans: Terminal velocity is given by

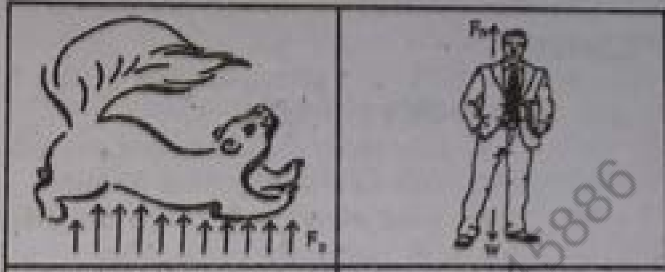
$$v_t=\frac{mg}{6\pi\eta r}$$



- Above equation shows that the terminal velocity of the body is directly proportional to the weight of the object for same radius.
- The lighter, air-filled table tennis ball reaches terminal velocity first. Its mass is less for the same shape and size, so the friction force of upward air resistance becomes equal to the downward force of mg sooner.
- As the force of gravity on the water-filled table-tennis ball (more mass) is larger, its terminal velocity is larger, and it strikes the ground first.

Q.2 Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Ans: A squirrel jumps from a tree branch to the ground and run away undamaged while a human could break a bone in such a fall.



Large surface area is exposed to dragging force	Less surface area is exposed to dragging force.

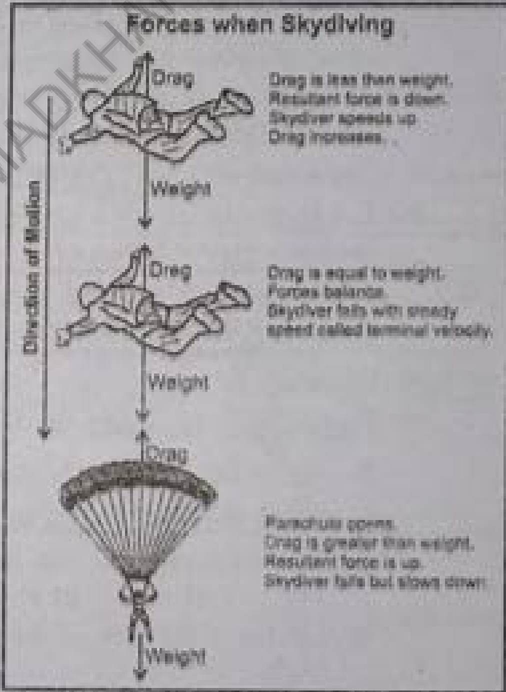
Reason:

- The squirrel jumps from a tree with extended legs.
- As a result his exposed surface area becomes maximum.
- The squirrel faces large drag force and his terminal velocity is smaller.
- The squirrel falls on the ground with smaller momentum and runs away undamaged.

$$V_T = \frac{W}{6\pi\eta r}$$

- The falling heavier human being experience smaller drag force and falls with high terminal velocity and larger momentum.
- So the bone of a human being may break when he falls on the ground.

Q.3 How does the terminal speed of a parachutist before opening parachute compare to the terminal speed afterward? Why is there a difference?



Ans: The terminal velocity of a parachutist before opening the parachute is different from its terminal velocity after opening the parachute.

Reasons and Explanation:

- The dragging force depends upon the expose surface area of the falling object. i.e. greater the exposed surface area of the body, greater will be the dragging force and low will be its terminal velocity and vice versa.
- When the parachutist is falling down without opening parachute, the exposed surface of parachutist to air resistance is small. So he experiences less drag force than his weight and falls with high terminal velocity.
- When the parachutist is falling down with open parachute, large surface area is exposed to air resistance.
- As a result the paratrooper experience large drag force and his terminal velocity is low.

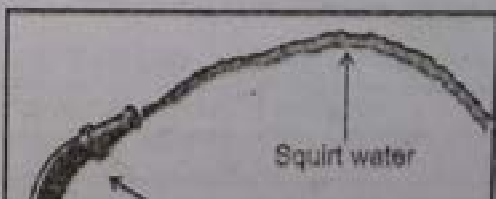
Q.4 you can squirt water over a greater distance by placing your thumb over the end of a garden hose, than by leaving it completely uncovered. Explain how this works?

Ans: Water squirt with greater distance by placing a. thumb on the end of garden hose than by leaving it completely uncovered.

Reasons:

According to the equation of continuity, we have

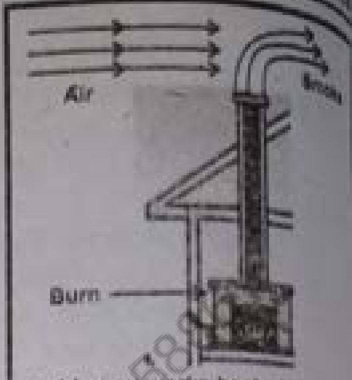
$$A_1v_1 = A_2v_2$$
$$Av = \text{constant}$$
$$v = \frac{constant}{A}$$



- This equation shows that the speed of liquid emerging out from an opening is inversely proportional to the area of cross-section of that opening. i.e. smaller the area of cross-section of the opening, greater will be the speed of outgoing fluid and vice versa.
- Now when we places our thumb over the end of a garden hose, its cross-sectional area decreases as a result, the speed of outgoing water increases.
- In this way, the water can be squirt over a larger distance.

Q.5 Why does smoke rise faster in a chimney on a windy day?

Ans: The smoke rises faster in a chimney on a windy day due to difference in pressure inside the chimney and outside the chimney.



A chimney works best when it is ... and exposed to air currents, which reduces the pressure at the top and force the upward flow of smoke.

Reason

- The Bernoulli's principle plays a vital role in this process. According to **Bernoulli's principle.**
  "Where the speed of fluid is high, the pressure will be low and where speed of fluid is low the pressure will be high"
- As the wind ***blows with high speed*** across the top of a ***Chimney***, the pressure is low there than the pressure inside the chimney.
- Thus the air and ***smoke*** are pushed upwards from high pressure towards the low pressure and rises faster in upwards.

Q.6 Two boats moving in parallel paths close to one another risk colliding. Why?

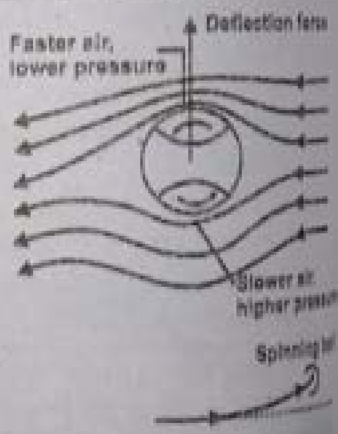
Ans: . **According to Bernoulli's relation**

"where the speed of fluid is high, its pressure will be low and where speed of fluid is low the pressure will be high".

- As speed of water and air between the two boats *is high*. So pressure between the two boats will be *low*.
- The speed of fluid is low on outer side and pressure will be high.
- The force acts on the boats from high pressure towards low pressure which pulls the boats towards each other the boats may collide.

Q.7 A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?

Ans: **Reason**



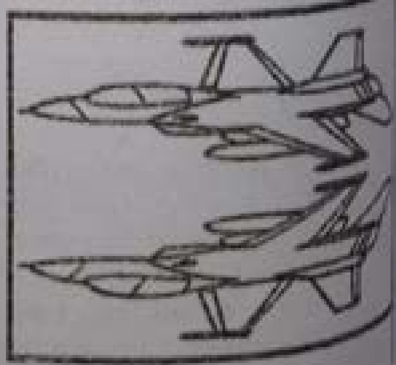
According to **Bernoulli's relation:**

- *where the speed of fluid is high, pressure will be low and where speed of fluid is low, pressure will be high*
- In a spinning ball, the speed of air on its one side becomes *high* and the pressure of air will be low.
- On the other side of the ball the speed of air is low and pressure will be high.
- Hence a force acts on ball from high pressure towards low pressure which gives an extra curvature to the ball.

This force is given by $F = \frac{1}{2}\rho A(v_2^2 - v_1^2)$

Q.8 If aero-foil lift the aero-plane in upright position, how do the pilots make the aero-planes upside down?

Ans:



- To fly upside down the wing is designed in such a way that it can still provide lift even when inverted.
- On a conventional aircraft, the aero foil is curved on the upper side and flat on the lower side. The air on the top of wing flows more quickly as compare to the air below the wing.
- The difference in pressure gives the wing lift upwards.
- But wings on aerobatic planes are curved on both the upper and lower sides.
- With this symmetric design, the plane can fly either normally or inverted.
- The pilot can flip from one to the other by altering the angle of attack.
- While great for stunts, the downside of this design is that it reduces the plane's fuel efficiency.

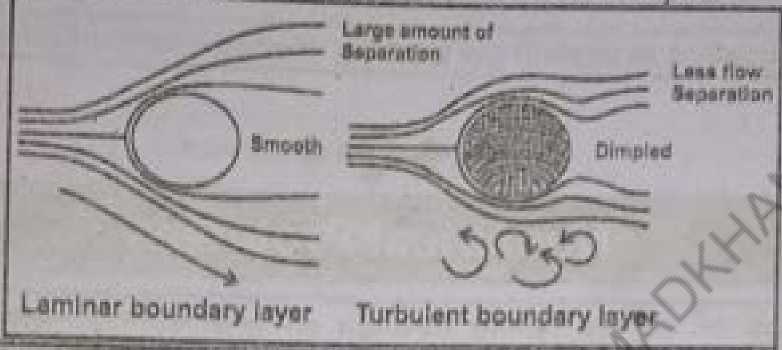
Q.9 Why do the golf balls have dimples?

Ans: Golf balls have dimples, so that can move up in the air very high and can travel larger distance.

Reasons and Explanation:

"Consider two balls Figure (a) represents a ball without dimples while figure (b) represents a ball with dimples."

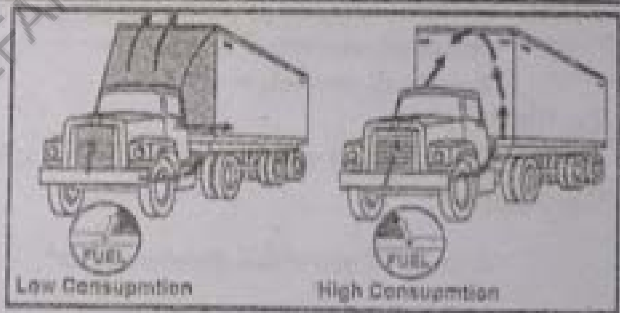
- In case of ball without dimples, the air passes smoothly with greater speed and drag force is greater. Due to larger drag force it slows down quickly and covers smaller distance.
- Dimples on a golf ball create a thin turbulent boundary layer of air that clings to the ball's surface and reduces the drag force. A dimpled ball has about half the drag of a smooth ball.
- Due to reduction of dragging force, the golf ball cannot be slowed down easily. As a result, the golf ball moves up in the air easily and travel larger distance as compare to a ball without dimples.



Q.10 How by using wind deflectors on the top truck cabs reduce fuel consumption?

Ans: Reason

- The portion of the trailer extending above the roof of the truck have a large surface area vertical and perpendicular to the motion of the truck.
- Without a wind deflector, this shape encounters large air friction from wind passing over the top of the truck and hitting the vertical front face of the trailer. Energy is lost due to friction and fuel consumption is greater.



- A wind deflector is a device that changes the airflow around a vehicle.
- An effective wind deflector will **reduce air friction** while the vehicle is in motion, and consequently increases stability and fuel efficiency.
- Due to low friction the speed of truck will be high.

Comprehensive Questions

Q3. Give a short response to the following questions.

1. What is viscous drag? State and explain Stokes Law.

Ans: See Q from book.

2. What is terminal velocity? Derive mathematical relation for terminal velocity by using Stokes law.

Ans: See Q from book.

3. Derive mathematically the equation of continuity, and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?

Ans: See Q from book.

4. Derive mathematical expression for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?

Ans: See Q from book.

5. Using Bernoulli's equation, what is the speed of efflux from a leak at the bottom of large storage tank?

Ans: See Q from book.

6. By Bernoulli's equation, how we can determine the speed of the fluid in a pipe?

Ans: See Q from book.

7. What is aero-foil? Explain aero-foil lift on the wing of an aero-plane.

Ans: See Q from book.

8. Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray.

Ans: See Q from book.

Numerical Problems

1. Eight equal drops of oil are falling through air with steady velocity of 0.1ms^{-1}, the drops recombine to form a single drop, what should be the new terminal velocity?

Data: Terminal velocity of each small droplet = $v_t = 0.1$ m/s
Radius of each small droplet = r
Number of droplets = n = 8

To Find:
Eight droplets combine and terminal velocity of one big drop = $v_t' = ?$

Calculation:

Volume of one small droplet $= \frac{4}{3}\pi r^3$

Volume of n small droplets $= V = n \times \frac{4}{3}\pi r^3$

Terminal velocity of each small droplet $= v_t = \frac{2\rho g r^2}{9\eta}$ ⟶ (1)

When 8 droplets coalesce to form a big drop then volume of big drop is–

$V' = \frac{4}{3}\pi R^3$ (where R is radius of big drop)

Volume of one big drop = Volume of 8 small droplets

$V' = V$

Putting values of V' and V

$\Rightarrow \frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi r^3\right)$

$R^3 = 8r^3$

Taking cube root of both sides

$R = 2r$

$\Rightarrow v_t' = \frac{2\rho g R^2}{9\eta}$ putting R = 2r

$v_t' = \frac{2\rho g(2r)^2}{9\eta}$

$v_t' = \frac{2\rho g(2r)^2}{9\eta} = 4\left(\frac{2\rho g r^2}{9\eta}\right)$ ⟶ (2)

Putting value from Eq (1) in (2)

$v_1' = 4v_1$

⇒ $v_1' = 4(0.1)$

⇒ $\boxed{v_1' = 0.4 m/s}$

---

2. Water travels through a 9.6 cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5 cm. (a) What is the speed of the water coming out of the nozzle? (b) What diameter nozzle is required to give water speed of 21 m/s?

((a)19 m/s, (b) 2.4 cm)

Given Data:

Diameter of fire hose = D_1 = 9.6cm = 0.096m

Speed of water at one end = v_1 = 1.3m/sec

Diameter of nozzle at other end D_2 = 2.5cm = 0.025m

Required:

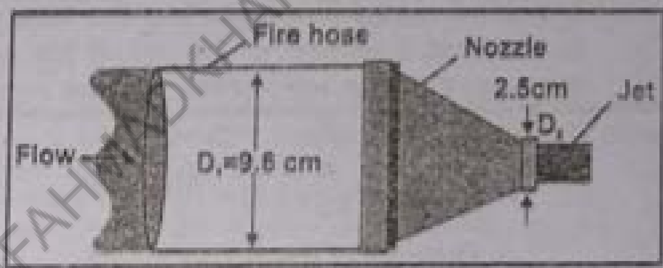
(a) Speed of water at other end = v_2' = 21m/s

(b) Diameter of nozzle = D_2' = ?

Calculation:

From equation of continuity, we have, $A_1v_1 = A_2v_2$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$



Putting $r_1 = \frac{D_1}{2}$ and $r_2 = \frac{D_2}{2}$ so above equation becomes,

$$v_2 = \frac{\frac{D_1^2}{4} \times v_1}{\frac{D_2^2}{4}} = \frac{D_1^2 v_1}{D_2^2}$$

$$v_2 = \frac{D_1^2 v_1}{D_2^2}$$ Putting value we get,

$$v_2 = \left[\frac{(0.096)^2 \times 1.3}{(0.025)^2}\right]$$

$$v_2 = 19.2 \text{ m/s}$$

(b) For speed $v_2' = 21$m/s

the diameter of nozzle D_2' is given by

$$D_2'^2 = \frac{D_1^2 v_1}{v_2'}$$ Putting the values, we get,

$$D_2'^2 = \left[\frac{(0.096)^2 \times 1.3}{(21)}\right]$$

$$D_2'^2 = 5.7 \times 10^{-4}$$

Taking square root of both sides

$$D_2' = 0.024 \text{ m}$$

$$D_2' = 0.024 \times 100 \text{ cm} = 2.4 \text{ cm}$$

---

3. A fish tank has dimensions 0.30 m wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 3.0 h, what should the flow speed be in the 3.0 cm diameter input tube for the filter?

Given Data: Width of tank = w = 0.30m

Length of tank = L = 1m

Height = h = 0.60 m

Density of water = ρ = 1000 kg/m^3
Diameter of hole = D = 3 cm = 0.03 m
Time = t = 3h
H = 3 x 3600 = 10800s

To Find: Velocity of water = v_1 = ?

Calculation: Volume of tank = ΔV = W.L.h

$\Delta V = 0.30 \times 1 \times 0.60 = 0.18\ m^3$

Area of cross-section of hole $= A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4} = \frac{3.14(0.03)^2}{4} = 7.065 \times 10^{-4} m^2$

Volume flow rate $= \frac{\Delta V}{\Delta t} = Av$

$v = \frac{\Delta V}{A\,\Delta t} = \frac{0.18}{7.065 \times 10^{-4} \times 10800} = 2.3 \times 10^{-2} m$

v = 2.3 cm/s

Note: For book answer put width = W = 0.36m

4. A venturi meter is measuring the flow of water; it has a main diameter of 3.5 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm-Hg, what is the speed of the water entering the venturi throat?

Given Data: Diameter of pipe = D_1 = 3.5 cm

Radius of pipe $= r_1 = \frac{D_1}{2} = \frac{3.5}{2}$ cm = 1.75 cm = 1.75×10^{-2}m

Diameter of throat = D_2 = 1 cm

Radius of throat $= r_2 = \frac{D_2}{2} = \frac{1}{2}$ cm = 0.5 cm = 0.5×10^{-2}m

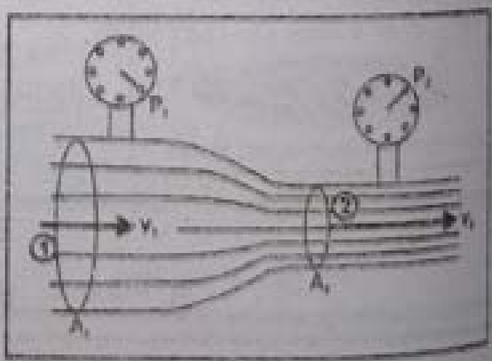
Pressure difference = $P_1 - P_2$ = 18 mm of Hg (1 mm of Hg = 133.3 Pa)
Pressure difference = $P_1 - P_2$ = 18x133.3 Pa = 2399.4 Pa
Speed of water in pipe = v_1 =?
Density of water = ρ =1000 kg/m^3

Calculation:

Area of one pipe $= A_1 = \pi r_1^2 = 3.142 \times (1.75 \times 10^{-2})^2 = 9.62 \times 10^{-4} m^2$

Area of one throat $= A_2 = \pi r_2^2 = 3.142 \times (0.5 \times 10^{-2})^2 = 7.85 \times 10^{-5} m^2$

$v_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$ Putting the values



$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{2(2399.4)}{1000[(0.62 \times 10^{-4})^2 - (7.85 \times 10^{-5})^2]}}$

$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4798.8}{9.2 \times 10^{-4}}}$

$v_1 = 7.85 \times 10^{-5} \sqrt{5216086.9}$

$v_1 = 7.85 \times 10^{-5} (2283.9)$

v_1 = 0.179 m/s

5. A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second.

Given Data: Diameter of hole = D = 6mm = 6×10^{-3}m

$r = \frac{D}{2} = 3$ mm $= 3 \times 10^{-3}$m

Height of tank = h = 14m

Required:

(a) Speed of efflux of water = v = ?

(b) Volume discharge per second $= \frac{\Delta V}{\Delta t} = ?$

Calculation:

(a) We know that $V = \sqrt{2gh}$

$V = (\sqrt{2 \times 9.8 \times 14})$ m/s

$V = 16.6$ m/s

(b) Volume flow rate $= \frac{\Delta V}{\Delta t} = Av$

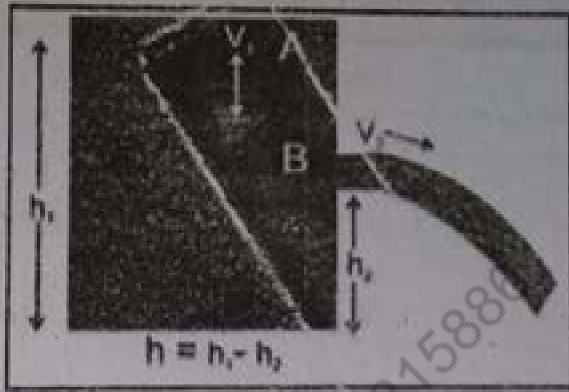
$\frac{\Delta V}{\Delta t} = \pi r^2 v$

$\frac{\Delta V}{\Delta t} = 3.142\,(3 \times 10^{-3})^2\, 16.6$

$\frac{\Delta V}{\Delta t} = 4.69 \times 10^{-4}\, m^3/s$.

$\frac{\Delta V}{\Delta t} = 4.69 \times 10^{-4}\, m^3/s$

$\frac{\Delta V}{\Delta t} = 0.000469\, m^3/s$



6. What is the Aero-foil lift (in newtons) due to Bernoulli's principle on a paper plane of wing area $0.01 m^2$ if the air passes over the top and bottom surfaces at speeds of 9 m/s and 7 m/s respectively? (Take the density of air as $1.28\ kg/m^3$.) (0.2 N)

Given Data: Area of wing = A = $0.01 m^2$

Speed of air below the wing = v_1 = 7 m/s

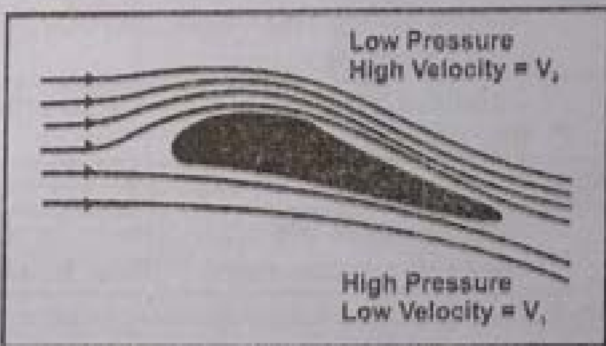
Speed of air at top of wing = v_2 = 9 m/s

Density of air = ρ = 1.28 kg/m³

$h_1 = h_2 = h$

To Find: Aero foil lift = F = ?

Calculation:



$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$

$\Rightarrow \quad P_1 - P_2 = \frac{F}{A} \Rightarrow F = A(P_1 - P_2)$ ------------------(1)

$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh$

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$------------------(2)

Putting equation (2) in equation (1), we get

$F = A \times \frac{1}{2}\rho(v_2^2 - v_1^2)$ Putting the values

$F = 0.01 \times \frac{1}{2} \times 1.28\, \{ (9)^2 - (7)^2 \}$

$F = 6.4 \times 10^{-3}\,(81-49)$

$F = 6.4 \times 10^{-3} \times 32N \;= 204.8 \times 10^{-3}$

$F = 0.2N$

7. During a windstorm, a 25 m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is $1.28\ kg/m^3$).

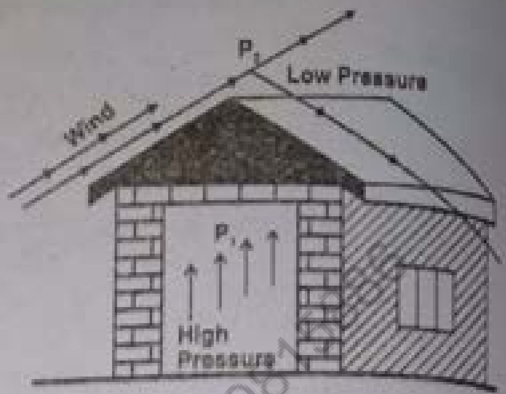
Given Data: Speed of air at top of roof = v_2 = 25m/sec

Density of air = ρ = 1.28 kg/m³

As the doors and windows of the house are closed, so we take the speed of inside air to be zero i.e. put $v_1 = 0$

Required:

Pressure difference = $P_1 - P_2$ = ?



Calculation:

We know that,

$$P_1 - P_2 = \frac{1}{2}(v_2^2 - v_1^2) \quad \text{Putting the value}$$

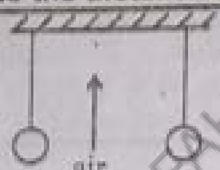
$$P_1 - P_2 = \left[\frac{1}{2} \times 1.28\,(25^2 - 0^2)\right]$$

$$P_1 - P_2 = 0.64 \times (625 - 0)$$

$$P_1 - P_2 = 400\ \text{Pa}$$

Additional Conceptual Short Questions With Answers

1. Two light balls are suspended as shown in the figure. When a stream of air passes through the space between them, what happens to the distance between the balls?



Ans: When air is blown between the balls, then due to high speed air; the **pressure decreases** there as compare to pressure on the other sides. Therefore, high pressure pushes the balls towards low pressure. Hence distance between balls decreases.

2. Consider a physical balance in equilibrium, if stream of air is blown under one of its pans, will this pan go up or down?

Ans: The pan will go down because pressure below the pan **decreases** as the speed of air below the pan **increases.**

3. Houses roofs are often lifted due to hurricanes at high altitude areas. Explain why? Why should we keep windows open under these conditions?

Ans: During the hurricanes, the speed of air outside the house is very fast, so pressure is extremely low due to it. While inside the house pressure is very high. So this **pressure difference produces a very high net force on the roof in the upward direction and roofs are often lifted up.** Opening the windows results in a smaller pressure difference between outside and inside of the house and therefore less net force on the roof. This reduces the chance of damaging the structure of the building.

4. How do you use straw to sip water? Can you sip water with straw on the moon?

Ans: When you drink a liquid through a straw, you reduce the pressure in your mouth and then high atmospheric pressure pushes the liquid towards the mouth **(low pressure)** through the straw.

We cannot use straw to sip water on **moon because there is no atmosphere** [Atmospheric pressure of moon is negligibly small], so we cannot produce the required pressure difference to sip water on the moon.

5. It is often seen that leaves lying on the road start following the fast-moving car when it passes through the road. Why?

Ans: When a car starts moving then speed of air closer to the car is high because stream lines come close to each other. So pressure decreases there. Therefore, **high pressure pushes the leaves towards the low pressure.** It is why it seems that leaves follow the fast-moving car.

6. Why the clouds appear to float in air?

Ans: As the clouds are composed of small droplets of water, so according to formula

$v_t = \frac{mg}{6\pi\eta r}$ $\frac{v}{t} \propto m$

Since mass of the droplets be small. So their terminal velocity is also very small. Hence, they keep floating in air.

7. Why fog droplets appear to be suspended in air?

Ans: Reason

Terminal velocity of a fog droplet is

$$v_t = \frac{mg}{6\pi\eta r}$$

OR $v_t \propto mg$

As the weight of a fog droplet is *very small*, the drag force become equal to the its weight very quickly. Thus, *terminal velocity is very small* and hence the droplet appears to be suspended in air.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **Which of the following is equal to one torr?**
(a) 1.333 Nm^{-2} (b) 13.33 Nm^{-2} (c) 133.3 Nm^{-2} (d) 1333 Nm^{-2}

2. **Which of the following is the S-I unit of viscosity?**
(a) Nsm^{-2} (b) Nsm^{2} (c) Nsm^{1} (d) $Nm^{-2}s^{2}$

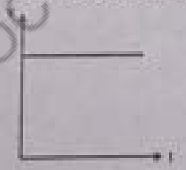
3. **Sphere of mass 1 kg falls through fluid with maximum constant velocity then drag force is**
(a) 98N (b) 49N (c) 9.8N (d) zero

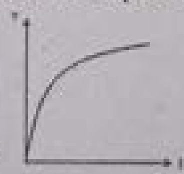
4. **Clouds float in atmosphere because of:**
(a) Low density (b) low viscosity (c) low temperature (d) low pressure

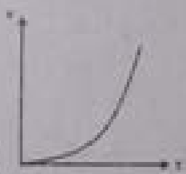
5. **The radius at two ends of a pipe is in the ratio 2:3, then the speed of liquid at the two ends is in the ratio of:**
(a) 3:2 (b) 2:3 (c) 4:9 (d) 9:4

6. **Which of the following represent motion of freely falling water droplet?**


(a)


(b)


(c)


(d)

7. **Which of the following is the expression for terminal velocity?**
(a) $V_t = \frac{2r^2\rho}{9\eta g}$ (b) $V_t = \frac{2gr^2}{9\eta\rho}$ (c) $V_t = \frac{2gr^2\rho}{9\eta}$ (d) $V_t = \frac{2g\eta r^2}{9\rho}$

8. **Venturi meter is used to find:**
(a) speed of the fluid (b) density of the fluid
(c) pressure of the fluid (d) viscosity of the fluid

9. **When the streamlines are far apart from each other, the pressure will be:**
(a) Low (b) High (c) Zero (d) Medium

10. **Which of the following is the S-I unit of flow rate?**
(a) m/s (b) m^3/s^2 (c) m^2/s (d) m^3/s

11. **Which of the following is the unit of co-efficient of viscosity?** (FBISE- 2017)
(a) $Kgm^{-1}s^{-1}$ (b) $Kgm^{-1}s^{-2}$ (c) $Kgm^{-2}s^{-1}$ (d) $Kgm^{-3}s^{-2}$

12. **Which of the following are the dimensions of co-efficient of viscosity?** (FBISE- 2017)
(a) $[ML^{-3}T^{-2}]$ (b) $[ML^{-1}T^{-1}]$ (c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-3}]$

13. **If radius of the droplet is doubled at same density then its terminal velocity increases:** (FBISE-201
(a) Four times (b) Eight times (c) Two times (d) Three times

14. **Which of the following is the device used to measure the speed of liquid flow?** (FBISE (ON) -201
(a) Speedometer (b) Spectrometer (c) Barometer (d) Venturi meter

15. **Which of the following are the dimensions of flow rate?** (FBISE (ON) -201
(a) $[L^{-1}T^{-1}]$ (b) $[L^{-1}T^{-2}]$ (c) $[L^{3}T^{-1}]$ (d) $[L^{2}T^{-1}]$

16. **The property of fluid by which its own molecules are attracted is said to be:** (FBISE (ON) 201
(a) Adhesion (b) Cohesion (c) Viscosity (d) Both A and B

Answers Key

1.	c	2.	a	3.	c	4.	a	5.	d
6.	c	7.	c	8.	a	9.	b	10.	d
11.	a	12.	b	13.	a	14.	d	15.	c
16.	b								

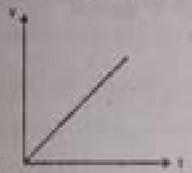
SELF – ASSESSMENT PAPER

Total Mark: 40

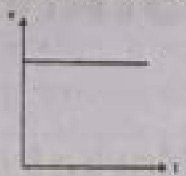
Question.No.1 **Choose the correct answer from the given options.** **(1 x 6 = 6)**

SECTION – A

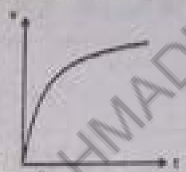
1. **Which of the following is equal to one torr?**
   (A) 1.333 Nm^{-2} (B) 13.33 Nm^{-2} (C) 133.3 Nm^{-2} (D) 1333 Nm^{-2}
2. **Which of the following is the SI unit of viscosity?**
   (A) Nsm^{-2} (B) Nsm^{2} (C) Nsm^{1} (D) $Nm^{-2}s^{2}$
3. **Sphere of mass 0.1 kg falls through fluid with maximum constant velocity then drag force is:**
   (A) 98N (B) 49N (C) 9.8N (D) 0.98 N
4. **The radius at two ends of a pipe is in the ratio 2:3, then the speed of liquid at the two ends is in the ratio of**
   (A) 3:2 (B) 2:3 (C) 4:9 (D) 9:4
5. **Which of the following represent motion of freely falling water droplet?**



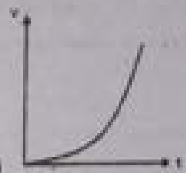
(A)



(B)



(C)



(D)

6. **Bernoulli's equation is based upon law of conservation of**
   (A) Mass (B) Momentum (C) Energy (D) None

Question.No.2 Give short answers of followings: **(3 x 7 =21)**

SECTION – B

(i) How does the terminal speed of a parachutist before opening parachute compare to the terminal speed afterward? Why is there a difference?
(ii) Two boats moving in parallel paths close to one another risk colliding. Why?
(iii) A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?
(iv) Applying Bernoulli's Equation, explain the lift of an aero-plane.
(v) Why do the golf balls have dimples?
(vi) Differentiate between laminar and turbulent flow of fluid.
(vii) Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1}, while at another point 3 m higher, the speed is 4ms^{-1}. At lower point the pressure is 80 kPa. Find the pressure at the upper end.

Question.No.3 **Extensive Questions.** **(13)**

SECTION – C

(a) State and explain Bernoulli's Equation in detail. (06)
(b) What is the Aero-foil lift (in newtons) due to Bernoulli's principle on a paper plane of wing area 0.01m^2 if the air passes over the top and bottom surfaces at speeds of 9 m/s and 7 m/s respectively? (Take the density of air as 1.28 kg/m^3.) (04)
(c) Applying Bernoulli's Equation Explain the working of Carburetor of car Engine? (03)

The End

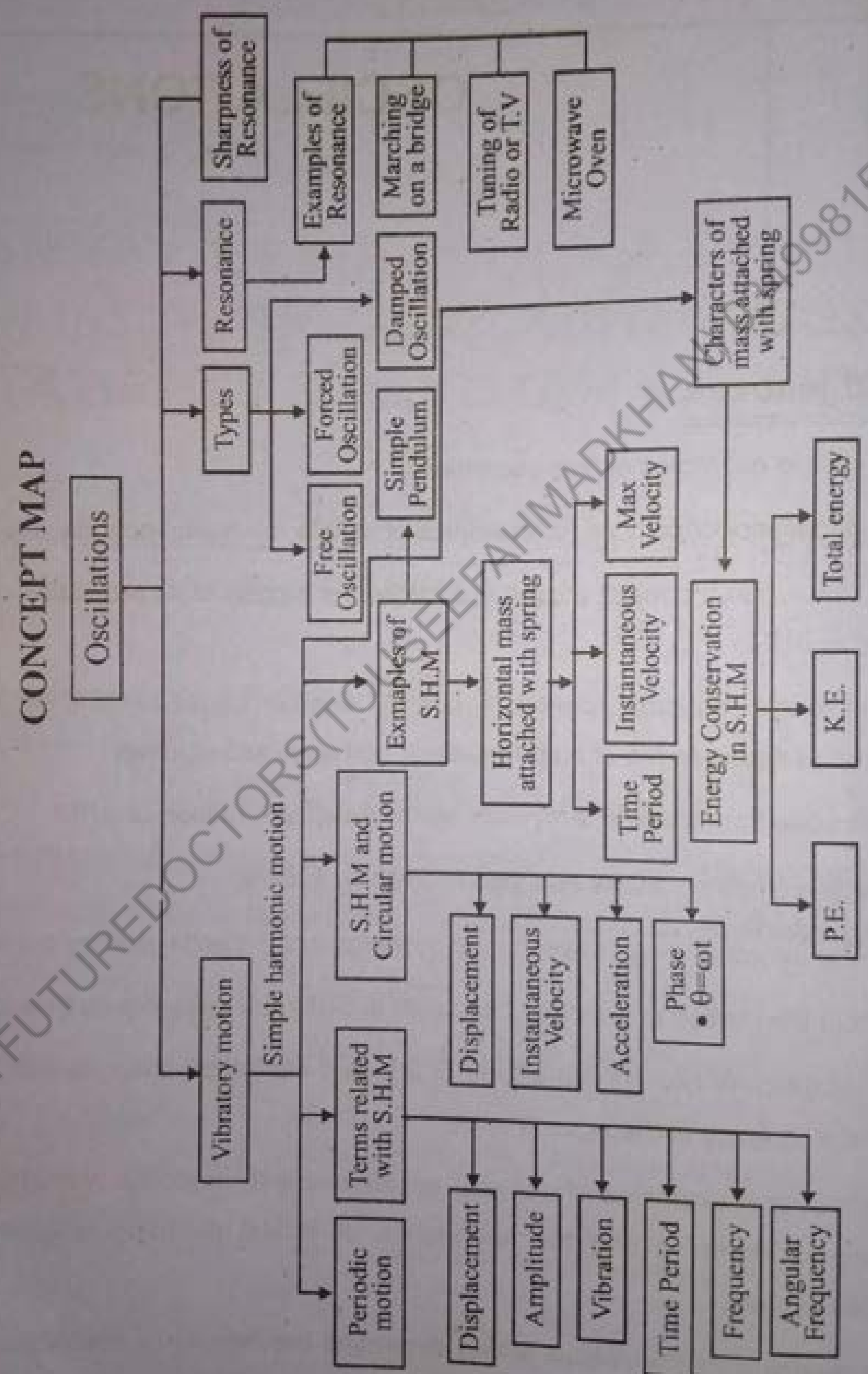
CHAPTER 7

OSCILLATIONS

Learning Objectives

- Describe simple examples of free oscillations.
- Describe necessary conditions for execution of simple harmonic motions.
- Describe that when an object moves in a circle, the motion of its projection on the diameter of the circles is SHM.
- Define the terms amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency.
- Identify and use the equation; $a = -\omega^2 x$ as the defining equation of SHM.
- Prove that the motion of mass attached to a spring is SHM.
- Describe the interchanging between kinetic energy and potential energy during SHM.
- Analyze that the motion of a simple pendulum is SHM and calculate its time period.
- Describe graphically how the amplitude of a forced oscillation changes with frequency near to the natural frequency of the system.
- Describe practical examples of damped oscillations with particular reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system.
- Describe qualitatively the factors which determine the frequency response and sharpness of the resonance.

Chapter No. 7


CONCEPT MAP
Oscillations
Vibratory motion
Types
Resonance
Sharpness of Resonance
Simple harmonic motion
Periodic motion
Terms related with S.H.M
Displacement
Amplitude
Vibration
Time Period
Frequency
Angular Frequency
S.H.M and Circular motion
Displacement
Instantaneous Velocity
Acceleration
Phase
• θ=ωt
Exmaples of S.H.M
Simple Pendulum
Horizontal mass attached with spring
Time Period
Instantaneous Velocity
Max Velocity
Free Oscillation
Forced Oscillation
Damped Oscillation
Examples of Resonance
Marching on a bridge
Tuning of Radio or T.V
Microwave Oven
Characters of mass attached with spring
Energy Conservation in S.H.M
P.E.
K.E.
Total energy

Oscillatory Motion

To and fro motion of a body about a mean position is called oscillatory or vibratory motion.

Periodic Motion

The oscillatory motion that repeats itself after equal intervals of time is called periodic motion.

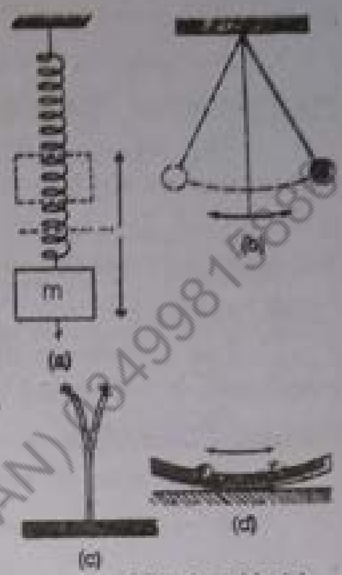


Fig. 7.1

Examples of vibrating bodies

1. The motion of mass suspended from a spring
2. The motion of bob of a simple pendulum
3. A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways.
4. A steel ball rolling in a curved dish

Restoring force

The force which brings the system back to its stable equilibrium position is called elastic restoring forçe.

Mathematically: $\vec{F} = -k\vec{x}$

It is equal and opposite to the applied force

Oscillation:

An oscillation can be a periodic motion that repeats itself in a regular cycle.

In order to get oscillation:

A body is **pulled** away on one side from its equilibrium position and then released. The body begins to oscillate (vibrate) due to **restoring force**. Under the action of this restoring force, the body **accelerates** and it passes over the rest position due to **inertia.** The restoring force pulls it back. Since restoring force is always directed towards the **mean position**, so the acceleration is also directed towards the mean position.

Note:

The vibrating bodies produce waves. e.g. A violin string produces waves.

There are many phenomena in nature, which are explained on the concept of vibration and waves. There are many large structures such as skyscrapers and bridges, which appears to be rigid, actually vibrate, so the architects and engineers take into account these vibrations, while designing and building of certain structure.

Terminology of Oscillatory Motion

Vibratory motion:

To and fro (back and forth) motion of a body about the mean position is called vibratory motion.

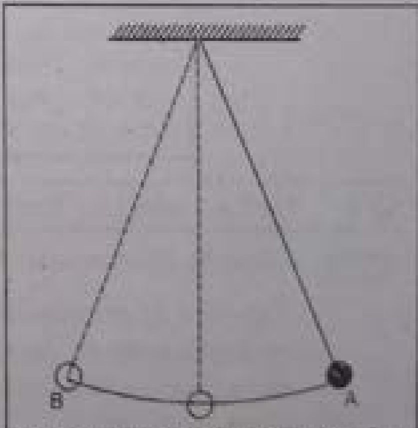


Fig:7.1(a) Vibrating simple pendulum

Vibration

One complete round trip of a vibrating body about its mean position is called one vibration.

For example, motion of bob of simple pendulum from A to B and back from B to A is one vibration as shown in fig 7.1

Time Period

The time required by a body to complete one vibration is called time period.

It is represented by T .Its unit is second.

Frequency

The number of vibrations completed in one second by the body is called frequency. It is the reciprocal of the time period.

It is represented by f. The unit of frequency is hertz or vibrations /second or cycles /second.

$$\boxed{f = \frac{1}{T}}$$

OR $\quad fT = 1$

(i.e. product of frequency and time period equals one)

Instantaneous Displacement (x)

The shortest distance of the vibrating body at any instant from its mean position is called instantaneous displacement.

It is usually denoted by x. The value of instantaneous displacement is zero at mean position while it has maximum value at the extreme positions.

Amplitude (x_o)

The maximum displacement of the vibrating body on either sides from its mean position is called amplitude. It is denoted by x_o.

Angular Frequency:

The number of revolution per second of a body is called angular frequency.

If T is the time period of a body executing SHM, its angular frequency (ω) is given as

$$\omega = \frac{\theta}{T}$$

$$\Rightarrow \quad \omega = \frac{2\pi}{T}$$

$$\Rightarrow \quad \omega = 2\pi\left(\frac{1}{T}\right) \qquad \left(\text{Putting } \frac{1}{T} = f\right)$$

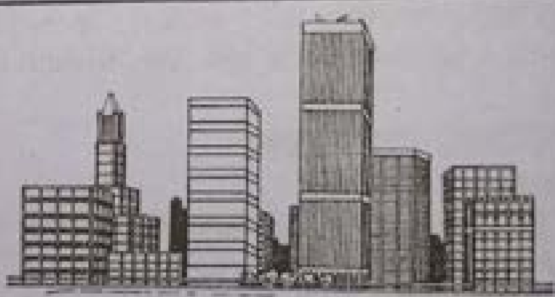
$$\Rightarrow \quad \omega = 2\pi f$$

Note:

Basically, angular frequency is the **property of circular motion**. In SHM, it provides an easy method to determine the instantaneous displacement and instantaneous velocity of body executing SHM.

For Your Information

The most relevant dynamic characteristics of a high-rise building are its natural oscillatory period, mass, stiffness, and damping coefficient. Tall buildings are characterized by low natural frequency; hence they can vibrate significantly under lateral dynamic earthquake loads.



Q.1 Define simple harmonic motion (S.H.M) and give conditions of SHM.

Ans: Simple Harmonic Motion

The type of oscillatory motion, in which acceleration of the body at any instant is directly proportional to displacement from the mean position and directed towards the mean position, is called simple harmonic motion (SHM).

Conditions for SHM

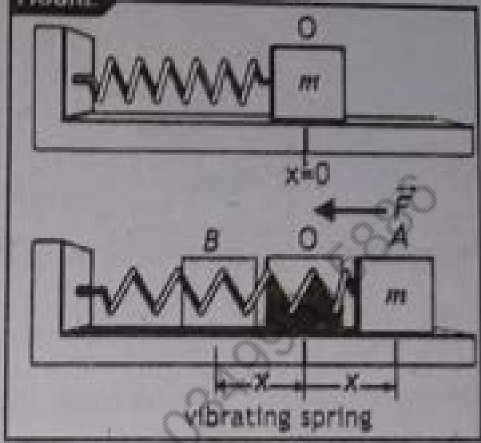
1. The system must have ***inertia***
2. The system should have ***restoring force***
3. $a \propto -x$
4. The system should be ***frictionless***

Q.2 Show that motion of mass attached with a spring is SHM.

Ans: Motion of Mass attached to a spring

Consider a mass 'm' attached with one end of the spring which can move freely on a frictionless horizontal surface as shown in figure.

FIGURE



vibrating spring

When mass m is displaced through a distance x from mean position by a force F then,

According to Hook's law the extension in spring is proportional to the force within elastic limit

$$F = kx$$

Where k is constant of proportionality, known as spring constant.

Spring Constant. $K = \frac{F}{x}$

- Spring constant is defined as *the force per unit extension.*
- Its SI unit is Nm^{-1} and dimension is $[MT^{-2}]$.

Due to elasticity, spring opposes the applied force. This opposing force is called ***restoring force.***

Elastic Restoring Force

The force which brings the body back towards its mean position is called elastic restoring force.

The restoring force represented F_r is

$$F_r = -kx \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

The negative sign shows that F_r is directed opposite to x.

When the mass is released, it begins to oscillate about the equilibrium position as shown in figure, such type of oscillations are due to restoring force and inertia. This type of oscillatory motion is called simple harmonic motion.

Expression for acceleration

The acceleration due to restoring force F

$$F = ma \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

Comparing equations (1) and (2), we get

$$ma = -kx$$

$$\boxed{a = -\frac{k}{m}x} \quad \text{\_\_\_\_\_\_\_\_ (3)}$$

$a = -\text{constant}\ x$ and $a \propto -x$

Angular Frequency

As we know that $a = -\omega^2 x$ ________ (4)

Comparing equations (3) and (4), we get

$$-\omega^2 x = -\frac{k}{m}x$$

OR

$$\omega^2 = \frac{k}{m}$$

OR

$$\boxed{\omega = \sqrt{\frac{k}{m}}} \quad \text{\_\_\_\_\_\_\_\_ (5)}$$

Which is the angular frequency.

Time period

As the time period of mass m having SHM can be expressed as,

$$T = \frac{2\pi}{\omega}$$ Putting values of ω from equation (5) we get

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

OR $$\boxed{T = 2\pi\sqrt{\frac{m}{k}}} \quad \text{\_\_\_\_\_\_\_\_\_\_ (6)}$$

Frequency: As the reciprocal of the time period is called frequency. So,

$$f = \frac{1}{T}$$

Putting value of T from equation (6), we get $f = \dfrac{1}{2\pi\sqrt{\frac{m}{k}}}$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Assignment 7.1:

A 2.0kg block hung from a vertical spring causes it to stretch by 20.0 cm. If the 2.0 kg block is replaced by a 0.50kg mass, and the spring is stretched and released, what are the frequency and period of the oscillations?

Given Data: Mass = m = 2.0 kg
Extension = x = 20.0cm = 0.2m
(a) f = ?
(b) T = ?

Solution: $k = \frac{F}{x} = \frac{W}{x} = \frac{mg}{x} = \frac{2.0 \times 9.8}{0.2} = 98\ N/m$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Hence $$f = \frac{1}{2(3.142)}\sqrt{\frac{98}{0.5}} = 2.2Hz$$

(b) $$T = \frac{1}{f} = \frac{1}{2.2} = 0.45s$$

MCQ's

1. Distance covered during one vibration of an oscillating body in terms of amplitude A is:
(A) A/2 (B) A (C) 2A (D) 4A
2. The wave form of SHM is:
(A) Sine wave (B) Square wave (C) Tangent (D) None
3. The acceleration of a body performing the SHM depends upon:
(A) Mass (B) Time period (C) Displacement (D) Amplitude
4. Which of the following is the relation between linear frequency and angular frequency?
(A) $\omega = 2\pi f$ (B) $\omega = \frac{2\pi}{f}$ (C) $\omega = \frac{2f}{\pi}$ (D) $\omega = \frac{\pi}{2f}$
5. The product of frequency and time period is always equal to:
(A) 1 (B) speed (C) wavelength (D) energy
6. Which of the following is true at mean position in SHM for P-E and K-E?
(A) P.E. is max and K.E is min. (B) P.E. is zero and K.E is max. (C) Both are max. (D) Both are min.

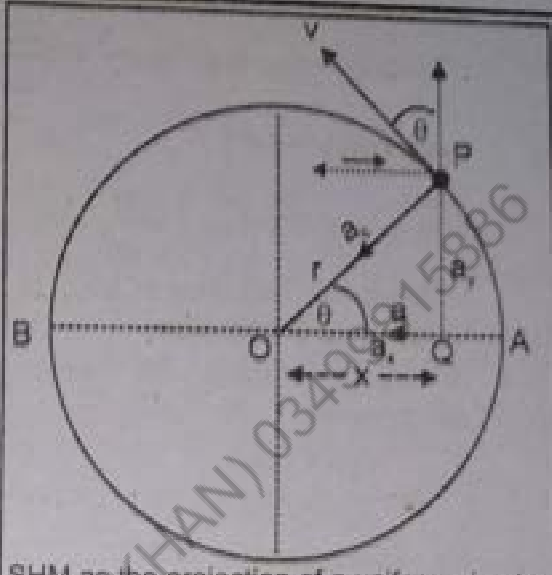
Answers Key

1. D	2. A	3. C	4. A	5. A	6. B

Q.3 Show that the motion of projection of body moving along a circular path is SHM and derive expression for acceleration and velocity of SHM.

Ans: **Circular Motion and S.H.M**

Let there be a particle of mass m whirling in a horizontal circle of radius r with angular velocity ω. A distant light cause a shadow of the stone on wall; the shadow executes simple harmonic motion. Similarly, when a body moves in vertical circle, its projection vibrates simple harmonic motion on the diameter of the circle, having time-period, frequency and acceleration.



SHM as the projection of a uniform circular motion on any diameter of the reference circle.

Fig:7.2

(1) **Acceleration of Projection**

Let $\overline{AB}$ is the diameter and O is the center of circle.

- When the body moves in circle, its projection 'Q' vibrates along the diameter of circle about the mean position O.
- When the body is at point 'P', its projection 'Q' is at distance 'x' from mean position.
- If $\vec{a_c}$ is the centripetal acceleration of body directing towards the mean position O. We have resolved the centripetal acceleration into its components.

From Δ POQ

$\frac{QO}{PO} = \cos\theta$ (Putting QO= a_x and PO = a_c)

$\frac{a_x}{a_c} = \cos\theta$ (cross multiplying)

$a_x = a_c \cos\theta$ .... (1)

a_x is component of a_c along the diameter AB.

As $a_c = \frac{v^2}{r} = r\omega^2$, where 'ω' is the angular velocity of body.

Then equation (1) becomes,

$a_x = r\omega^2(\cos\theta)$ .... (2)

From Fig. 7.2, we see that ΔOQP is a right angle triangle. So

$\frac{x}{r} = \cos\theta$ ---------------- (3)

$x = r\cos\theta$

- This is expression for instantaneous displacement of SHM.

Putting value of $\cos\theta$ from equation (3) in Eq. (2)

$a_x = r\omega^2(\frac{x}{r})$

$a_x = \omega^2 x$

- As a_x directed towards the mean position then we take its sign negative and above equation becomes,

$a_x = -\omega^2 x$

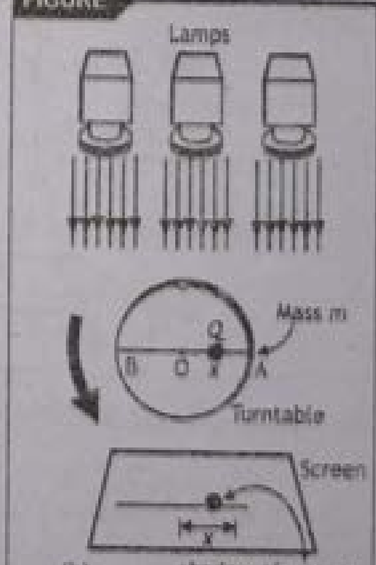
$a_x = -\text{constant } x$

Where ω^2 = constant

$a_x \propto -x$

Which is the equation of S.H.M.

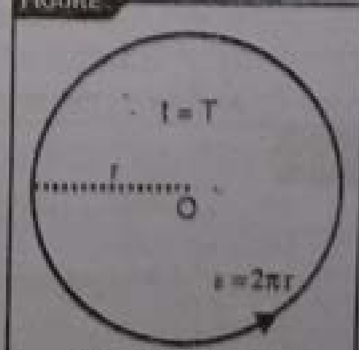
FIGURE



(b) shadow of mass m

Projection of moving particle on a circle on a diameter

FIGURE



(a) One Vibration

(2) **Time-period = T**
The time required by a body to complete one vibration, is called time-period. In given diagram, the vibration of a body is shown.

The time period is given by $T = \frac{2\pi}{\omega}$

$\omega = \frac{\theta}{t}$
For one vibration
$\omega = \frac{2\pi}{T}$

(3) **Frequency = $f = \frac{1}{T}$**

Putting for 'T', then $f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f$
which is the frequency of body in S.H.M.

(4) **Velocity of Projection:**

► In figure, resolve the velocity in to rectangular components.

$v_y = v\cos\theta$
$v_x = v\sin\theta \longrightarrow (1)$

By trigonometry
$\cos^2\theta + \sin^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

From figure $\cos\theta = \frac{x}{r}$

So $\sin^2\theta = 1 - \frac{x^2}{r^2}$

$\sin^2\theta = \frac{r^2 - x^2}{r^2}$

Taking square root of both sides.

$\sin\theta = \sqrt{\frac{r^2 - x^2}{r^2}} = \frac{1}{r}\sqrt{r^2 - x^2}$

putting $\sin\theta = \frac{\sqrt{r^2 - x^2}}{r}$ in equation (1)

$v_x = v\left(\frac{\sqrt{r^2 - x^2}}{r}\right)$

Putting $v = r\omega$

$V = r\omega\left(\frac{\sqrt{r^2 - x^2}}{r}\right)$

$V = \omega\sqrt{r^2 - x^2} \longrightarrow (2)$

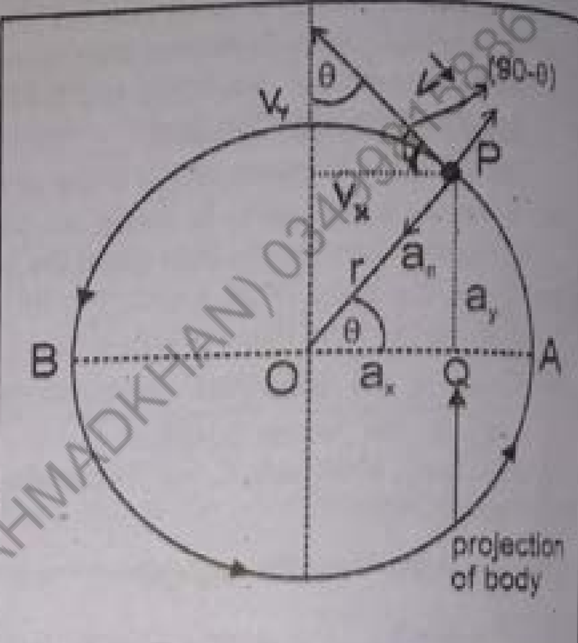


Fig:7.3 (b) Circular motion of a mass

$\frac{v_x}{v} = \cos(90° - \theta)$
$v_x = v\cos(90° - \theta)$
Putting $\cos(90° - \theta) = \sin\theta$
$v_x = v\sin\theta$
$\frac{v_y}{v} = \sin(90° - \theta)$
$v_y = v\sin(90° - \theta)$
Putting $\sin(90° - \theta) = \cos\theta$
$v_y = v\cos\theta$

where 'r' is radius of circle, 'x' is the instantaneous displacement of projection 'Q' from mean position 'O' and 'ω' is the angular velocity of projection of particle.

MCQ's

1. **When a particle is moving along a circular path, its projection along the diameter executes:**
(A) Linear motion (B) Vibratory motion (C) Rotatory motion (D) SHM

2. **Acceleration of projection of a particle moving around a circle is given by relation:**
(A) $a = g\frac{x}{l}$ (B) $a = -\omega^2 x$ (C) $a = -kx/m$ (D) $a = -g\sin\theta$

3. Which of the following is SI unit of plane angle?
(A) Radian (B) Degree (C) Steradian (D) Revolution

4 Which of the following is the expression for frequency of mass m attached to a spring of constant k?

(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{k}{m}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

5 **10 cm extension is produced in a string due to a force of 20 N.** Which of the following is the spring constant?

(A) $2Nm^{-1}$ (B) $20\ N\ m^{-1}$ (C) $200\ N\ m^{-1}$ (D) $2000\ N\ m^{-1}$

6 Which of the following is the **S.I unit of spring constant?**

(A) m^{-1} (B) Nm^{-1} (C) N m (D) Nm^2

7 Which of the following are **the dimension of spring constant?**

(A) $[MLT^{-2}]$ (B) $[MLT^{-3}]$ (C) $[MT^{-2}]$ (D) [MLT]

8 **If F = 0.06N and x = 4cm then** Which of the following is the value of spring constant k?

(A) $56\ Nm^{-1}$ (B) $5.6\ N\ m^{-1}$ (C) $23\ N\ m^{-1}$ (D) $2\ N\ m^{-1}$

9 **Which of the following quantity can be expressed in $kg\ s^{-2}$:**

(A) Spring constant (B) Density (C) Momentum (D) Force

10 The time period of an oscillating mass spring system is 10 second. If mass attached to spring is doubled then which of the following is its time period?

(A) 10 s (B) 20 s (C) 5 s (D) $10\sqrt{2}$ s

Answers Key

1. D	2. B	3. A	4. C	5. C	6. B	7. C	8. D	9. A	10. D

Q.4 What is simple pendulum? Show that the motion of pendulum is SHM. Also derive expression for its time period and frequency.

Ans: Simple Pendulum

A simple pendulum consists of a small heavy mass attached with light and inextensible string suspended with a frictionless support.

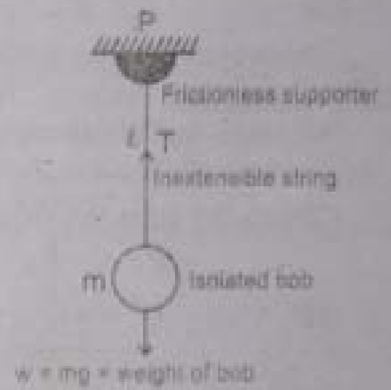


Fig: 7.4 mass m attached to a string

Motion of simple Pendulum is SHM

- Consider an object of mass m attached with the end of a light weight string.

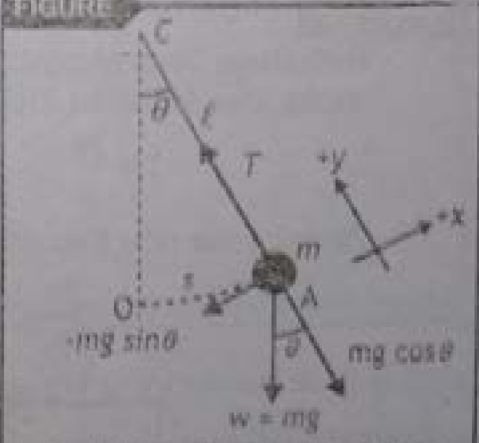
Length of the pendulum

- The length of the simple pendulum ℓ is the distance between the point of suspension and the center of the bob.

Working

- When the simple pendulum is displaced from its mean position through a small angle θ and released then it starts to oscillate about mean position O.

FIGURE



Oscillating Simple Pendulum

Components of weight

- Resolve the weight w of simple pendulum into two rectangular components w cos θ and w sin θ.

Tension in string = w cos θ

⇒ Tension = mg cos θ

Restoring force

- The only force responsible for motion of the simple pendulum is w sin θ which brings the bob back towards its mean position and acts as the restoring force for the bob.

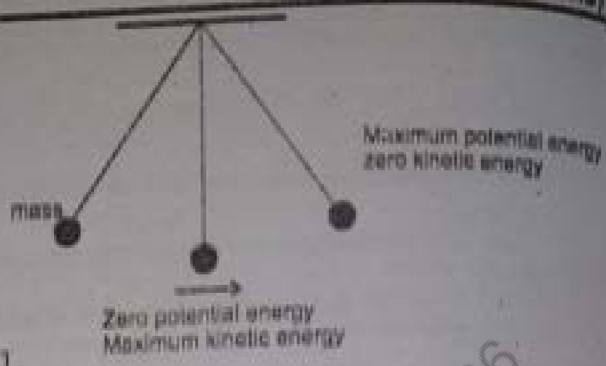
Restoring force = F = − w sin θ

$$F = -mg\sin\theta \quad \_\_\_\_\_\_ (1)$$

Negative sign shows that restoring force is directed towards mean position and direction of restoring force is opposite to displacement.

Also we know that

$$F = ma \quad \_\_\_\_\_\_ (2)$$



Comparing above two equations (1) and (2) we get

$$ma = -mg\sin\theta$$

OR $$a = -g\sin\theta$$

For small value of angle θ, $\sin\theta \approx \theta$

So, $$a = -g\theta \quad \_\_\_\_\_\_ (3)$$

From figure $\theta = \frac{\text{arc } OA}{l}$ $\quad [\because S = r\theta \Rightarrow \theta = \frac{S}{r}]$

$$\theta = \frac{x}{\ell} \quad [\because \theta \text{ is small so arc } OA \approx x]$$

Putting $\theta = \frac{x}{\ell}$ in equation (3)

$$a = -g\left(\frac{x}{\ell}\right)$$

$$\boxed{a = -\left(\frac{g}{\ell}\right)x} \quad \_\_\_\_\_\_ (4)$$

This is expression for the acceleration of simple pendulum.

$$a = -\text{constant}(x) \quad [\because \frac{g}{\ell} = \text{constant}]$$

OR $$\vec{a} \propto -\vec{x}$$

This proves that the motion of simple pendulum is SHM.

Angular Frequency

We know that acceleration for a body executing SHM is

$$a = -\omega^2 x \quad \_\_\_\_\_\_ (5)$$

Comparing equation (4) and (5), we have

$$-\omega^2 x = -\frac{g}{l}x$$

OR $$\omega^2 = \frac{g}{l}$$

OR $$\boxed{\omega = \sqrt{\frac{g}{\ell}}} \quad \_\_\_\_\_\_ (6)$$

This is expression for the angular frequency of simple pendulum.

Time period

Definition: Time required to complete one vibration is called time period.

As the time period for SHM can be expressed as,

$$T = \frac{2\pi}{\omega}$$

Putting value of ω from equation (6) we get

$$T = \frac{2\pi}{\sqrt{g/\ell}}$$

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}} \quad \_\_\_\_\_\_ (7)$$

Dependence of Time period

This equation (7) shows that time period of the simple pendulum depends upon

- ***Length*** of pendulum
- ***Acceleration due to gravity (g)***

Note

Time period of the simple pendulum is independent of

- Mass of bob.
- Amplitude of vibration.

Frequency

As the reciprocal of the time period is called frequency. So,

$$f = \frac{1}{T}$$

Putting value of T, we get

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}} \qquad \text{———— (8)}$$

Q.5 What is second pendulum? Calculate its frequency.

Ans: Second Pendulum

The pendulum whose time period is 2 seconds is called second pendulum.

So $\quad T = 2 \text{ sec}$

Frequency of second Pendulum

$$f = \frac{1}{T}$$

$$f = \frac{1}{2}$$

$$f = 0.5 \text{ Hz}$$

QUIZ

A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is measured. If the elevator moves with constant velocity, does the period (a) increase, (b) decrease, or (c) remain the same? If the elevator accelerates upward, does the period (a) increase, (b) decrease, or (c) remain the same?

Ans:

Part-1: We know that,

$$T = 2\pi\sqrt{\frac{l}{(g+a)}} \quad (1)$$

When the elevator is moving with constant velocity, its acceleration "a" is zero. So according to equation (1), time period will remain the same i-e $T = 2\pi\sqrt{\frac{l}{g}}$. Thus in part (I) the option (c) is correct i.e. "T" remain the same.

Part-2: When the elevator is moving upward with acceleration "a" then net acceleration (g + a) ". So time period is given by

$$T = 2\pi\sqrt{\frac{l}{(g+a)}}$$

From this equation $(g + a) > g$ so time period will decrease. Thus option (b) is correct i.e. "T" decreases.

Assignment 7.2:

A pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower?

Given Data: Time period pendulum T = 12s

Length of tower = ℓ = ?

Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad \text{and} \quad T^2 = 4\pi^2\frac{\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2}$$

$$\ell = \frac{9.8\times(12)^2}{4\times(3.14)^2} = 36m$$

MCQ's From Past Board Papers

1. When the bob of simple pendulum is at extreme position its kinetic energy is:
   (A) Maximum (B) Minimum (C) Zero (D) Small
2. Which expression is correct for the time period of a simple pendulum?
   (A) $T \propto \sqrt{\ell}$ (B) $T \propto l$ (C) $T \propto m$ (D) $T \propto g$
3. In case of oscillating simple pendulum, its acceleration "a" is:
   (A) $a \propto \theta$ (B) $a \propto$ length of pendulum (C) $a \propto$ mass of bob (D) $a \propto$ time period
4. The frequency of Simple Pendulum is given by:
   (A) $\frac{1}{2\pi}\sqrt{\frac{g}{l}}$ (B) $2\pi\sqrt{\frac{g}{l}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{l}{g}}$ (D) $2\pi\sqrt{\frac{l}{g}}$
5. The frequency of second pendulum is:
   (A) 1 Hz (B) 0.5 Hz (C) 1.5 Hz (D) 2 Hz
6. If amplitude of a simple pendulum is increased by 4 times the time period will be:
   (A) four times (B) Half (C) Same (D) Two times
7. If Mass of Pendulum becomes double, then its time period will be:
   (A) Doubled (B) Half (C) Four Time (D) remains same
8. If a simple pendulum vibrates with frequency 0.5 Hz, then its length will be:
   (A) 10 cm (B) 50 cm (C) 80 cm (D) 100 cm
9. Which of the following force is responsible for the vibratory motion of simple pendulum?
   (A) mg Cosθ (B) mg Secθ (C) mg Sinθ (D) mg Tanθ
10. If the time period of simple pendulum is 2 second, its frequency will be:
    (A) 1 Hz (B) 0.5 Hz (C) 1.5 Hz (D) 2 Hz
11. A simple pendulum is moved from the Earth to the Moon. How does it change the period of oscillation? (Acceleration due to gravity on moon = 1.6 m/s^2)
    (A) Time period remains the same (B) Time period is increased by factor $\sqrt{6}$
    (C) Time period is increased by factor four (D) Time period is decreased by factor $\sqrt{6}$
12. At which place the motion of a simple pendulum will be slowest?
    (A) Karachi (B) K−2 (C) Murree (D) Lahore

Answers Key

1. C	2. A	3. A	4. A	5. B	6. C	7. D	8. D	9. C	10. B	11. B	12. B

Q.5 Explain energy conservation in SHM. Prove that the total energy is conserved in SHM.

Ans: **Energy Conservation in SHM**

- Let us consider a mass 'm' attached with a spring of spring constant 'k' executing S.H.M, as shown in fig 7.5 The mass oscillates with amplitude x_0.
- Let oscillating mass is at displacement x from mean position at any instant of time.
- According to Hook's law the applied force is directly proportional to displacement x.

Instantaneous K.E.in SHM

$$K.E = \frac{1}{2}mv^2$$

$$V = \omega\sqrt{x_0^2 - x^2}$$

$$K.E = \frac{1}{2}m\left(\omega\sqrt{x_0^2 - x^2}\right)^2$$

$$K.E = \frac{1}{2}m\omega^2(x_0^2 - x^2) \quad \text{.................(1)}$$

$\vec{a} = -\frac{k}{m}\vec{x}$ and $\vec{a} = -\omega^2\vec{x}$

Comparing these equations

$$-\omega^2\vec{x} = -\left(\frac{k}{m}\right)\vec{x}$$

$$\omega^2 = \frac{k}{m} \quad \text{(or)} \quad m\omega^2 = k$$

Putting $m\omega^2 = k$ in above equation

$$K.E = \frac{1}{2}k(x_0^2 - x^2) \text{ .... (2)}$$

This is expression for instantaneous K.E of mass spring system.

Maximum K.E.

- The K.E is maximum at mean position where x = 0 (putting x = 0 in equation (1)

$$K.E = \frac{1}{2}k\{(x_o^2 - (0)^2\}$$

$$(K.E)_{max} = \frac{1}{2}kx_o^2 \text{ ....(3)}$$

Minimum K.E.

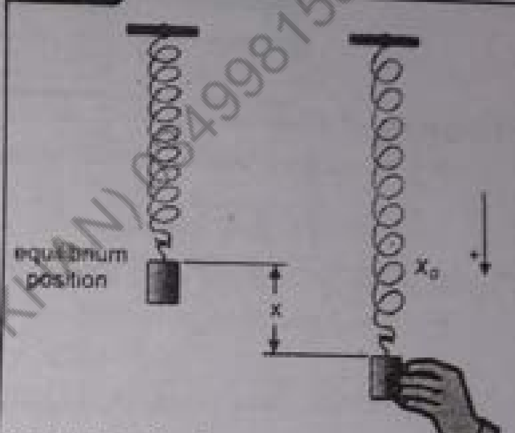
- The K.E is minimum at extreme position where

$x = x_0$ (putting $x = x_0$ in equation (1)

$$K.E = \frac{1}{2}k\{x_o^2 - (x_0)^2\}$$

$$(K.E)_{min} = 0$$

FIGURE



Oscillating Spring

Instantaneous P.E.in SHM

- The restoring force of simple harmonic oscillator at displacement x is

$F_r = -kx$ putting $k = m\omega^2$

$F_r = -m\omega^2 x$ where $\omega = \sqrt{\frac{k}{m}}$

As the applied force is $F = -F_r$

$F = -(-m\omega^2 x)$

$F = m\omega^2 x$

- Initially when body is at rest at mean position then displacement, x = 0

And Initial force = $F_i = 0$

- When displacement is x then final force is $F = m\omega^2 x$

So average force is $F_{av} = \frac{F_i + F_f}{2}$

$$F_{av} = \frac{0 + m\omega^2 x}{2}$$

$$\Rightarrow F_{av} = \frac{1}{2}m\omega^2 x$$

- The work done against the restoring force for displacement x is

$W = F_{av}(x)$ Putting value of F_{av}

$$W = \frac{1}{2}m\omega^2 x\,(x)$$

$$\Rightarrow W = \frac{1}{2}m\omega^2 x^2$$

- This Work done is stored as elastic P.E stored in oscillating mass-spring system. So,

$$P.E = \frac{1}{2}\, m\omega^2 x^2 \quad .... (4)$$

Putting $m\omega^2 = k$

$$P.E = \frac{1}{2}\, Kx^2 \quad .... (5)$$

This is expression for instantaneous P-E

- At extreme position, $x = x_0$ the P.E is maximum,

So. $$(P.E)_{max} = \frac{1}{2} k\, x_o^2 \quad ....(6)$$

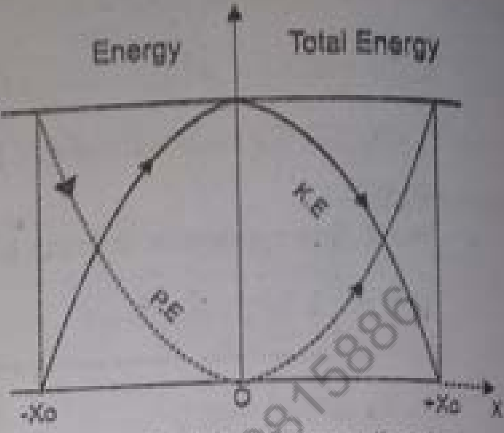


Fig : 7.9 graphical view of conservation of energy

Minimum P.E.

- At mean position x = 0, putting x = 0 in equation (1) P.E. is zero

$$P.E. = \frac{1}{2} k(0)^2$$

$$\vec{a} = -\frac{k}{m}\vec{x}$$

$$\vec{a} = -\omega^2\,\vec{x}$$

Comparing these equations

$$-\omega^2\vec{x} = -\left(\frac{k}{m}\right)\vec{x}$$

OR $\omega^2 = \dfrac{k}{m}$ (or) $m\omega^2 = k$

OR $\boxed{\omega = \sqrt{\dfrac{k}{m}}}$

Total Energy in SHM and Energy conservation

- The total energy is sum of P.E and K.E. at any position.

$$E = P.E + K.E \quad .... (7)$$

Putting values from equation (1) and (4) in equation (5)

$$E = \frac{1}{2} m\omega^2 (x_0^2 - x^2) + \frac{1}{2}\, m\omega^2 x^2$$

$$E = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2}\, m\omega^2 x^2 + \frac{1}{2}\, m\omega^2 x^2$$

$$E = \frac{1}{2} m\omega^2 x_0^2$$

$$\boxed{E = \frac{1}{2} k\, x_o^2}$$

Thus the total energy of vibrating mass spring system always remains constant.

Note: During the oscillatory motion

At mean position

- K.E is maximum.
- P.E is zero.
- Total energy = E = K-E_{Max} = $\frac{1}{2} k\, x_o^2$

At extreme position

- P.E is maximum
- K.E is zero.
- Total energy = E = $(P.E)_{max}$ = $\frac{1}{2} k\, x_0^2$
- The change of P.E and K.E with displacement is required for maintaining the oscillation.

Thus, *periodic exchange of energy is the property of all oscillatory systems but total energy is conserved.*

Assignment 7.3:

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of 6.5 × 104 N/m after hitting a bump. Assume the shock absorbers are poor , so the car really oscillates up and down.

Data: Mass of car =1400kg Spring constant = k = 6.5×10^4 N/m

(a) T = ?

(b) F = ?

Solution:

(a) Since $T = 2\pi\sqrt{\frac{k}{m}} = 2\pi\sqrt{\frac{1400}{6.5\times10^{4}}} = 0.92s$

(b) $f = \frac{1}{T} = \frac{1}{0.92} = 1.08\ Hz = 1.0Hz$

IMPORTANT POINTS

Physical Quantity	Mean Position	Extreme Position
Displacement	Zero	Maximum
Velocity	Maximum	zero
Acceleration	Zero	Maximum
Momentum	Maximum	Zero
K-E	Maximum	Zero
P-E	Zero	Maximum
Total Energy	Equal to maximum K–E	Equal to maximum P–E

MCQ's

1. **When the amplitude of vibration becomes double, its energy becomes:**
(A) Double (B) 4 times (C) One half (D) remains same

2. **Mass attached to spring is pulled slowly from mean position to x_0. Then the work done will be:**
(A) $\frac{1}{2}kx_0$ (B) $\frac{1}{2}k(x_0)^2$ (C) kx_0 (D) ω^2x^2

3. **A body is executing the SHM. What fraction of its total energy will be kinetic energy when its displacement from the mean position is half of its amplitude?**
(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

4. The P.E. stored by a mass spring system at an extension of 2cm is 10J.Which of the following is the P.E. stored by the same system at an extension of 4cm?
(A) 10J (B) 20J (C) 30J (D) 40J

5. Which of the following type of energy stored in compressed or stretched spring?
(A) Elastic P.E. (B) Gravitational P.E. (C) K.E. (D) Chemical P.E.

6. **The velocity of a particle having SHM is v at mean position. If its amplitude is doubled then which of the following will be velocity at mean position?**
(A) $\frac{v}{2}$ (B) v (C) 2 v (D) 4 v

7. **The total energy of horizontal mass spring system is independent of:**
(A) Mass of the body (B) Amplitude
(C) Spring constant (D) Nature of material of spring

8. **A spring of spring constant 10 $\frac{N}{m}$, after loading the amplitude is 2m. Which of the following is the maximum P.E?**
(A) 10 J (B) 20 J (C) 30 J (D) 40 J

9. **Which of the following is the expression for total energy of a particle executing S.H.M?**
(A) $\frac{1}{2}kx^2$ (B) $\frac{1}{2}k(x_0^2 - x^2)$ (C) $\frac{1}{2}kx_0^2$ (D) $\frac{1}{2}k(x^2 - x_0^2)$

Answers Key

1. B	2. B	3. C	4. D	5. A	6. C	7. A	8. B	9. C

Q.6 What are free and forced oscillations? Also define driven harmonic oscillator.

Ans: **Free Oscillations**

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example

A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

A body is said to be executing forced vibrations if it oscillates with the interference of an external force.

For example

- If the mass of vibrating pendulum is struck repeatedly, then forced vibrations are produced.
- The vibrations of factory floor caused by the running of heavy machinery is another example.

Driven harmonic oscillator

The physical system undergoing forced vibrations is known as driven harmonic oscillator.

Q.7 What is resonance phenomenon? Explain it with examples?

Ans: **Resonance**

The increase in amplitude of oscillation of oscillating system exposed to a periodic force whose frequency is equal to the natural frequency of the system is called resonance."

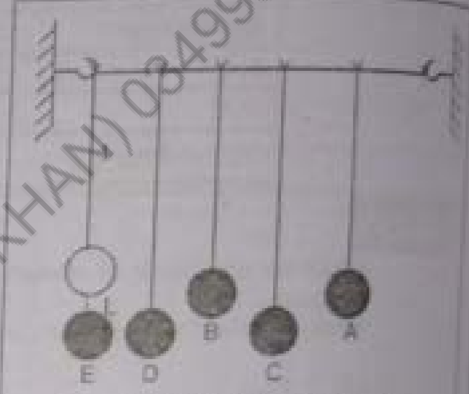


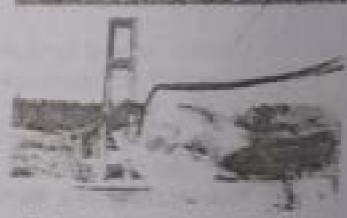
Fig: 7.10 Pendulum A, B, C, D & E

- ► Resonance also occurs when the applied force has frequency an integral multiple of the natural frequency of body.
- ► If 'f_1' is the natural frequency of a body, then resonance takes place at:

  $f_2 = 2f_1$, $f_3 = 3f_1$, $f_4 = 4f_1$, ..., $f_n = nf_1$
- ► A vibrating swing is a good example of ***mechanical resonance.***

  We apply a ***periodic force*** on swing. When the frequency of periodic force ***becomes equal*** to the natural frequency of the swing, resonance is produced. So energy absorption is ***maximum***. Hence, the amplitude of vibration is ***increased***.

Interesting Information


The collapse of Tacoma Narrow bridge (USA) is suspected to be due to violent resonance oscillations

Collapse of suspended bridge

While crossing suspension bridge the soldiers are ordered to break their steps. If the frequency of the force due to steps becomes equal to the natural frequency of the bridge than there is a chance to collapse the bridge due to resonance.

Experiment: Five simple pendulums A, B, C, D, E are suspended with string /rubber cord stretched in between the two hooks.as shown diagram 7.10.

- ► Lengths of simple pendulums A, B are same and is equal to ℓ, and lengths of "C" and 'D' are same and is equal to L.

Case (a):

When length of 'E' is adjusted equal to the lengths of 'A' and 'B' and if 'E' is set into vibrations in a direction perpendicular to the plane of the paper, then after sometime, we see that 'A' and 'B' will start vibrations automatically (because of same length and frequencies) but 'C' and 'D' will remain at rest (because of the difference is lengths of 'E' with 'C' and 'D')

Case (b):

If the length of 'E' is made equal to the lengths of 'C' and 'D' and if 'E' is set into vibrations by applying some force, we see in this case, 'C' and 'D' will start vibrations while 'A' and 'B' will remain at rest.

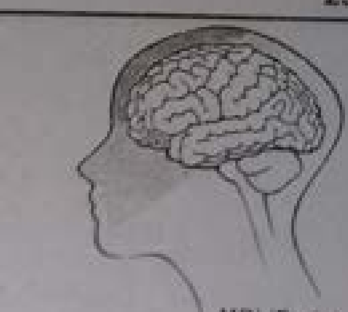
1. Radio and Resonance:


Radio
Fig: 7.11

- ► Tuning a radio is the best example of electrical resonance.
- ► When we turn the knob of a radio to tune a station, we are changing the **natural frequency** of the electric circuit of the receiver, to make it **equal to the transmission frequency** of the radio station.

- When the two frequencies match, energy absorption is maximum and this is the only station we hear.

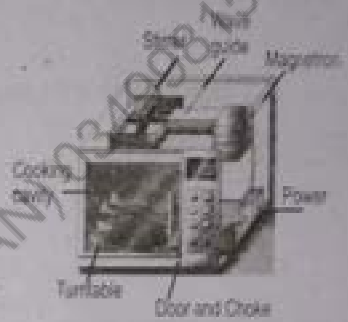
2. **Magnetic Resonance Image (M.R.I)**



MRI (Brain)
Fig: 7.12

- Magnetic resonance scanning has greatly improved medical diagnoses.
- Strong radio frequency radiations are used to cause nuclei of atoms to oscillate.
- When **resonance** occurs, **energy** is **absorbed** by the molecules. The pattern of energy absorption can be used to produce a computer enhanced photograph.

3. **Cooking of Food and Resonance:**



- In a microwave oven, microwave with a frequency similar to the natural frequency of vibration of water or fat molecules are used.
- When food which contains water molecules is placed in the oven, the **water molecules resonates, absorbing energy** from the microwaves and consequently gets **heated up.**
- The plastic or glass containers do not heat up since they do not contain water molecules.

Q.8 Explain wave form of SHM.

Ans: **Waveform of simple harmonic motion**

Wave Form of SHM

The displacement-time (x − t) graph of a simple harmonic oscillator is known as the wave form of SHM. To study the wave form of SHM we know that the instantaneous displacement of SHM is:

$$x = r\cos\theta$$

Putting $r = x_0$ and $\theta = \omega t$ in above equation

$$x = x_0 \cos \omega t \;\ldots\ldots (1)$$

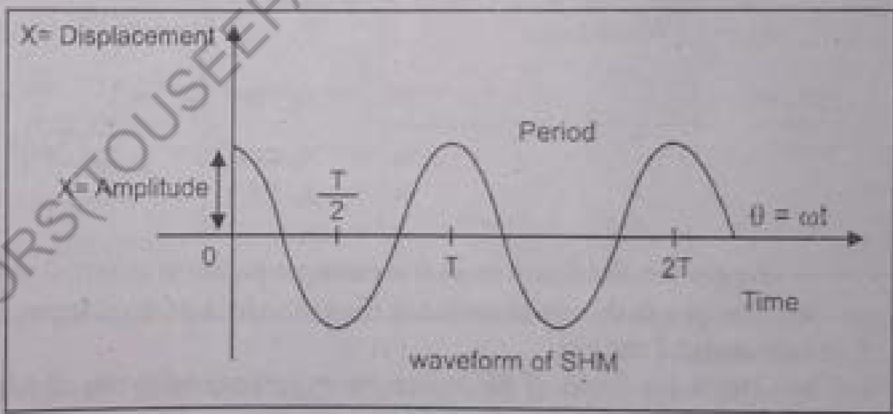


Fig: 7.14

Putting $\omega = \dfrac{2\pi}{T}$

$$x = x_0 \cos\left(\frac{2\pi}{T}\right) t \;\ldots\ldots (2)$$

a) When $t = 0$,

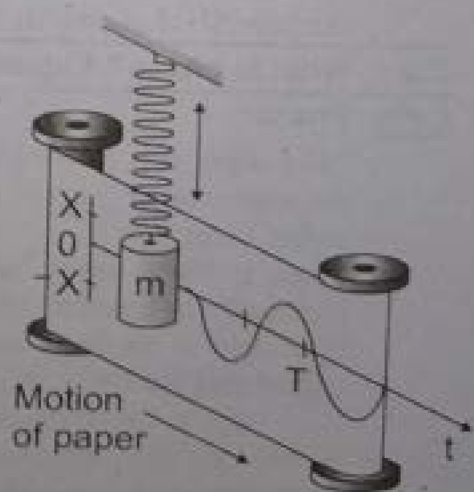
Then from equation (2), we have

$$x = x_0 \cos\left(\frac{2\pi}{T}\right) \times 0$$

$$x = x_0 \cos 0^o$$

$$x = x_0 \quad \text{(maximum)}$$

The oscillator starts motion from the extreme position.



b) When $t = \dfrac{T}{4}$, then from equation (2) we have

$$x = x_0 \cos\left(\frac{2\pi}{T}\right) \times \frac{T}{4}$$

$$x = x_0 \cos(\frac{\pi}{2}) = x_0 \cos 90^\circ = x_0 (0)$$

$$x = 0$$

So the oscillator is passing through the mean position.

c) When $t = \frac{T}{2}$, then from equation (2), we have

$$x = x_0 \cos(\frac{2\pi}{T}) \times \frac{T}{2}$$

$$x = x_0 \cos(\pi) = x_0 \cos 180^\circ$$

$$x = x_0(-1)$$

$$x = -x_0$$

The oscillator is passing through the lower extreme position.

d) When $t = \frac{3T}{4}$, then from equation (2), we have

$$x = x_0 \cos(\frac{2\pi}{T}) \times \frac{3T}{4}$$

$$x = x_0 \cos(\frac{3\pi}{2})$$

$$x = x_0 \cos 270^\circ = x_0 \times 0$$

$$x = 0$$

Again the oscillator will be at the mean position.

e) When time $t = T$, then from eq (3) we have

$$x = x_0 \cos(\frac{2\pi}{T}) \times T$$

$$x = x_0 \cos 2\pi = x_0 \cos 360^\circ$$

$$x = x_0 (1)$$

$$x = x_0$$

Again the oscillator is at the extreme position at initial side.

- The graph drawn between different values of displacement x and time t is called wave form.

Experimental Tracing:

- The wave form of SHM can be experimentally traced when a mass m with a pen is hung from a vertical spring.
- A sheet of paper is placed behind the mass and there is an arrangement to move the paper at a constant speed, as shown. When the mass is stretched down through displacement of x_0 and then released it executes SHM of amplitude x_0 and the pen will trace out the wave form of SHM, as shown.

Q.9 What is phase? Explain.

Ans: Phase:

"The angle $\theta = \omega t$ which specifies the displacement x as well as the direction of motion of the point oscillating with S.H.M. is called Phase".

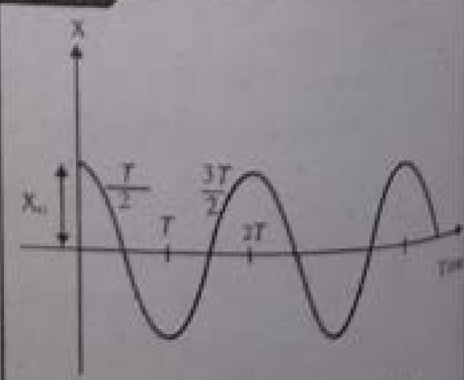
Phase is the quantity which shows the state of motion of an oscillator. In circular motion, we studied, the displacement or projection of the body moving in circle, executing S.H.M on the diameter of the circle, as given by:

$$x = x_0 \cos\theta = x_0 \cos\omega t \qquad \_\_\_\_ (1)$$

Where x is instantaneous displacement, x_0 is maximum displacement; ω is the angular velocity its graphical representation is shown in figure. The general way of showing this equation is

$$x = x_0 \cos(\omega t + \Phi) \qquad \_\_\_\_ (2)$$

FIGURE



graphical representation of oscillator when φ = 0

Where $\theta = (\omega t + \Phi)$ is the phase angle.

- Φ is called starting or initial phase of an oscillator
- Φ also represents the phase difference between the states of motion of two oscillators.

Let us explain, this by the help of graph drawn between 'x' and 't'.

- If $\Phi = 0$ then equation (2) become $x = x_0 \cos \omega t$.

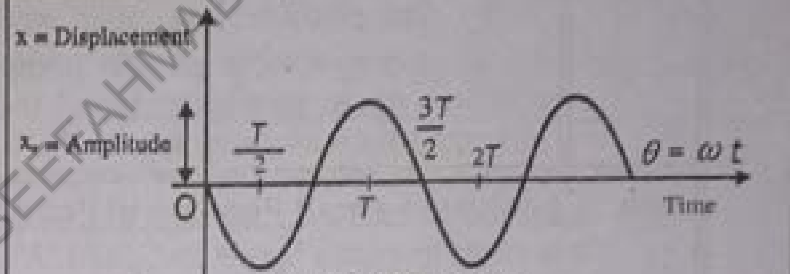
Then putting different values of $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$ we get a graph shown in Fig. 7.13.

- If $\Phi = 90°$, then from equation (2), we get

$$x = x_0 \cos(\omega t + 90°) \quad \_\_\_\_\_\_ (3)$$

Putting different values of $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$ in equation (3) we get a graph shown in Fig. (7.14)

FIGURE



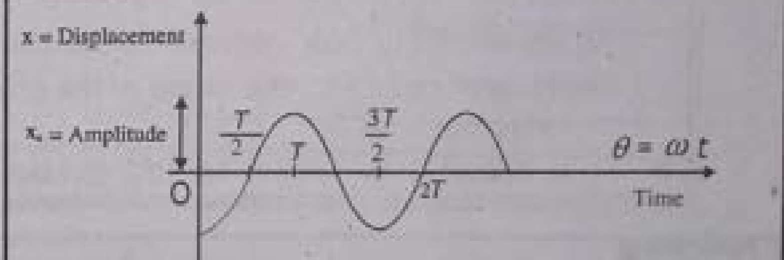
Waveform of SHM when φ = 90°

- Comparing curves in Fig.7.16 and Fig.7.17 we say that, the curve in Fig.7.16 leads in phase the curve in Fig.7.17 by 90°.
- Similarly, if $\Phi = 180°$ then from equation (2) we get

$$x = x_0 \cos(\omega t + 180^0) \quad \_\_\_\_\_\_ (4)$$

Putting different values of $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$ in equation (4) we get a graph shown in Fig. (7.15)

FIGURE



Waveform of SHM when φ = 180°

- Comparing fig. 7.13 and fig. 7.15, we see that in fig. 7.13 the displacement of the oscillation reaches position maximum value x_0, whereas at the same instant the other oscillation reaches a negative maximum value $(-x_0)$. Thus, the two oscillations are said to be out of phase.

Assignment 7.4:

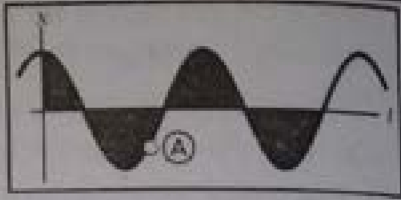
An object vibrates with an amplitude of 6cm and a frequency of 0.490 Hz. Starting from maximum displacement in the positive direction, when will be the first time that its displacement is 2cm?

QUIZ

Consider a graphical representation of simple harmonic motion as described mathematically in Equation 7.22. When the particle is at point A on the graph, what can you say about its position and velocity?

(a) The position and velocity are both positive.
(b) The position and velocity are both negative.
(c) The position is positive, and the velocity is zero.
(d) The position is negative, and the velocity is zero.
(e) The position is positive, and the velocity is negative.
(f) The position is negative, and the velocity is positive.

An x – t graph for a particle undergoing simple harmonic motion. At a particular time, the particle's position is indicated by A in the graph.



Ans: **Conclusion about Position of Point "A":**

It is clear from the figure that point "A" is in negative x-direction. Therefore, the position of point "A" is negative.

Conclusion about Velocity of Point "A":

Now the velocity of the particle is equal to the slope of position time graph at any point. So we have,

$$v = \frac{\Delta x}{\Delta t} \quad \text{——— (1)}$$

In equation (1), "Δx" represents the change in position and "Δt" represents the time interval.

- Now at point "A", the slope of the graph is positive. Therefore, the velocity at point "A" will be also positive.
- Thus option (f) is correct. i.e. the position is negative and velocity is positive.

MCQ's

1. **A quantity which indicates the state and direction of motion of a vibrating body is known as:**
(A) Time (B) Amplitude (C) Phase (D) Frequency
2. **The phase angle $\theta = \omega t$ of a body performing the SHM indicates**
(A) Only the direction of amplitude (B) Only the magnitude of displacement
(C) Both magnitude and direction (D) None
3. **If mass of the pendulum becomes double, its time period will:**
(A) become doubled (B) become half (C) become one fourth (D) remains same

Answers Key

1. C	2. C	3. D

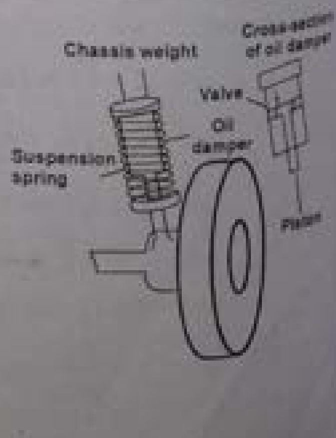
Q.10 What are damped and un-damped oscillations? What is damping?

Ans: **Damped Oscillations**

Oscillations in which amplitude decreases with time due to energy dissipation are called damped oscillations.

Explanation

- The amplitude of the oscillating body gradually becomes smaller and smaller because of friction and air resistance.
- As the energy of the oscillator is used up in doing work against the resistive forces, that is why the amplitude decreases with time till it becomes zero as shown in figure 7.20.



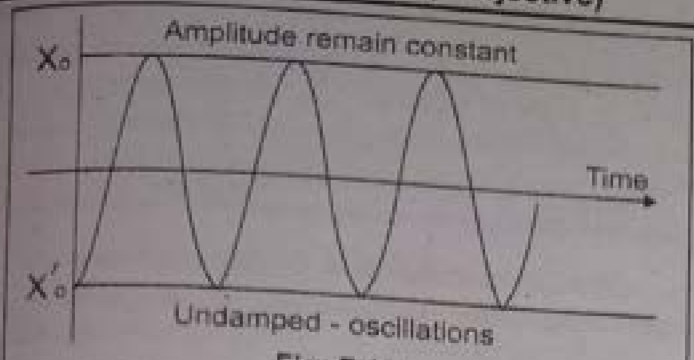


Fig: 7.19

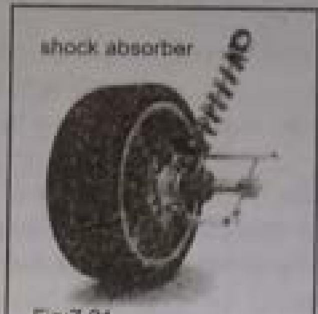


Fig:7.21

Application

An application of damped oscillation is the ***shock absorber*** of a car which provides a damping force to stop the excessive oscillations.

Damping

Damping is the process by which energy is lost by the oscillating system

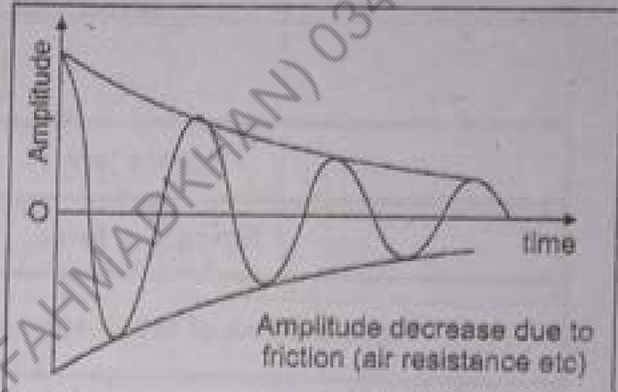


Fig: 7.20

Un-damped oscillations

Oscillations in which the amplitude remains same with time are called un-damped oscillations.

In un-damped oscillations energy is not dissipated from the oscillating system as shown in fig 7.19.

Example

Oscillations of an ideal simple pendulum is the example of un-damped oscillation.

Q.11 What is the effect of damping on the sharpness of resonance?

Ans: Sharpness of Resonance

- The amplitude of vibration of a body increases when the damping is small.
- Thus, the presence of damping prevents the amplitude from becoming sufficiently large.
- The amplitude decreases rapidly at a frequency slightly different from resonance frequency.
- The amplitude as well as sharpness depends upon damping.
- A heavily damped system has fairly flat resonance curve.

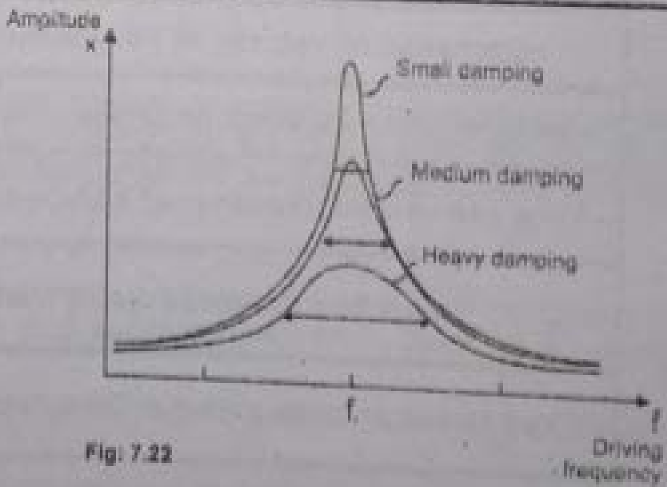
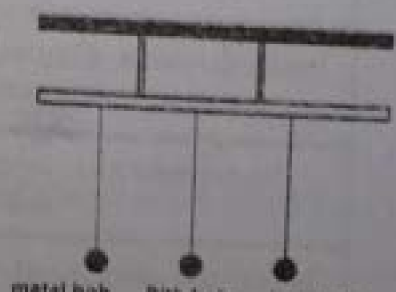


Fig: 7.22

Example to see the effect of damping

- Attach a pendulum having very light mass such a pith ball and another of same length with a heavy mass of equal size such as lead ball.
- Set them into vibrations by third pendulum of equal length and attached to the same rod.
- It is observed that the amplitude of the heavy ball is much greater than the light ball.
- So the sharpness of the resonance curve of resonating system depends on energy loss due to friction.



MCQ's

1. The frequency of waves produced in microwave oven is:
(A) 2250 MHz (B) 2450 MHz (C) 2650 MHz (D) 2850 MHz
2. The wavelength of waves produced by microwave oven is:
(A) 0.12 cm (B) 1 cm (C) 6 cm (D) 12 cm
3. Turning a radio is the best example of ________
(A) Mechanical resonance (B) Electrical resonance (C) Magnetic resonance (D) Musical resonance
4. The process by which the energy is dissipated from an oscillating system called
(A) Resonance (B) Damping (C) Forced vibrations (D) Harmonic oscillations
5. Oscillation of shock absorber of a car is a practical example of:
(A) simple harmonic motion (B) forced oscillations (C) damped oscillations (D) un-damped oscillations

Answers Key

1. B	2. D	3. B	4. B	5. C

FORMULAE

Hook's law	$\vec{F} = -k\vec{x}$	
Restoring force	$\vec{F} = -k\vec{x}$	
Acceleration of mass-spring system	$\vec{a} = -\frac{k}{m}\vec{x}$	
Angular frequency	$\omega = \frac{2\pi}{T}$	$\omega = 2\pi f$
Instantaneous displacement of body executing SHM	$x = x_0 \cos\theta$	
Instantaneous velocity of body executing SHM	$v = \omega\sqrt{x_0^2 - x^2}$	
Instantaneous acceleration of body executing SHM	$\vec{a} = -\omega^2 \vec{x}$	
Time period and frequency of body executing SHM	$T = \frac{2\pi}{\omega}$	$f = \frac{\omega}{2\pi}$
Value of ω for mass spring system	$\omega = \sqrt{\frac{k}{m}}$	
Time period of mass spring system executing SHM	$T = 2\pi\sqrt{\frac{m}{k}}$	
Frequency of mass spring system executing SHM	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	
Instantaneous displacement of mass spring system executing SHM	$x = x_0 \sin\left(\sqrt{\frac{k}{m}}\right)t$	
Instantaneous velocity of mass spring system executing SHM	$v = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$	
Maximum velocity of mass spring system executing SHM	$v_0 = x_0\sqrt{\frac{k}{m}}$	

Instantaneous velocity of mass spring system in terms of maximum velocity executing SHM	$v = v_0\sqrt{1-\frac{x^2}{x_0^2}}$	
Restoring force for simple pendulum	$F = -mg\sin\theta$	
Acceleration of simple pendulum	$a = -\left(\frac{g}{\ell}\right)x$	
Value of ω for simple pendulum	$\omega = \sqrt{\frac{g}{\ell}}$	
Time period of simple pendulum	$T = 2\pi\sqrt{\frac{\ell}{g}}$	
Frequency of simple pendulum	$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$	
Instantaneous P.E. of mass spring system	$(P.E)_{ins} = \frac{1}{2}kx^2$	
Maximum P.E. of mass spring system	$(P.E)_{max} = \frac{1}{2}kx_0^2$	
Instantaneous K.E. of mass spring system	$K.E = \frac{1}{2}Kx_0^2 - \frac{1}{2}Kx^2$	
Maximum K.E. of mass spring system	$(K.E)_{max} = \frac{1}{2}kx_0^2$	
Total energy of mass spring system	$E = \frac{1}{2}kx_0^2$	

Key Points

- The to and fro motion of a body about its mean position is called oscillations.
- Amplitude is the maximum displacement of particles from their normal position.
- Damping is the dissipation of energy during oscillation, which prevents an object from continuing in simple harmonic motion and will eventually force it to stop oscillating altogether. Damping is usually caused by friction.
- The angle ($\theta = \omega t$) which specifies the displacement x as well as the direction of motion of SHM is called phase.
- For a particle experiencing oscillation, frequency is the number of cycles that take place during one second. Frequency is measured in hertz.
- The repeated movement of a particle about a position of equilibrium is called Harmonic Motion.
- hertz is unit of frequency. It may be expressed in units of cycle s^{-1}.
- Period is the amount of time required for one cycle in oscillating motion.
- Motion that is repeated at regular intervals is called periodic motion.
- Resonance is the vibrations of a body under the action of a force having frequency equal to the natural frequency of body.

Solved Examples

Example 7.1: A mass of 0.5 kg is suspended from a spring. The spring is stretched by 0.098m. Calculate the spring constant of the mass when it is given a small displacement.

Solution:

Mass = 0.5 kg

Force = weight = mg = 0.5 x 9.8 = 4.9 N

Extension = x = 0.098 m

In case of spring: $k = \frac{F}{x}$

$$k = \frac{4.9}{0.098} = 50\ Nm^{-1}$$

Example 7.2: A spider swings in the breeze from a silk thread with a period of 0.6s. How long is the spider's strand of silk?

Given: Time period T = 0.60s

Gravitational acceleration, $g = 9.8\ m/s^2$

Required: length $\ell = ?$

Solution: $T\ 2\pi\sqrt{\frac{\ell}{g}} \quad \Rightarrow T^2 = 4\pi^2\left(\frac{\ell}{?}\right)$

$\ell = 4\pi^2 g T^2$

$$= \frac{(9.8)(0.60)^2}{4\pi^2} = 0.089293m$$

$\boxed{T = 0.089293m}$

Example 7.3: A mass at the end of a spring vibrates up and down with a frequency of 0.600 Hz and an amplitude of 5cm. What is its displacement 2.56s after it reaches a maximum?

Given: amplitude $x_0 = 5cm$

Frequency, $f = 0.600\ Hz$

Required: displacement x = ?

Solution: $x = x_0 \cos(2\pi ft)$

$= (5cm)\cos[2\pi\ (0.6\ Hz)(2.56s)];$

$\boxed{x = -4.89m}$

Text Book Exercises

Q.1 Select the correct answer of the following questions.

(1) **Tuning of a radio set is an example of**
(a) Mechanical resonance (b) Musical resonance (c) Electrical resonance (d) Free vibrations

(2) **The heating and cooking of food evenly by microwave oven is an example of:**
(a) S.H.M (b) Resonance (c) Damped Oscillation (d) Free oscillation

(3) **The time period of the same pendulum at Karachi and Murree are related as** (FBISE-2019)
(a) $T_k = T_M$ (b) $T_k > T_M$ (c) $T_k < T_M$ (d) $2T_k = 3T_M$

(4) **In an isolated system the total energy of vibrating mass and spring is:**
(a) Variable (b) Low (c) High (d) Constant

(5) **While deriving the equation of time period for simple pendulum which quantity should be kept small:**

(a) Length of simple pendulum (b) Amplitude

(c) Mass of simple pendulum (d) Gravitational acceleration $\vec{g}$

(6) **If the period of oscillation of mass (M) suspended from a spring is 2s, then the period of mass 4M will be (FBISE-(ON)-2019)**

(a) 1s (b) 2s (c) 3s (d) 4s

(7) **The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times, then its period becomes**

(a) 16s (b) 12s (c) 8s (d) 4s

(8) **To make the frequency double of a spring oscillation, we have to:**

(a) Reduce the mass to one fourth (b) Quadruple the mass

(c) Double the mass (d) Half the mass

(9) **The restoring force of SHM is maximum when particle:**

(a) Displacement is maximum (b) Half way between them

(c) Crossing mean position (d) At rest

(10) **Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by**

(a) $(k_1 + k_2)/2$ (b) $k_1 + k_2$ (c) $k_1k_2/(k_1 + k_2)$ (d) $\sqrt{k_1k_2}$

No.	Option	ANSWER	EXPLANATION
1	(c)	Electrical resonance	
2	(b)	Resonance	
3	(c)	$T_k < T_M$	$T_k = 2\pi\sqrt{\frac{\ell}{g_k}}$ $T_M = 2\pi\sqrt{\frac{\ell}{g_M}}$ $g_k > g_M$ $T_k < T_M$
4	(d)	Constant	
5	(b)	Amplitude	When amplitude is smaller then friction force is smaller and time period will be more accurate.
6	(d)	4s	$T = 2\pi\sqrt{\frac{M}{k}} \longrightarrow (1)$ $T' = 2\pi\sqrt{\frac{M'}{k}}$ Putting M' = 4M $T' = 2\pi\sqrt{\frac{4M}{k}}$ $T' = 2\left[2\pi\sqrt{\frac{M}{k}}\right] \longrightarrow (2)$ Putting value from equation (1) in (2) $T' = 2T$ $T' = 2(2)$ $T' = 4s$
7	(d)	4s	$T = 2\pi\sqrt{\frac{\ell}{g}} \longrightarrow (1)$ $T' = 2\pi\sqrt{\frac{\ell'}{g}}$ Putting $\ell' = 4\ell$

			$T' = 2\pi\sqrt{\frac{4\ell}{g}}$ $T' = 2\left(2\pi\sqrt{\frac{\ell}{g}}\right) \longrightarrow (2)$ Putting value from eq (1) in (2) $T' = 2T$ Putting $T = 2s$ $T' = 2(2) = 4s$
8	(a)	Reduce the mass to one fourth	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \longrightarrow (1)$ $f' = \frac{1}{2\pi}\sqrt{\frac{k}{m'}} \longrightarrow (2)$ $f' = 2f$ Putting value from eq (1) & (2) $\frac{1}{2\pi}\sqrt{\frac{k}{m'}} = 2\left[\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right]$ $\frac{1}{\sqrt{m'}} = \frac{2}{\sqrt{m}}$ Squaring $\frac{1}{m'} = \frac{4}{m}$ (OR) $m' = \frac{m}{4}$
9	(a)	Displacement is maximum	$F = -kx$ When x is maximum then F is maximum
10	(c)	$\left(\frac{k_1 k_2}{k_1 + k_2}\right)$	For 1[st] spring $F = k_1x_1$ (OR) $x_1 = \frac{F}{k_1}$ For 2[nd] spring $F = k_2x_2$ (OR) $x_2 = \frac{F}{k_2}$ Total displacement $x = x_1 + x_2$ $\frac{F}{k_{eff}} = \frac{F}{k_1} + \frac{F}{k_2}$ $\frac{F}{k_{eff}} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$ $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_2 + k_1}{k_1k_2}$ $k_{eff} = \frac{k_1k_2}{k_1 + k_2}$

Short Answers of the Exercise

Q.2 Write short answers of the following questions.

Q.1 Give two applications in which resonance plays an important role.

Ans: (1) Tuning a radio (Electrical resonance)

Tuning of radio is a good example of ***electrical resonance.*** To tune a radio,

- We turn the knob of a radio.
- It ***changes*** the natural frequency of electrical circuit of receiver until it becomes equal to the frequency of transmitter.
- Now the resonance is produced and energy absorption is ***maximum***.
- The radio begins to receive that waves which produce resonance in it.
- Hence a station is tuned.

(2) Cooking by microwave oven

- Resonance plays an important role in heating and cooking food by microwave oven.
- The microwaves produced by oven are ***absorbed due to resonance*** by water and fat molecules in the food. This increases the internal energy of the molecules.
- They get ***heat up*** and so food is cooked quickly.

Q.2 What happens to the time period of a simple pendulum if its length is doubled?

Ans: The time period of a simple pendulum is,

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad \text{\_\_\_\_\_ (1)}$$

Explanation

▶ When length is doubled, $\ell' = 2\ell$

▶ New time period $T' = 2\pi\sqrt{\frac{\ell'}{g}}$

putting $\ell' = 2\ell$

$$T' = 2\pi\sqrt{\frac{2\ell}{g}}$$

$$T' = \sqrt{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right) \quad \text{\_\_\_\_\_ (2)}$$

Putting value from equation (1) in (2)

$$T' = \sqrt{2}\,T$$

▶ When length becomes double, the time period increases $\sqrt{2}$ *times*.

Q.3 What will be the frequency of a simple pendulum if its length is '1 m'.

Ans:

$$\ell = 1\text{m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\times 3.14}\sqrt{\frac{9.8}{1}}$$

$$f = \frac{1}{2\times 3.14}(3.13)$$

$$f = 0.5\ \text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{0.5}$$

$$T = 2\ \text{s} \quad \text{(approximately)}$$

Q.4 Give one practical example each of free and forced oscillation.

Ans: **Free Oscillations**

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example

▶ A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

A body is said to be executing forced vibrations if it oscillates with the interference of an external force.

For example

- If the mass of vibrating pendulum is struck repeatedly, then forced vibrations are produced.
- The vibrations of factory floor caused by the running of heavy machinery is another example.

Q.5 How can you compare the masses of two bodies by observing their frequencies of oscillation when suspended by a spring.

Ans: Frequency of mass-spring system is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Let f_1 and f_2 be the frequencies of masses m_1 and m_2 respectively

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{K}{m_1}} \qquad \ldots\ldots(1)$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{K}{m_2}} \qquad \ldots\ldots(2)$$

Divide eq. (1) by (2)

$$\frac{f_1}{f_2} = \frac{\frac{1}{2\pi}\sqrt{\frac{K}{m_1}}}{\frac{1}{2\pi}\sqrt{\frac{K}{m_2}}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}}$$

Squaring both sides

$$\frac{f_1^2}{f_2^2} = \frac{m_2}{m_1} \qquad \ldots\ldots(3)$$

From eq. (3) masses of two bodies attached at the end of a spring in terms of their frequencies f_1 and f_2 can be compared.

Q.6 A wire hangs from the top of dark high tower' so that the top of tower is not visible. How you would be able to determine the height of that tower?

Ans: In order to find the height of tower we have to determine the length of hanging wire.

- First we attach a bob at the lower end of the wire and make it to vibrate like a simple pendulum.
- Now find the time period of simple pendulum using the stop watch. Let 'ℓ' be the length of hanging wire, the time period T of the bob is

$$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad \text{squaring both sides}$$

$$T^2 = 4\pi^2\frac{\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2}$$

Putting the values of T and g, ℓ can be calculated and the height of tower can be determined.

7 **Why in S.H.M the acceleration is zero when the velocity is greatest.**

Acceleration is zero when the velocity is greatest

The velocity and acceleration of a body executing SHM is given by

$$v=\omega\sqrt{x_o^2-x^2} \qquad \_\_\_\_\_\_ (1)$$

$$a=-\omega^2 x \qquad \_\_\_\_\_\_ (2)$$

At mean position $x=0$

- Putting $x=0$ in equations (1)

$$v=\omega\sqrt{x_o^2-(0)^2}=\omega x_o \quad \text{[maximum]}$$

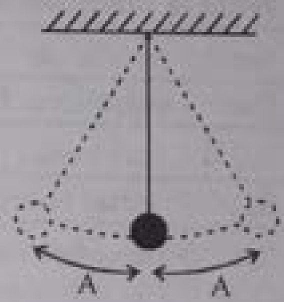
- Putting $x=0$ in equations (2)

$$a=\omega^2(0)=0 \qquad \text{[zero]}$$

- **In S.H.M at mean position the acceleration is zero but velocity is maximum.**

8 **What is the total distance traveled by a simple harmonic oscillator in a time equal to its period. The of amplitude is A?**

The total distance covered by the body is ***4A***.



Explanation

- Time period is the time during which the vibrating body completes one round trip.
- In one round trip total distance covered by simple harmonic oscillator is four times greater than its amplitude.
- Total distance covered = A + A + A + A = 4A

9 **What happens to the frequency of a simple pendulum as it oscillations die down from large amplitude to small?**

- The large amplitude certainly does affect the frequency.
- But for sufficiently small amplitudes, say angle less than 5°, it is a good approximation to say that the frequency does not depend on the amplitude.
- In this small-amplitude limit, the frequency is $f=\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$
- This equation shows that frequency of simple pendulum is independent of amplitude.
- For larger amplitudes, the frequency will be less than that the above equation predicts.

10 **A singer, holding a note of right frequency, can shatter a glass. Explain.**

- The glass has a certain frequency of vibration.
- When a singer sings a song of the **same frequency** as the **natural frequency** of atoms of glass, then two **frequencies match** and **resonance** will take place.
- Due to resonance the **amplitude** of vibration **increases**, energy transferred is maximum and the glass can be shattered.
- To break the glass, you need to broadcast not only a sound that is just the right frequency, but also has a high enough amplitude (loudness).

Comprehensive Questions

Q3. **Give a short response to the following questions.**

1. **Show that motion of a mass attached with a spring executes S.H.M.**

Ans: See Q.2 from book.

2. **Prove that the projection of a body motion in a circle describes S.H.M.**

Ans: See Q.3 from book.

3. **Show that energy is conserved in case of S.H.M.**

Ans: See Q.5 from book.

4. **Differentiate free and forced oscillations.**

Ans: See Q.6 from book.

5. **What is resonance give three of its applications in our daily life.**

Ans: See Q.7 from book.

6. **Derive equations for kinetic and potential energy of a body of mass m executing S.H.M.**

Ans: See Q.5 from book.

7. **Explain what is mean by damped oscillations.**

Ans: See Q.10 from book.

Numerical Problems

1. **A force of 0.4N is required to displace a body attached to a spring through 0.1m from its mean position. Calculate the spring constant of spring.**

Data: Force = F = 0.4N
Displacement = x = 0.1m

To Find: Spring constant = k = ?

Solution:

$$F = K x$$

$$K = \frac{F}{x} = \frac{0.4}{0.1} = 4\ \text{Nm}^{-1}$$

2. **A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly 9.8 m/s^2. When the clock is moved to higher altitude, it loses 80 s per day. Find the value of g at new location.**

Data: Acceleration due to gravity = 9.8 ms^{-2}
Time lost per day = 80 s/day
Increase in time period = 80.0 s / 24 x 3600 = 0.000926 s
New value of acceleration due to gravity g = ?

Solution: Let the original period be; T = 1 s

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{.............. (1)}$$

If the clock is losing 80.0 s per day, it must be swinging slower i.e it has greater time period.
Hence the new period is T' = 1.0 s + 0.000926 s = 1.000926s

The new period is; $$T' = 2\pi\sqrt{\frac{L}{g'}} \quad \text{............ (2)}$$

Divide equation (1) by (2)

Then; $\frac{T}{T'} = \sqrt{\frac{g'}{g}}$ squaring both sides

$$\left(\frac{T}{T'}\right)^2 = \frac{g'}{g}$$

$$g' = \left(\frac{T}{T'}\right)^2 \times g$$

$$g' = \left(\frac{1.0\ s}{1.000926\ s}\right)^2 \times 9.8 = 9.78\ m/s^2$$

3. Calculate the length of second pendulum having time period of 2 s at a place where g=9.8 m/s²

Data: Time period = T = 2s

$g = 9.8\ m/s^2$

To Find: $\ell = ?$

Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Squaring both sides

$$T^2 = 4\pi^2\frac{\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2}$$

$$= \frac{9.8\,(2)^2}{4\times(3.142)^2}$$

$$\ell = 0.993\ m$$

4. A body of mass 'm', suspended from a spring with force constant k, vibrates with 'f_1'. When its length is cut into half and the same body is suspended from one of the halves, the frequency is 'f_2'.

Find out $f_1 f_2^{-1}$.

Data: mass = m

Spring constant = K

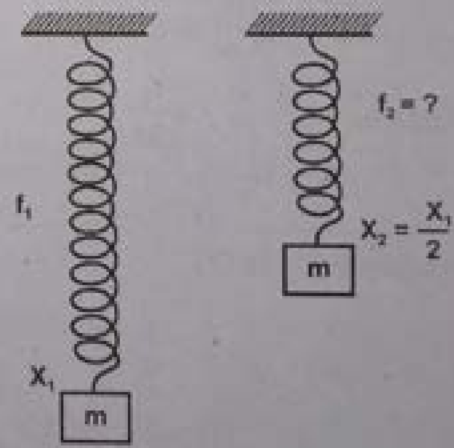
Frequency of complete spring = f_1

Displacement of complete spring = x_1

Frequency of half-spring = f_2

Displacement of half spring = x_2

$$f_1 f_2^{-1} = \frac{f_1}{f_2} = ?$$



Solution:

Frequency of mass spring system is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Putting $K = \frac{F}{x} = \frac{mg}{x}$

$$f = \frac{1}{2\pi}\sqrt{\frac{mg}{mx}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{x}}$$

Frequency of complete spring is

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{g}{x_1}} \longrightarrow (1)$$

When spring is cut into two equal parts and same mass is suspended with half spring, then

Frequency of half spring is

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{g}{x_2}}$$

Putting $x_2 = \frac{X_1}{2}$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{g}{X_1/2}}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{2g}{X_1}}$$

$$f_2 = \sqrt{2}\left[\frac{1}{2\pi}\sqrt{\frac{g}{x_1}}\right] \longrightarrow (2)$$

Putting value eq (1) in (2)

$$f_2 = \sqrt{2}\,[f_1]$$

$$\frac{f_1}{f_2} = \frac{1}{\sqrt{2}}$$

$$\boxed{\frac{f_1}{f_2} = 0.707}$$

Alternate method

Frequency with full spring

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{..........(1)}$$

If the length of spring is cut in to half then its extension also become half. So $x' = \frac{x}{2}$

So $k' = F/x'$

$$k' = \frac{F}{x/2}$$

$$k' = 2\left(\frac{F}{x}\right) \Rightarrow k' = 2k$$

New frequency $f_2 = \frac{1}{2\pi}\sqrt{\frac{k'}{m}}$ putting $k' = 2k$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \quad \text{..................(2)}$$

Divide equation (1) by (2)

$$\frac{f_1}{f_2} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{m}}}{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

$$\frac{f_1}{f_2} = \frac{1}{\sqrt{2}} = 0.707$$

5. A mass at the end of spring describes S.H.M with T = 0.40 s. Find out $\vec{a}$ when the displacement is 0.04m.

Data: Displacement = x = 0.04 m

Time period = T = 0.40 s

To Find: Acceleration = a = ?

Solution:

$$a = -\omega^2 x$$

Putting $\omega = \frac{2\pi}{T}$

$$a = -\left(\frac{2\pi}{T}\right)^2 x$$

$$a = -\frac{4\pi^2}{T^2}x$$

$$a = -\frac{4\times(3.14)^2(0.04)}{(0.4)^2}$$

$$a = -9.86\ m\ s^{-2}$$

A block weighing 4.0 kg extends a spring by 0.16 m from its un-stretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and then released. What is its period of vibration?

Data: mass of block = m′ = 4 kg

Extension = X = 0.16 m

To Find: Spring constant = k = ?

Data: mass of body = m = 0.50 kg

To Find: Time period = T = ?

Solution:

(a) $F = Kx$

$$K = \frac{F}{x}$$

Putting F = W = m′g

$$K = \frac{m'g}{x}$$

$$K = \frac{4\times 9.8}{0.16}$$

$$\boxed{K = 245\ Nm^{-1}}$$

(b) Time period is given by

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\times 3.14\sqrt{\frac{0.50}{245}}$$

$$\boxed{T = 0.28\ s}$$

What should be the length of simple pendulum whose time period is one second? What is its frequency?

Data: Time period = T = 1s

g = 9.8 m s^{-2}

To Find: Length = ℓ = ?

Frequency = f = ?

Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Squaring both sides

$$T^2 = \frac{4\pi^2\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2}$$

$$\ell = \frac{9.8\times(1)^2}{4(3.14)^2}$$

$$\ell = 0.25\ m$$

$$f = \frac{1}{T} = \frac{1}{1} = 1\ Hz$$

8. A spring, whose spring constant is 80.0 Nm^{-1} vertically supports a mass of 1.0 kg is at rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with velocity of one meter per second.

Data: Spring constant = K = 80 Nm^{-1}

Mass = m = 1 kg

Maximum velocity = v_o = 1 ms^{-1}

To Find: Maximum displacement = x_o = ?

Solution:

$$v_o = x_o\sqrt{\frac{K}{m}}$$

$$1 = x_o\sqrt{\frac{80}{1}}$$

$$1 = x_o(8.944)$$

$$x_o = \frac{1}{8.944}$$

$$x_o = 0.11 \text{ m}$$

9. A 8000 g body vibrates S.H.M with amplitude 0.30m. The restoring force is 60 N and the displacement is 0.3m. Find out (i) T (ii) $\vec{a}$ (iii) $\vec{v}$ (iv) K.E (v) P.E when the displacement is 12cm.

Given Data: Mass of body = m = 8.0 kg

Amplitude = x_o = 30 cm = 0.30 m

Restoring force = F = 60 N

Displacement = x = 30 cm = 0.30 m

To Find:

(i) Period = T = ?

(ii) Acceleration = a = ?

(iii) Speed = v = ?

(iv) K.E = ?

(v) P.E = ?

When the displacement, x = 12 cm = 0.12 m

Calculation:

(i) Time period

According to Hooks law, $F = kx_0$

Or $$k = \frac{F}{x_o}$$

$$k = \frac{60}{0.30}$$

Or $k = 200\ Nm^{-1}$

Now using the formula for time period of mass spring system

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Putting values, we get

$$T = 2\times 3.14\sqrt{\frac{8}{200}}$$

$$T = 6.28\times\sqrt{0.04}$$

$$T = 6.28\times 0.2$$

$$T = 1.256 \text{ s}$$

Or $\boxed{T = 1.3 \text{ s}}$

ii- Alternate method

$$a = -\omega^2 x$$

Putting $\omega = \frac{2\pi}{T}$

$$a = -\left(\frac{2\pi}{T}\right)^2 x = -\left(\frac{4\pi^2}{T^2}\right)x$$

$$a = -\frac{4(3.142)^2\ 0.12}{(1.256)^2}$$

$$a = -3 \text{ m/s}^2$$

(ii) Acceleration:

$$a = -\omega^2 x \qquad \text{As} \quad \omega^2 = \frac{k}{m}$$

Or

$$a = -\left(\frac{k}{m}\right)x$$

Putting values, we get

$$a = -\frac{200}{8} \times 0.12$$

$$\boxed{a = -3.0\,ms^{-2}}$$

Negative sign shows that acceleration in directed towards the mean position.

(iii) Speed:

The speed of the body executing SHM is given by

$$v = \omega\sqrt{x_0^2 - x^2}$$

Thus,

$$v = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2} \qquad \text{As} \quad \left(\omega = \sqrt{\frac{k}{m}}\right)$$

Putting values, we get

$$v = \sqrt{\frac{200}{8}}\sqrt{(0.3)^2 - (0.12)^2}$$

$$v = 5\sqrt{0.09 - 0.0144}$$

$$v = 5\sqrt{0.0756}$$

$$v = 1.37\ ms^{-1}$$

$$\boxed{v = 1.4\ ms^{-1}\ \text{(approx)}}$$

(iv) K.E:

Using the formula, for calculating K.E.

$$K.E = \frac{1}{2}mv^2$$

Putting values, we get

$$K.E = \frac{1}{2} \times 8 \times (1.37)^2$$

$$K.E = 4 \times 1.89$$

$$K.E = 7.56\ J$$

Or

$$\boxed{K.E = 7.6\ J}$$

(v) P.E:

The formula for P.E. is

$$P.E = \frac{1}{2}kx^2$$

$$P.E = \frac{1}{2} \times 200 \times (0.12)^2$$

$$P.E = 100 \times 0.0144$$

$$\boxed{P.E = 1.44\ J}$$

10. Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant t is given by $x = 0.25\cos\left(\frac{\pi}{8}\right)t$. Find the displacement of the object after 2.0s.

Given data: $x = 0.25\cos\left(\frac{\pi}{8}\right)t$

Time = t = 2 s

To Find:

Amplitude = x_o = ?

Frequency = f = ?

Period = T = ?

Displacement = x = ?

Calculation:

Amplitude: As given displacement is

$$x = 0.25\cos\left(\frac{\pi}{8}\right)t \;\dots\dots\dots(1)$$

And general equation for displacement is

$$x = x_o \cos \omega t \;\dots\dots\dots\dots(2)$$

Comparing equation (1) and (2), we get

$$\boxed{x_o = 0.25\text{ m}}$$

From above equation , angular frequency is

Frequency: $\omega = \frac{\pi}{8}t$ comparing angle of Eq (1) and (2)

$$\omega = \frac{\pi}{8}$$

Or $2\pi f = \frac{\pi}{8}$ $(as \quad \omega = 2\pi f)$

$$\boxed{f = \frac{1}{16}\text{ Hz}}$$

Time period:

Now $T = \frac{1}{f}$

$$T = \frac{1}{1/16}$$

$$\boxed{T = 16\text{ s}}$$

Now calculation of displacement when t = 2 s

Putting value in equation (1)

$$x = 0.25\cos\left(\frac{\pi}{8}\right)\times 2$$

Or $x = 0.25\cos\left(\frac{\pi}{4}\right)$

Or $x = 0.25\cos 45°$

Or $x = 0.25 \times 0.707$

Or $x = 0.1768$

Hence $\boxed{x = 0.18\ m}$

Additional Conceptual Short Questions With Answers

1. If a pendulum clock keeps perfect time at the base of mountain, will it keep perfect time when moved to the top of the mountain?

Ans: No, we know that time period of simple pendulum depends upon gravitational acceleration "g"

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad \Rightarrow T \propto \frac{1}{\sqrt{g}}$$

We also know that value of g decreases with increase in height. so pendulum will not keep correct time at the top of mountain.

2. If x_0 is amplitude of vibration then at which displacement, K.E. and P.E. will be equal in SHM?

Ans: According to the given condition:

$$K.E. = P.E.$$

$$\frac{1}{2}K(x_o^2 - x^2) = \frac{1}{2}Kx^2$$

$$\frac{1}{2}Kx_o^2 - \frac{1}{2}Kx^2 = \frac{1}{2}Kx^2$$

$$\frac{1}{2}Kx_o^2 = \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2$$

$$\frac{1}{2}Kx_o^2 = k\,X_0^2$$

$$\Rightarrow \quad \frac{1}{2}x_o^2 = x^2 \qquad \text{(or)} \quad x^2 = \frac{1}{2}x_o^2$$

Taking square root of both sides

$$\Rightarrow \quad x = \sqrt{\frac{x_o^2}{2}}$$

$$\Rightarrow \quad \boxed{x = \frac{x_o}{\sqrt{2}}}$$

3. If the time period of a pendulum is 2 second on the earth surface. Then what is its time period at the center of earth? What is its frequency?

Ans: Time period of simple pendulum is:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

At the center of earth $\quad g = 0$

So $\quad T = 2\pi\sqrt{\frac{\ell}{0}}$

$$\Rightarrow \quad \boxed{T = \infty}$$

Time period will be infinite.

We know that $\quad f = \frac{1}{T}$

$$\Rightarrow \quad f = \frac{1}{\infty} = 0$$

Frequency is 0 Hz.

4. If pendulum of time period T is taken from earth to moon, then what will be its time period on moon?

Ans: A second pendulum is one whose time period is 2 second. Since value of g on moon is 6 times less than its value on earth. And

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

* Time period of simple pendulum on moon is calculated as:

$$T' = 2\pi\sqrt{\frac{\ell}{g_m}}$$

On moon $g_m = g/6$

$\Rightarrow$ $$T' = 2\pi\sqrt{\frac{\ell}{g/6}} = 2\pi\sqrt{\frac{6\ell}{g}}$$

$\Rightarrow$ $$T' = \sqrt{6}\left(2\pi\sqrt{\frac{\ell}{g}}\right)$$

[where $\left[T = 2\pi\sqrt{\frac{\ell}{g}}\right]$ is time period of the pendulum on earth.]

$\Rightarrow$ $$T' = \sqrt{6}T$$

5. A simple harmonic motion has an amplitude A and time period T. Then how much time will take to travel from $x = A$ to $x = \frac{A}{2}$?

Ans: The displacement equation for a body performing SHM is:

$$x = x_o \cos \omega t$$

$\Rightarrow$ $$x = A \cos \omega t$$

where $x_o = A$ amplitude

Put $$x = \left(\frac{A}{2}\right)$$

$\Rightarrow$ $$x = A \cos \omega t$$

$$\frac{A}{2} = A \cos \omega t$$

$\Rightarrow$ $$\frac{1}{2} = \cos \omega t$$

$\Rightarrow$ $$\omega t = \cos^{-1}\left(\frac{1}{2}\right)$$

$\Rightarrow$ $$\omega t = \frac{\pi}{3}$$

$\Rightarrow$ $$\left(\frac{2\pi}{T}\right)t = \left(\frac{\pi}{3}\right)$$

$\Rightarrow$ $$t = \frac{T}{2(3)}$$

$\Rightarrow$ $$t = \left(\frac{T}{6}\right)$$

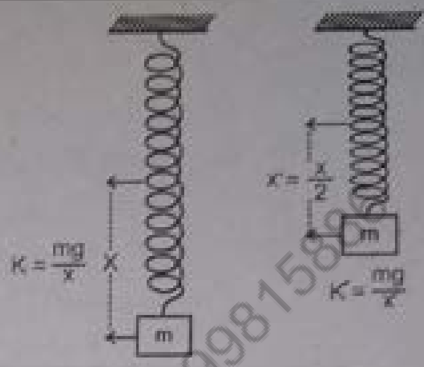
6. A spring of spring constant 'k' is cut into two pieces of equal lengths then what will be the spring constant of each part?

Ans: According to Hook's law

$$F = kx$$

$$k = F/x \qquad \text{putting } F = w = mg$$

$$k = \frac{mg}{x} \quad \ldots\ldots\ldots (1)$$



If the length of spring is reduced half then its extension also becomes half.

So $\quad x' = \frac{x}{2}$

So $\quad k' = \frac{mg}{x'} \qquad$ Putting $x' = \frac{x}{2}$

$$k' = \frac{mg}{X/2}$$

$$k' = 2\left(\frac{mg}{x}\right) \quad \ldots\ldots\ldots\ldots (2)$$

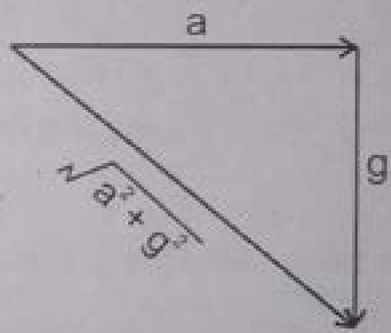
Putting value from equation (1) in (2)

$\Rightarrow \quad k' = 2k$

7. A simple pendulum of length *l* is suspended from the roof of a train which moves in a horizontal direction with acceleration "a" then what will be its time period?

Ans: The time period of pendulum T when the train is at rest is:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$



When the train moves with horizontal acceleration "a" then effective acceleration is $\sqrt{a^2+g^2}$

So new time period is:

$$\boxed{T' = 2\pi\sqrt{\frac{\ell}{\sqrt{a^2+g^2}}}}$$

$\Rightarrow$ The time period of pendulum will decrease

- $$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{a^2+g^2}}{\ell}}$$

$\Rightarrow$ Frequency of the pendulum will increase.

8. Two particles A and B of equal masses are suspended from two massless springs of spring constant K_1 and K_2. If maximum velocity during oscillation is equal. Find the ratio of amplitude.

Ans: Maximum velocity is

$$V_o = x_o\omega$$

$$V_o = x_o\sqrt{\frac{K}{m}}$$

OR $$V_o = A\sqrt{\frac{K}{m}}$$

$$V_1 = A_1\sqrt{\frac{K_1}{m}}$$

$$V_2 = A_2\sqrt{\frac{K_2}{m}}$$

According to given condition

$$V_1 = V_2$$

$$A_1\sqrt{\frac{K_1}{m}} = A_2\sqrt{\frac{K_2}{m}}$$

$$\frac{A_1}{A_2} = \frac{\sqrt{\frac{K_2}{m}}}{\sqrt{\frac{K_1}{m}}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{K_2}{K_1}}$$

9. If 'x' is instantaneous displacement and 'x_0' is amplitude of vibration of a mass-spring system then prove that $X = X_0\sqrt{1-\frac{V^2}{V_o^2}}$

Ans: The velocity of mass-spring system at any time in terms of maximum velocity is given by

$$V = V_o\sqrt{1-\frac{X^2}{X_o^2}}$$

$$\frac{V}{V_o} = \sqrt{1-\frac{X^2}{X_o^2}}$$

Squaring both sides

$$\frac{V^2}{V_o^2} = 1-\frac{X^2}{X_o^2}$$

$$\frac{X^2}{X_o^2} = 1-\frac{V^2}{V_o^2}$$

Taking square root of both sides

$$\frac{X}{X_o} = \sqrt{1-\frac{V^2}{V_o^2}}$$

$$X = X_o\sqrt{1-\frac{V^2}{V_o^2}}$$

This is required relation.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **The component of weight mg sinθ in simple pendulum is:**
   (a) Along the center (b) perpendicular to string (c) along the string (d) none of these
2. **If the amplitude of vibrating body is doubled, its velocity at mean position:**
   (a) Remains same (b) becomes double (c) becomes four time (d) becomes √2 times
3. **Keeping length constant If mass of bob of simple pendulum is doubled then its time period becomes:**
   (a) Double (b) half (c) remain same (d) four times

4. **Which of the following is the expression for instantaneous velocity of a body executing SHM?**

(a) ωx_o^2 (b) $\omega\sqrt{x^2 - x_o^2}$ (c) ωx^2 (d) $\omega\sqrt{x_o^2 - x^2}$

5. **The unit of spring constant is same as that of:**

(a) Force (b) pressure (c) flow rate (d) surface tension

6. **If mass of mass spring system becomes four times, then its time period 'T' becomes:**

(a) T (b) 2T (c) T/2 (d) 4T

7. **Which of the following is the length of second pendulum is approximately?**

(a) 1m (b) 100cm (c) 2m (d) Both a and b

8. **Which of the following is the length of pendulum having time period of 1s ?**

(a) 1m (b) 0.75m (c) 0.50m (d) 0.25m

9. **When length of pendulum is doubled, ratio of new frequency to old frequency is:**

(a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{2}$

10. **Which of the following is the frequency of SHM?**

(a) $2\pi\omega$ (b) $\frac{\omega}{2\pi}$ (c) $\frac{2\pi}{\omega}$ (d) $\frac{1}{\omega}$

11. **The waves produced in a microwave oven have frequency:**

(a) 2450 Hz (b) 2450 kHz (c) 2450 MHz (d) 2450 GHz

12. **Tuning a radio Set is the best example of:** (FBISE-2016)

(a) Mechanical resonance (b) electrical resonance

(c) Magnetic resonance (d) musical resonance

13. **A simple pendulum is moved from the Earth to the Moon. How does it change the period of oscillations? (Acceleration due to gravity on moon = 1.6 m/s2)** (FBISE- 2017)

(a) The period is increased by factor $\sqrt{6}$ (b) The period is increased by factor four

(c) The period is decreased by factor $\sqrt{6}$ (d) The period remains the same

14. **A simple pendulum on earth has period of 6.0s. What is the approximate period of this pendulum on the moon where the acceleration due to gravity is roughly $\frac{1}{6}$ th of earth's gravity?** [FBISE (ON) - 2017]

(a) 15 s (b) 1.0 s (c) 36 s (d) 2.4 s

15. **To double the period of simple pendulum, its length must be:** [FBISE (ON) - 2018]

(a) Increased two times (b) Increased four times (c) Decreased by $\frac{1}{3}$ (d) Decreased $\frac{1}{2}$

Answers Key

1.	b	2.	b	3.	c	4.	d	5.	d
6.	b	7.	d	8.	d	9.	b	10.	b
11.	c	12.	b	13.	a	14.	a	15.	b

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 Choose the correct answer from the given options. (1 x 6 = 6)

SECTION – A

1. **The maximum velocity of 1 kg mass attached to a spring of force constant of 1 Nm^{-1} up to the displacement of 5 cm is:**
   (A) 1 ms^{-1} (B) 0.01 ms^{-1} (C) 5 ms^{-1} (D) 0.05 ms^{-1}
2. **Which of the following is the wave form of S.H.M?**
   (A) Sine wave (B) tan wave (C) square wave (D) saw tooth wave
3. **Which of the following is the length of the second pendulum?**
   (A) 99.3 mm (B) 99.3 cm (C) 99.2dm (D) 9.92 m
4. **A process whereby energy is dissipated from the oscillating system is called:**
   (A) resonance (B) un-damping (C) damping (D) reflection
5. **For a mass spring system placed on a smooth horizontal surface oscillating with amplitude 'x_0'. At what displacement from the mean position its kinetic energy is equal to its elastic potential energy?**
   (A) $\frac{x_0}{\sqrt{2}}$ (B) $\frac{x_0}{4}$ (C) x_0 (D) $\frac{x_0}{2}$
6. **A simple pendulum has a period T. what will be the percentage change in period if the amplitude is decreased by 6%?**
   (A) 6% (B) 3% (C) 1.5% (D) No change

Question.No.2 Give short answers of followings: (3 x 7 = 21)

SECTION – B

(i) What happens to the time period of the simple pendulum if its length is doubled?
(ii) Show that in SHM the acceleration is zero when the velocity is greatest.
(iii) How can you compare the masses of two bodies by observing their frequencies of oscillation when suspended by a spring.
(iv) A singer, holding a note of right frequency, can shatter a glass. Explain.
(v) Differentiate between free and forced oscillation.
(vi) A pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower?
(vii) A mass at the end of spring describes S.H.M with T = 0.40 s. Find out acceleration when the displacement is 0.04m.

Question.No.3 Extensive Questions. (13)

SECTION – C

(a) What is simple pendulum? Show that motion of simple pendulum is simple harmonic motion and derive expression for its time period. (6)
(b) A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly 9.8 m/s^2. When the clock is moved to higher altitude, it loses 80 s per day. Find the value of g at new location. (4)
(c) Give two applications in which resonance plays an important role. (3)

The End

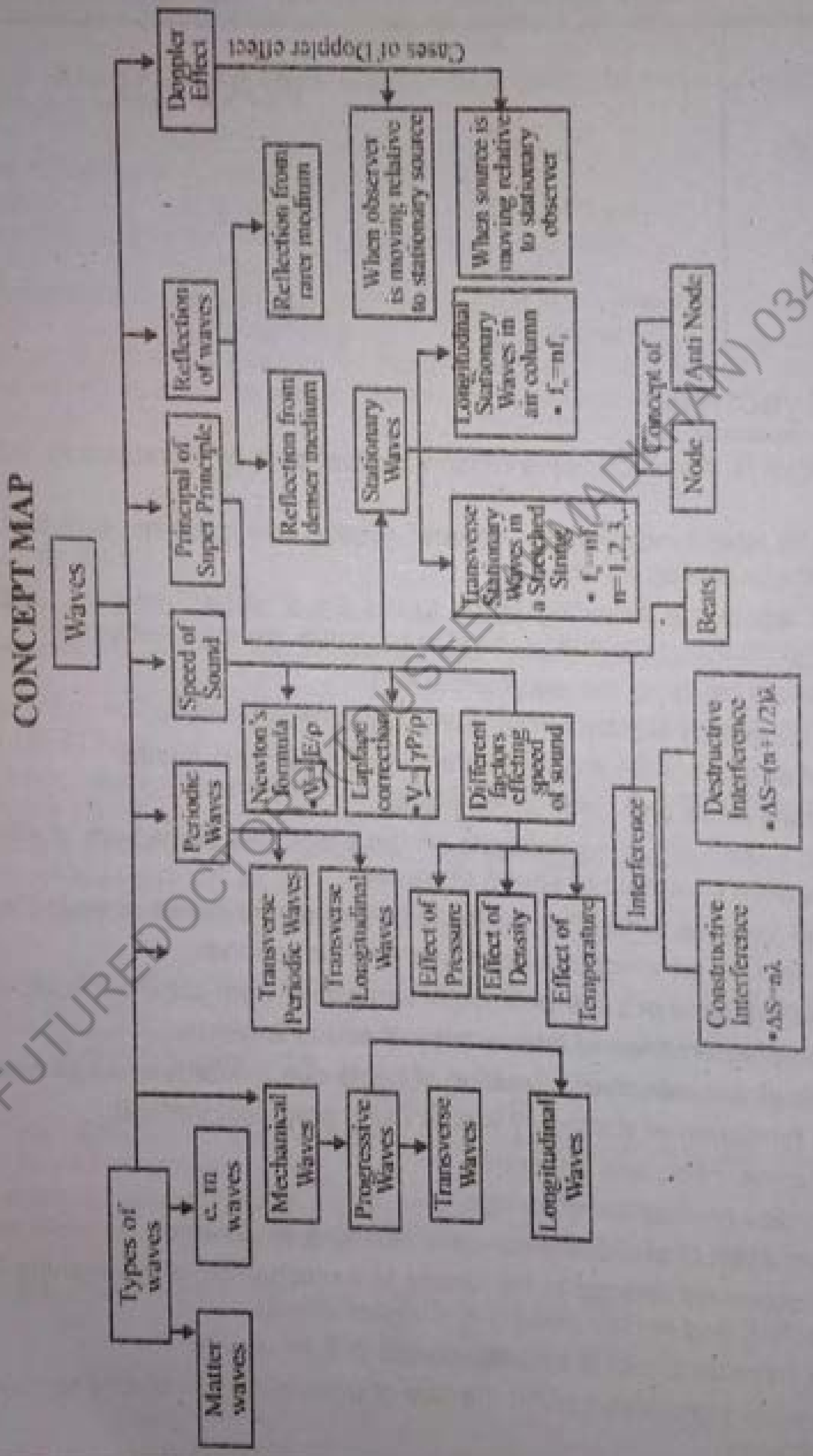
CHAPTER 8

WAVES

Learning Objectives

- Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and ripple tank?
- Demonstrate that mechanical waves require a medium for their propagation while electromagnetic waves do not.
- Define and apply the following terms to the wave model; medium, displacement, amplitude, period, compression, rarefaction, crest, trough, wavelength, velocity.
- Solve problems by using the equation: $v = f\lambda$.
- Describe that energy is transferred due to a progressive wave.
- Identify that sound waves are vibrations of particles in a medium.
- Compare transverse and longitudinal waves.
- Explain that speed of sound depends on the properties of medium in which it propagates and describe Newton's formula of speed of waves.
- Describe the Laplace correction in Newton's formula for speed of sound in air.
- Identify the factors on which speed of sound in air depends.
- Describe the principle of superposition of two waves from coherent sources.
- Describe the phenomenon of interference of sound waves.
- Describe the phenomenon of formation of beats due to interference of non -coherent sources.
- Explain the formation of stationary waves using graphical method.
- Define the terms, node and antinodes.
- Describe modes of vibration of strings.
- Describe formation of stationary waves in vibrating air columns.
- Explain the observed change in frequency of a mechanical wave coming from a moving object as it approaches and moves away (i.e. Doppler Effect).
- Explain that Doppler Effect is also applicable to E.M. waves.
- Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal structures.

Chapter No. 8


CONCEPT MAP
Waves
Types of waves
Matter waves
e.m waves
Mechanical Waves
Progressive Waves
Transverse Waves
Longitudinal Waves
Periodic Waves
Transverse Periodic Waves
Transverse Longitudinal Waves
Speed of Sound
Newton's formula
Laplace correction
Different factors effecting speed of sound
Effect of Pressure
Effect of Density
Effect of Temperature
Principal of Super Principle
Interference
Constructive Interference
Destructive Interference
Beats
Stationary Waves
Transverse Stationary Waves in a Stretched String
Longitudinal Stationary Waves in air column
Concept of
Node
Anti Node
Reflection of waves
Reflection from denser medium
Reflection from rarer medium
Doppler Effect
Cases of Doppler effect
When observer is moving relative to stationary source
When source is moving relative to stationary observer

Q.1 What are waves? Describe its types.

Ans: **Waves**

A wave is the mechanism by which energy is transferred from one place to another.

The mechanical waves transport energy without transporting matter.

- **A wave is a disturbance in a medium which causes the particles of the medium to oscillate.**

The nature of wave may be different, but the mechanism by which it transports energy is the same.

Types of Waves

Waves are of three types.

1) **Mechanical waves**

The waves which require a ***material medium*** *for their propagation are called mechanical waves.*

OR

The waves which propagate by the ***oscillation of material particles*** *are called mechanical waves*

For example

Water waves, sound waves, string waves etc.

2) **Electromagnetic waves**

The waves which are produced due to ***oscillations of electric and magnetic fields*** *and they require* ***no medium*** *for their propagation are called electromagnetic waves.*

- These waves propagate due to oscillations of electric and magnetic field.

For example

Radio waves, light waves, micro waves, x-rays etc.

3) **Matter waves**

The waves, associated with material particles in motion are called mater waves or de Broglie waves.

For example

Wave associated with the motion of electron.

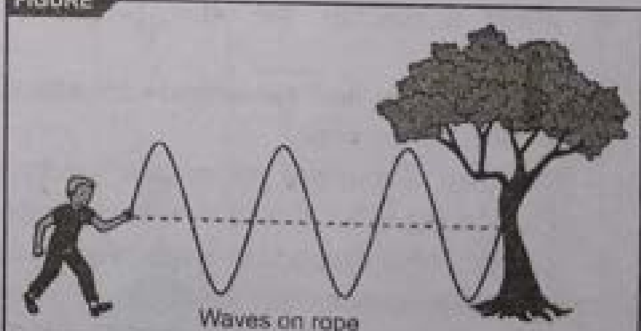
Q.2 What are periodic waves? How they are produced.

Ans: **Periodic Waves**

The waves which are produced by continuous and rhythmic disturbances in a medium are called periodic waves.

Continuous periodic waves can be produced by a source oscillating periodically in a medium. As the source oscillates, it disturbs the particles of the medium and set them to vibrate with same amplitude and frequency, causing wave motion.

FIGURE

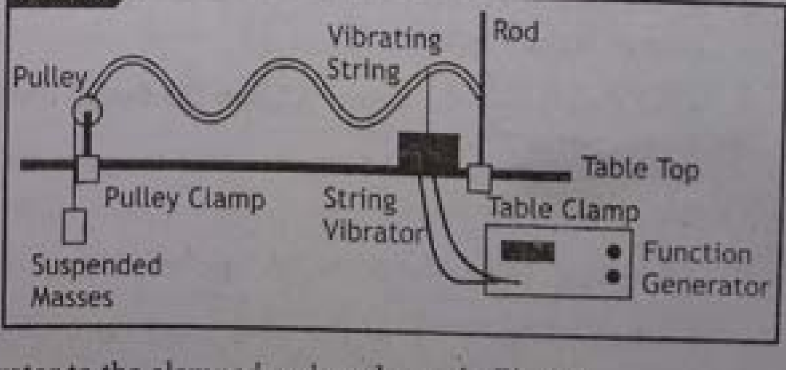


Waves on rope

i. A string whose one end is fastened to fixed support and the other free end is moved suddenly up and down periodically, a train of transverse waves will be produced in it, moving down the rope.

- Each portion of the rope moves up and down periodically as shown in fig 8.1.

FIGURE



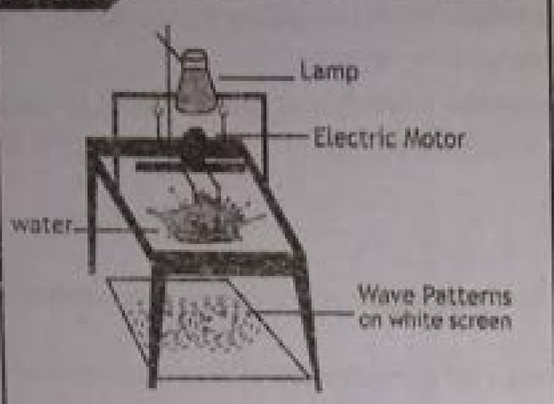
ii. When the frequency of the electromagnetic vibrator is increased then transverse pulses are produced in the string.

- These waves move down the cord from the vibrator to the clamped end as shown in Fig 8.2.

iii. The vibrating spherical dippers of ripple tank just touching the water surface driven by a small electric motor produce spherical periodic waves.

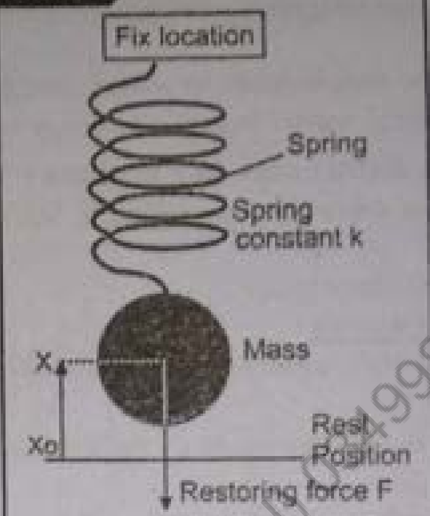
The pattern of waves obtained at any instant of time is shown in fig. 8.3.

FIGURE



Waves in ripple tank

FIGURE



iv. A pen attached to the oscillating mass-spring system will trace periodic waves on the paper which is moved at constant rate.

Q.3 What are progressive waves? And explain wave motion.

Ans: **Progressive Waves**

The waves which transfer energy by moving away from the source of disturbance are called progressive or travelling waves.

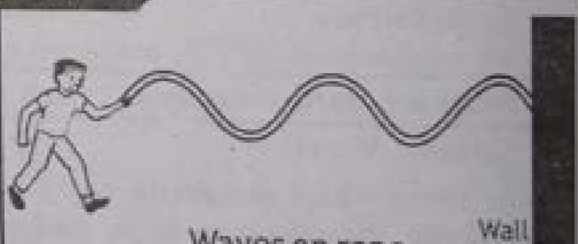
Example:

- Consider two persons holding the opposite ends of the rope.
- Suddenly one person gives a jerk to the rope, the disturbance in the rope produces a pulse which moves toward other person.
- When it reaches the other person it pushes his hand upward.
- So the ***energy and momentum*** transferred from one person to the other person.

This is an example of progressive wave.

The jerking of hand is its source and rope is the medium for the propagation of the waves.

FIGURE



Waves on rope

- The pebble is dropped in water produces disturbance and water waves Produce.

Motion of a Wave:

To understand that how a wave moves along a vibrating cord, consider the Fig (8.6).

i. At the time t = 0 the end "A" of the string is at the mean position while point "B" on the string is at a crest or highest point.

ii. After time $t = \frac{T}{4}$ the point "A" is at the trough while "B" is at the equilibrium position. During the same time the crest of the wave has moved to the right.

iii. After time $t = \frac{T}{2}$ the point "A" passes through the equilibrium position while "B" is at the trough and the crest of the wave is further moved to the right.

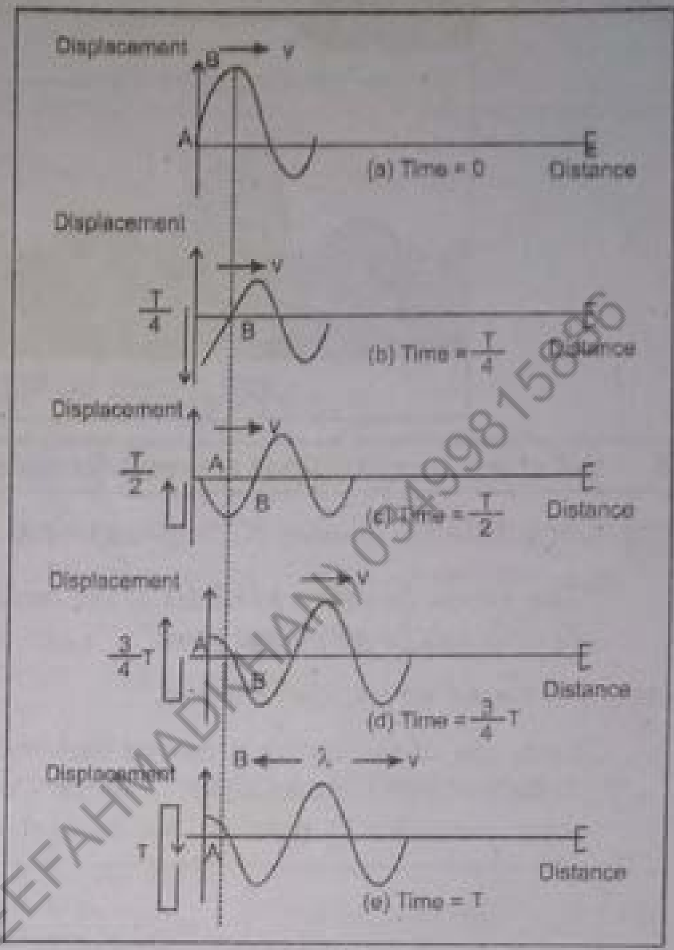
iv. At time $t = \frac{3T}{4}$ the point "A" is at the crest while "B" is at the mean position and the crest is further moved.

FIGURE

water waves

v. After time period "T" both the points "A" and "B" has completed exactly one oscillation and the crest moves through a distance equal to wavelength "λ".

- The particles of the medium oscillates with the same frequency when a wave passes through them.
- No particle has moved far from its initial position. Only the disturbance moves through the cord.
- The particles of the cord simply oscillate about their mean position due to which a wave moves through the medium.



Necessary Conditions for Wave Motion

- The following conditions are necessary for the propagation of waves.

i. The medium must be elastic.

ii. The particles of the medium should not be independent of each other, so that they could exert force on each other. Transverse and longitudinal wave can be setup in solid. In fluids, however, transverse wave die out very quickly and usually cannot be produced.

Classification of progressive waves

There are two kinds of progressive waves

(i) Transverse waves (ii) Longitudinal waves

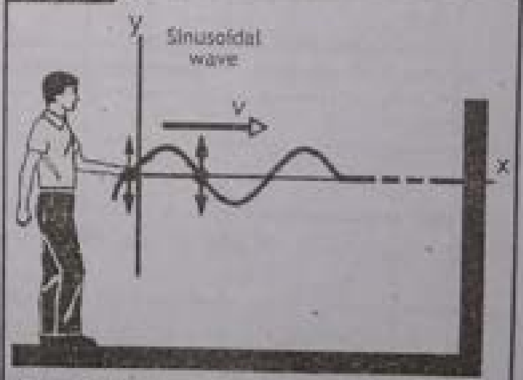
Q.4 What are transverse waves? Explain with examples.

Ans: **Transverse Waves**

The waves, in which particles of the medium vibrate perpendicular to the direction of propagation of waves, are called as transverse waves.

- ***Slinky spring is the soft spring which has small initial length but relatively large extended length.***
- Consider a horizontal spring system with its one end fixed. When the free end is moved from side to side, a pulse of wave having a displacement pattern, as shown in figure, will move along the spring.
- This shows that displacement of particles is perpendicular to the direction of propagation of wave, hence transverse waves are produced.

FIGURE



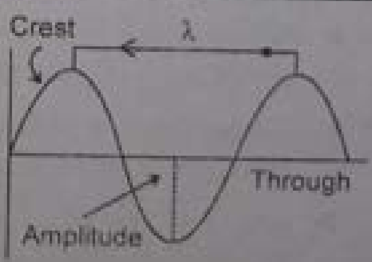
A typical string element (marked with a dot) moves up once and then down as the pulse passes. The element's motion is perpendicular to the wave's direction of travel, so the pulse is a transverse wave.

Wave Crest

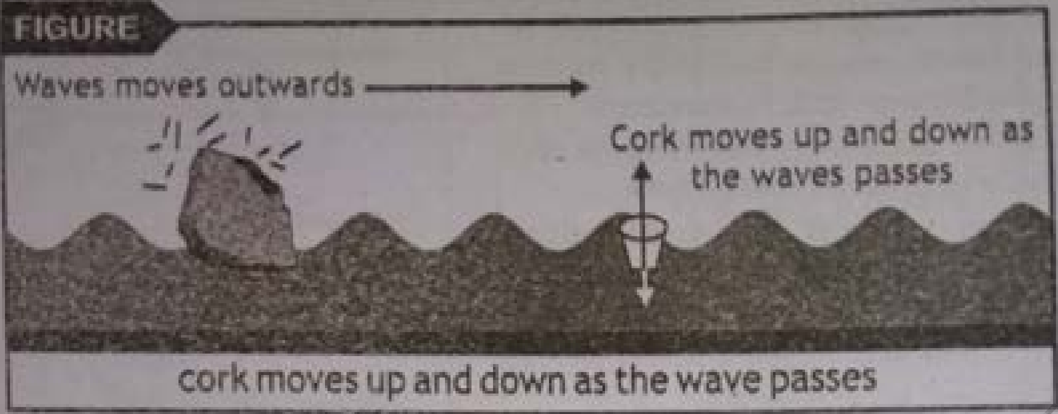
The portion of the wave above the mean level is called wave crest.

Trough:

The portion of the wave below its mean level is called wave trough.



Example: Consider a cord whose one end is attached to a strong support and the other end is free. If the free end is moved up and down periodically, a series of transverse waves is produced as shown is figure.



cork moves up and down as the wave passes

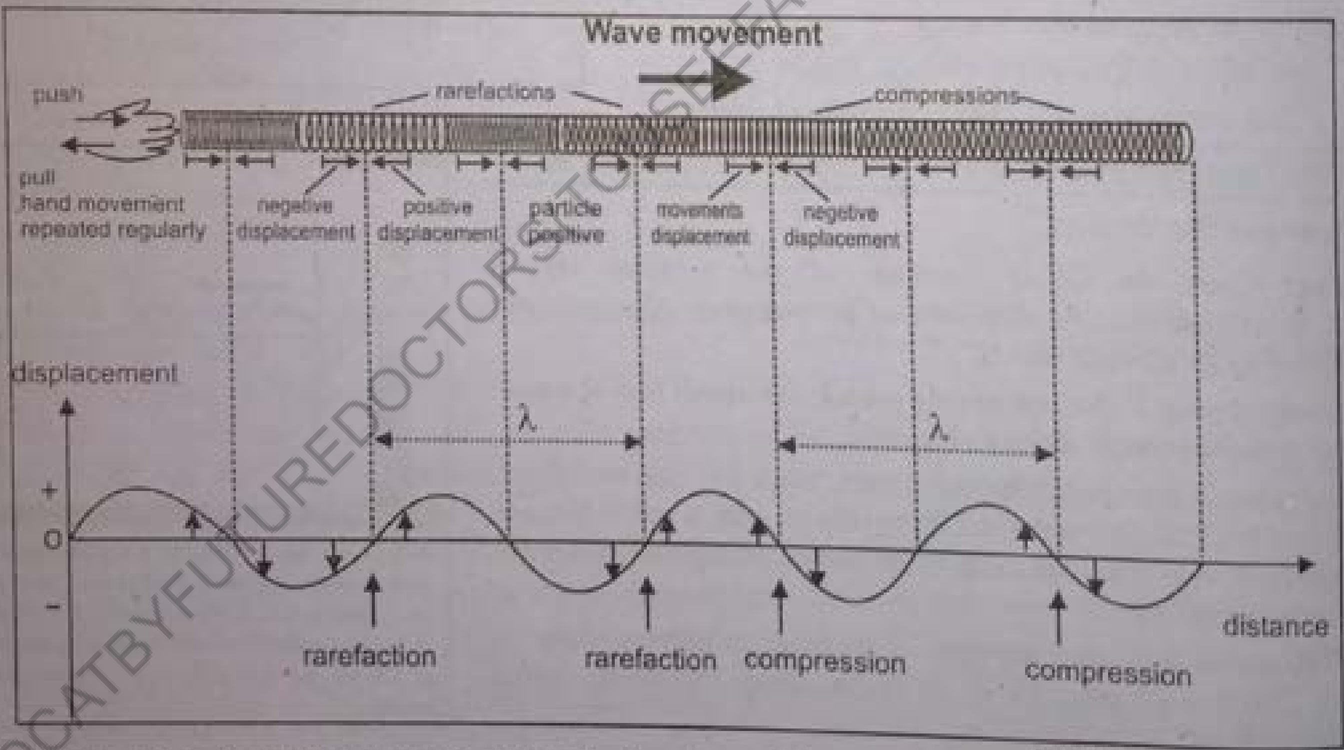
Q.5 What are longitudinal waves? Explain with example.

Ans: **Longitudinal Waves (Compressional waves)**

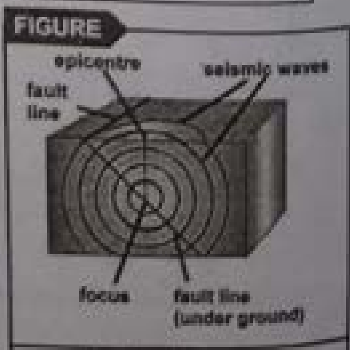
"The waves, in which particles of the medium vibrate along the direction of propagation of the waves, are called as longitudinal wave".

Examples: *sound waves, shock waves.*

If one end of the spring is moved back and forth, along the direction of the spring. Then the waves are produced in which displacement of the spring is along to the direction of propagation of wave and such waves are called longitudinal waves, as shown in figure.



- When spring is pushed towards right side then turn of spring come closer and compression produces.
- When spring is pulled towards left side then distance between turn of spring increases and rarefaction produces.
- other example of longitudinal waves are sound waves and shock waves produced during an earthquake which are also called seismic waves as shown in figure.



Seismic waves

MCQ's

1. Longitudinal waves are also known as:
(A) Stationary waves (B) Transverse waves (C) Compressional waves (D) Electromagnetic waves
2. The distance covered by wave in 1 second is:
(A) Wavelength (B) Wave number (C) Frequency (D) Wave speed
3. The waves associated with the particles in motion are called:
(A) Light waves (B) Electromagnetic waves (C) Matter waves (D) Both a and c
4. Distance between the crest and adjacent trough is:
(A) λ (B) $\frac{\lambda}{2}$ (C) $\frac{\lambda}{4}$ (D) 2λ
5. A wave which transfers energy in moving away from the source of disturbance is called:
(A) Progressive wave (B) travelling wave (C) both a & b (D) none of these
6. The time period of wave 0.2 s. Which of the following is its frequency?
(A) 2 Hz (B) 3 Hz (C) 4 Hz (D) 5 Hz
7. Two waves can interfere only if they have:
(A) Phase coherence (B) Same velocity (C) Different frequencies (D) Different wavelength
8. The louder the sound, the greater will be its:
(A) Amplitude (B) Wavelength (C) Speed (D) Frequency
9. If 20 waves pass through a medium in 1 sec. with a speed of 20m/s then the wavelength is:
(A) 20 m (B) 400m (C) 40m (D) 1 m
10. The waves that do not need any medium for their propagation are called:
(A) Mechanical waves (B) Matter waves (C) Electromagnetic waves (D) Compressional waves
11. Half wavelength corresponds to:
(A) 0° (B) 90° (C) 180° (D) 360°
12. Transverse waves are distinguished from longitudinal waves by the property:
(A) Interference (B) Diffraction (C) Reflection (D) Polarization
13. The profile of periodic waves generated by a source executing S.H.M. is represented by a:
(A) Circle (B) Sine curve (C) Tangent curve (D) Cosecant curve
14. If 332 waves pass through a medium in one second with speed of 332 ms^{-1} then wavelength will be:
(A) 7m (B) 332m (C) 664m (D) 1m

Answers Key

1. C	2. D	3. C	4. B	5. C	6. D	7. A	8. A	9. D	10. C	11. C	12. D
13. B	14. D										

Q.6 Explain the terms wave speed, frequency of waves, time period, wavelength, amplitude and intensity.

Ans: Characteristics of Wave

A wave is specified by the following parameters;

wave speed, frequency, time period, phase, wavelength, amplitude and intensity.

i. **Wave Speed**

The speed of a wave is defined as the distance traveled by a wave per unit time. The speed of a wave depends upon the type of wave as well as the properties of the medium. For example the speed of a transverse wave pulse in an elastic stretched string or spring is given by:

$$v = \sqrt{\frac{T \times L}{M}} \longrightarrow (1)$$

$$V = \sqrt{\frac{T}{M/L}} \quad \ldots\ldots\ldots\ldots (2)$$

Where M is the mass, L is the length of the string respectively and 'T' is the tension in it.

- If m is the mass per unit length of the string then putting $\frac{M}{L} = m$ in equation (2)

$$v = \sqrt{\frac{T}{m}} \longrightarrow (3)$$

- Thus the speed of transverse wave in a well stretched and thin string is greater as compared to a loosely and thick one.

The speed of a compressional or longitudinal wave depends upon the modulus of elasticity E and density ρ of the medium, which is given by

$$v = \sqrt{\frac{E}{\rho}} \longrightarrow (4)$$

Hence longitudinal waves travel more slowly in gases than in solids because gases are more compressible and hence having a smaller elastic modulus E.

ii. **Frequency of Waves**

The number of waves passing through a certain point in unit time is called frequency of the wave.

(OR)

The number of cycles completed in one second is called frequency.

$$\text{Frequency} = \frac{\text{Number of waves passing through a point}}{\text{Time taken by waves}}$$

iii. **Time Period of Wave**

The time during which a wave passes through a certain point is called the time period of the wave.

(OR)

The time required to complete one cycle is called time period.

It is equal to the reciprocal of the frequency.

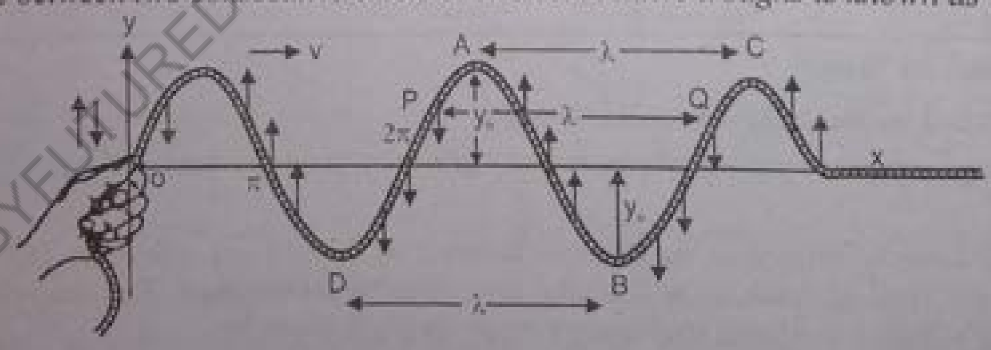
$$T = \frac{1}{f}$$

iv. **Wave Length λ**

The distance between the two successive particles which are exactly in the same state of vibration is called wavelength indicated by "λ".

(OR)

The distance between two consecutive crests or two consecutive troughs is known as wave length.



- In the fig 8.12, the point "A" and "C" or "P" and "Q" or "B" and "D" are in the same phase, because they are in the same state of vibration.

A very important relation exists between wavelength and frequency. One wavelength of a wave is sent out by the wave generating source as it completes one vibration. The time for 1 vibration is called time period T. The wave covers distance ℓ in time period T and we find the speed v of the wave as

$$v = \frac{\ell}{t} = \frac{\text{Distance covered}}{\text{Time Taken}}$$

Here $v = \frac{\lambda}{T}$

$v = \left(\frac{1}{T}\right)\lambda$ Putting $\frac{1}{T} = f$

$\Rightarrow$ $v = f\lambda$

For one wave
$\ell = \lambda$
Time = time period(T)

This relation is true for all waves.

v. **Amplitude of Wave**

The maximum displacement covered by a vibrating particle from its equilibrium position on either side is called amplitude.

(OR)

The amplitude is the maximum displacement of point in a crest or in a trough of the wave.

vi. **Intensity of Wave**

The amount of energy transmitted per second per unit area placed perpendicular to the direction of propagation of waves is called intensity of the waves indicated by I.

$$\text{Intensity} = \frac{\text{Energy}}{\text{Time X Area}}$$

Intensity $\propto$ (amplitude)^2

S-I unit of intensity is $\frac{J}{sm^2} = Wm^{-2}$ $\left[\because \frac{J}{s} = W\right]$

Q.7 What are the factors on which the speed of sound depends upon? What was Newton's formula for the speed of sound? What was drawback in it, how it was corrected by Laplace?

Ans. Speed of Sound in Air

The distance covered by sound waves per unit time is called speed of sound waves.

Sound waves are longitudinal waves and their speed depends upon

- Elasticity of the medium (E)
- Density of the medium (ρ)

If E be the modules of elasticity (reciprocal of compressibility) and ρ be the density of the medium, then the speed v can be expressed as,

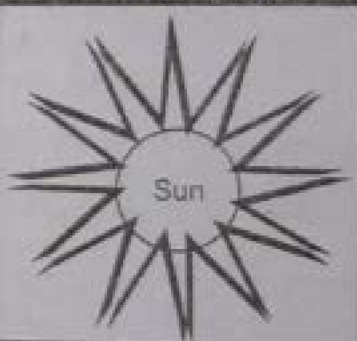
$$v = \sqrt{\frac{E}{\rho}}$$

- Speed of sound in solids is much greater than in gases.

Reason

- Since molecules are closer in solids than in the gases, so they respond more **quickly** to a disturbance.
- In other words, so the speed of sound in gases is smaller than in solids because the gases are **more compressible** and thus have smaller modulus of elasticity.

Do you Know?



Explosions, occurs on the surface of the sun due to fusion reactions but we can't hear, why?

Table

Speed of sound in different media

Medium	Speed ms^{-1}
Solids at 20°C	
Lead	1320
Copper	3600
Aluminium	5100
Iron	5130
Glass	5500
Liquids at 20°C	
Methanol	1120
Water	1483
Gases at S.T.P.	
Carbon dioxide	258
Oxygen	315
Air	332
Helium	972
Hydogen	1286

Newton's formula for the speed of sound in air

If E be the modulus of elasticity and ρ be the density of the medium, then the speed v is

$$v = \sqrt{\frac{E}{\rho}} \quad \text{———} \quad (1)$$

Calculation of modulus of elasticity

Newton's Assumption

- In order to calculate the elastic modulus for air, Newton assumed that *the temperature of the air during a compression remains constant.* (i.e. an isothermal change). So from Boyle's law:

PV = constant

- When the pressure increases from P to P+ΔP
- then the volume decreases from V to V − ΔV.

According to Boyle's Law,

$$P_1V_1 = P_2V_2$$

$$PV = (P + \Delta P)(V - \Delta V)$$

$$PV = PV - P\Delta V + \Delta P\, V - \Delta P\Delta V \quad \_\_\_\_\_\_ (2)$$

Since changes ΔP and ΔV represent the small. So their product ΔPΔV can be neglected. Hence above equation becomes,

For Your Information	
Values of constant	
Types of Gas	γ
Monoatomic	1.67
Diatomic	1.40
Polyatomic	1.29

OR $0 = -P\Delta V + V\Delta P$

$$P\Delta V = V\Delta P$$

OR $P = \frac{V\Delta P}{\Delta V}$

OR $P = \frac{\Delta P}{\Delta V/V}$ [where $= \frac{\Delta P}{\Delta V/V} = \frac{\text{volumetric stress}}{\text{volumetric strain}} = E$]

OR $P = E$ ______ (3)

So equation (1) becomes

$$v = \sqrt{\frac{P}{\rho}}$$

Putting $P = \rho_m g h$

$$v = \sqrt{\frac{\rho_m gh}{\rho}} \quad \text{-----------} (3)$$

ρ_m is density of mercury

$\rho_m = 13.6\ g/cm^3$

$g = 980\ cm/s^2$

$h = 76\ cm$

$$v = \sqrt{\frac{13.6 \times 980 \times 76}{0.001293}}$$

$v = 281\ m/s$

- The experimental value of speed of sound is **332m/sec.**
- The theoretical value is about **16%** less than the experiment value.

Drawback in Newton's Formula

- During a compression the temperature of air does not remain constant but *increases* .i.e. it is an adiabatic change.

Laplace Correction

- Laplace assumed that compressions and rarefactions in air take place so rapidly that heat of compression does not able to transfer to the neighboring cooler regions.
- Therefore the temperature of the medium rises in compression and fall in rarefaction does not remain constant. i.e. it is an **adiabatic change**.

$$PV^{\gamma} = \text{constant}$$

Where $\gamma = \frac{C_p}{C_v} = \frac{\text{molar spesific heat at constant pressure}}{\text{molar spesific heat at constant volume}}$

When the pressure increases from P to P+ΔP then the volume decreases from V to (V − ΔV), so

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma}$$

$$PV^{\gamma} = (P+\Delta P)\, V^{\gamma} \left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

$$P = (P+\Delta P)\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

By Binomial expansion $\quad (1+x)^n = 1 + nx + n\frac{(n-1)x^2}{2!} + \ldots\ldots\ldots\ldots$

$$P = (P+\Delta P)\left[1 - \gamma\frac{\Delta V}{V} + \text{neglecting square and higher powers of } \frac{\Delta V}{V}\right]$$

$$P = (P+\Delta P)\left(1 - \gamma\frac{\Delta V}{V}\right)$$

$$P = P - \gamma P\frac{\Delta V}{V} + \Delta p - \gamma \Delta P\frac{\Delta V}{V}$$

Since ΔP and ΔV both are small so the term $\gamma \Delta P \frac{\Delta V}{V}$ can be neglected

$$P = P - \gamma P\frac{\Delta V}{V} + \Delta p$$

$$P - P + \gamma P\frac{\Delta V}{V} = \Delta p$$

$$\gamma P\frac{\Delta V}{V} = \Delta p$$

$$\gamma P = \frac{\Delta P}{\Delta V/V}$$

Where $\frac{\Delta P}{\Delta V/V} = \frac{\text{volumetric stress}}{\text{volumetric strain}} = E$ (modulus of elasticity)

$$\gamma P = E \text{ ---------- } (4)$$

Putting value from equation (4) in equation (1) we get

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Putting $P = \rho_m gh$

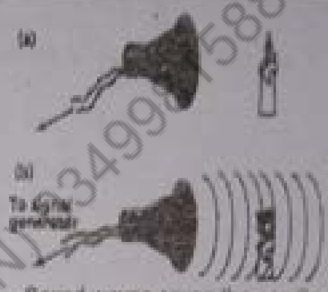
$$v = \sqrt{\frac{\gamma \rho_m gh}{\rho}}$$

$$v = \sqrt{\frac{1.42 \times 13.6 \times 980 \times 76}{0.001293}}$$

$$v = 33310 \text{ cm/s} = 333.10 \text{ m s}^{-1}$$

Which is close to the experimental value of 332 m s^{-1}.

Tidbits



Sound waves cause the candle flame to flicker.

For Your Information
Range of Hearing

Organisms	Frequencies (Hz)
Dolphin	150 - 150,000
Bat	1000 - 120,000
Cat	60 - 70,000
Dog	15 - 50,000
Human	20 - 20,000

Q.8 What is effect of density, moisture, pressure, temperature and wind on speed of sound?

Ans. Effect of Various Factors on Speed of Sound in air

Sound waves are compressional mechanical waves propagating in gas or air with a speed of

$$v = \sqrt{\frac{\gamma P}{\rho}} \longrightarrow (1)$$

The following factors affect the speed of sound in a gas.

1. **Density:** The speed of sound in a gas varies inversely as the square root of the density of the gas.

$$v \propto \frac{1}{\sqrt{\rho}}$$

2. **Moisture:** The presence of moisture in the air reduces the resultant density of air.
   - Therefore the speed of sound increases with humidity.
   - Hence the velocity of sound in damp air is greater than its value in dry air.

3. **Pressure:** For one mole of an ideal gas having volume V and pressure P at temperature T, we can write

$$PV = nRT \qquad (\because n = 1)$$

$$\Rightarrow \quad PV = RT$$

$$V = \frac{RT}{P} \longrightarrow (2)$$

Where R is a general gas constant.
If m is the mass of the gas then its density is

$$\rho = \frac{m}{V} \longrightarrow (3)$$

Putting value of V from Eq (2) in (3)

$$\rho = \frac{m}{RT/P}$$

$$\rho = \frac{mP}{RT}$$

$$\frac{P}{\rho} = \frac{RT}{m} \longrightarrow (4)$$

Putting value of $\frac{P}{\rho} = \frac{RT}{m}$ from equation (4) in equation (1)

$$v = \sqrt{\frac{\gamma RT}{m}} \longrightarrow (6)$$

- Hence the speed of sound in a gas is independent of its pressure.

4. **Temperature:** For solids and liquids the change in the speed of sound with temperature is very small and can be neglected.
   - But for gases the change in speed of sound with temperature is very large.
   - The increase in speed of sound with temperature in gas is about 0.61 m s^{-1} for each 1°C rise in temperature.

   Since the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma RT}{m}}$$

Therefore, $v \propto \sqrt{T}$

That is the speed of sound in a gas is directly proportional to the square root of the absolute temperature of the gas.

If v_o and v are the speeds of sound at temperatures T_o and T respectively then we can write.

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} \longrightarrow (1)$$

As $T_o = (0°C + 273)K$ and $T = (t°C + 273)K$

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} = \sqrt{\frac{(t°C + 273)K}{273K}} = \sqrt{\frac{273}{273} + \frac{t}{273}}$$

$$\frac{v}{v_o} = \sqrt{1 + \frac{t}{273}}$$

Or $$\frac{v_t}{v_o} = \left(1 + \frac{t}{273}\right)^{1/2}$$

Applying Binomial theorem and neglecting higher power terms we get

$$(1 + x)^n = 1 + nx + n\frac{(n-1)x^2}{2!} + \ldots\ldots\ldots\ldots$$

$$\frac{v}{v_o} = \left[1 + \frac{t^\circ C}{2\times 273}\right]$$

$$v = v_o + \frac{v_o t^\circ C}{546}$$

Since at 0°C, $v_o = 332\ ms^{-1}$.

$$v = v_o + \frac{332 t^\circ C}{546}$$

$$v = v_o + 0.61\ t^\circ C$$

$$v_o = v - 0.61\ t^\circ C$$

Thus the increase in the speed of sound for each degree rise in temperature is $0.61\ ms^{-1}$.

5. **Wind:** If the air carrying sound waves, is itself moving i.e., there is wind.

- The speed of sound in the direction of wind relative to the ground is $(v + v_w)$ i.e increases
- While the speed of sound against the wind is $(v - v_w)$ and decreases, where v_w is the speed of wind and v is the speed of sound.

Assignment 8.1:

Compute the speed in sea water, if its density is 1025 kg m^{-3} and Elastic modulus is 2.1×10^9 Nm^{-2}.

Given Data: $\rho = 1025\ kgm^{-3}$
$E = 2.1 \times 10^9\ Nm^{-2}$
Speed of sound v = ?

Solution: The speed of sound in water is;

$$V = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2.1\times 10^9}{1025}} = 1430\ m/s$$

MCQ's

1. The speed of sound in air depends upon:
(A) Temperature (B) Humidity (C) Density (D) All

2. If the speed of sound in air at given pressure is 'v', then doubling the pressure, the new speed becomes equal to:
(A) 2v (5) 0.5v (C) v (D) 4v

3. Newton's formula for the velocity of sound in gas /air is related as:
(A) $v = \sqrt{\rho E}$ (B) $v = \sqrt{\frac{\rho}{E}}$ (C) $v = \sqrt{\frac{P}{\rho}}$ (D) $v = \rho E$

4. The speed of sound is greater in solids due to their high:
(A) Density (B) Pressure (C) Temperature (D) Elasticity

5. The speed of sound in air does not depend upon:
(A) Density (B) Pressure (C) Humidity (D) Temperature

6. Increase in the speed of sound for 1 °C rise in temperature is:
(A) 0.61 m/s (B) 0.81cm/s (C) 6 m/s (D) 6.1m/s

7. The speed of sound is greater in:
(A) Air (B) Steel (C) Ammonia (D) Water

8. Appropriate range of audible frequencies for younger person:
(A) 20 - 200 Hz (B) 20 - 2000 Hz (C) 20 - 20,000 Hz (D) 2000 – 20000 Hz

9. Increase in speed of sound at t°C is given as:
(A) $v_t = v_o - 61t$ (B) $v_t = v_o + 61t$ (C) $v_t = v_o - 0.61t$ (D) $v_t = v_o + 0.61t$

10. The velocity of sound in vacuum:
(A) 332 m/s (B) 333 m/s (C) 280 m/s (D) Zero

11. The pressure exerted by the column of mercury 76 cm high and 0°C called:
(A) 1 atm (B) 1 N/m^2 (C) 1 pascal (D) None

12. Which of the following are the dimensions of elastic modulus 'E'?
(A) $[ML^{-1}T^{-2}]$ (B) $[ML^{-2}T^{-2}]$ (C) $[MLT^{-2}]$ (D) $[ML^{2}T^{-2}]$

13. Sound travel fasters in:
(A) CO_2 (B) H_2 (C) O_2 (D) He

14. Sound waves cannot be:
(A) Reflected (B) Refracted (C) Polarized (D) diffracted

15. Laplace's expression for speed of sound in air is:
(A) $v = \sqrt{\frac{P}{\rho}}$ (B) $v = \frac{P}{\rho}$ (C) $v = \sqrt{\frac{\gamma P}{\rho}}$ (D) $v = \sqrt{\frac{\rho}{\gamma P}}$

16. The speed of sound in air would become double than its speed at 10°C at a temperature of:
(A) 313°C (B) 586°C (C) 859°C (D) 899°C

17. The correct relation between speed of sound and absolute temperatures is:
(A) $\frac{v_1}{v_0} = \sqrt{\frac{T}{T_0}}$ (B) $\frac{v_0}{v_1} = \frac{T}{T_0}$ (C) $\frac{v_0}{v_1}\sqrt{\frac{T}{T_0}}$ (D) $\sqrt{\frac{v_0}{v_1}} = \frac{T}{T_0}$

18. Speed of sound in aluminium at 20 °C is:
(A) 3600 m/s (B) 5100 m/s (C) 5130 m/s (D) 5500 m/s

19. Speed of sound in copper is:
(A) 36000 m s^{-1} (B) 3600 m s^{-1} (C) 3500 m s^{-1} (D) 3400 m s^{-1}

20. The ration of $\frac{C_p}{C_v} = \gamma$ for diatomic gas like air is:
(A) 1.40 (B) 1.30 (C) 1.29 (D) 1.67

21. The velocity of sound is maximum at 20°C in:
(A) Lead (B) Copper (C) Glass (D) Iron

22. In which medium the speed of sound is greater?
(A) Oxygen (B) Air (C) Water (D) Copper

23. The louder the sound, the greater will be its:
(A) Speed (B) Frequency (C) Amplitude (D) Wavelength

Answers Key

1. D	2. C	3. C	4. D	5. B	6. A	7. B	8. C	9. D	10. D	11. A	12. A
13. B	14. C	15. C	16. C	17. A	18. B	19. B	20. A	21. C	22. D	23. C	

Q.9 State and explain principle of super position.

Ans: **Superposition Principle**

If a particle of medium is simultaneously acted upon by a number of waves then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called superposition principle.

Let the displacement of the individual waves be $y_1, y_2, y_3, \ldots y_n$.

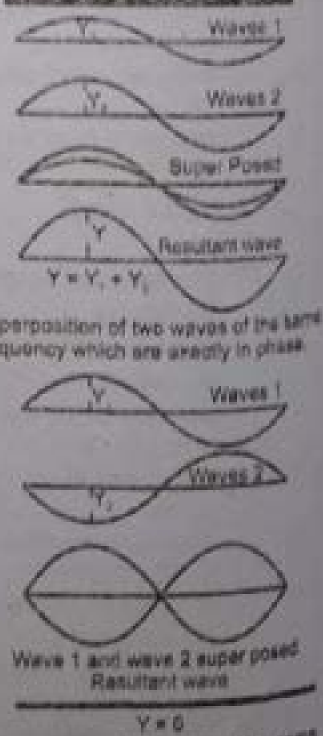
Then by super position principle the resultant displacement be.

$$y = y_1 + y_2 + y_3 + \ldots\ldots + y_n$$

Consider the two waves coming from opposite direction through a coil of spring, as shown in figure.

- When theses waves combine with each other then during the time of overlapping, the displacements of waves are added up as shown in figure (c).
- After having crossed each other they again adopt their original shapes and continue their motion along the spring in their respective directions as shown in figure (d).
- When crest of one wave falls on the trough of other wave then waves cancel the effect of each other and resultant displacement becomes zero.

For Your Information



Superposition of two waves of the same frequency which are exactly out of phase

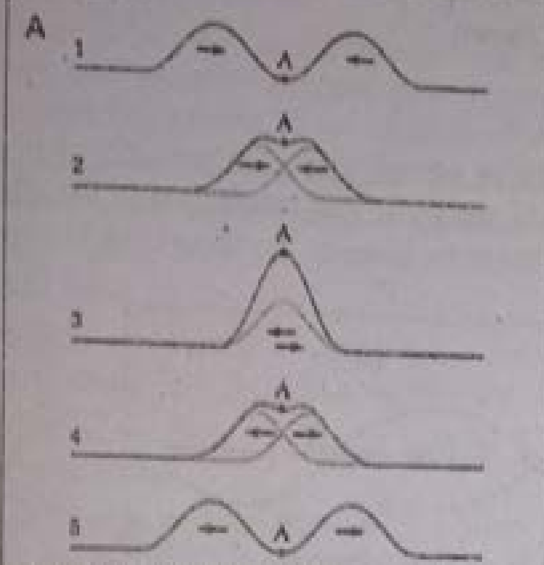
Cases of Superposition principle

(i) Two waves having same frequency and traveling in the same direction produce the phenomenon of **interference**.

(ii) Two waves of slightly different frequencies and traveling in the same direction produce **beats**.

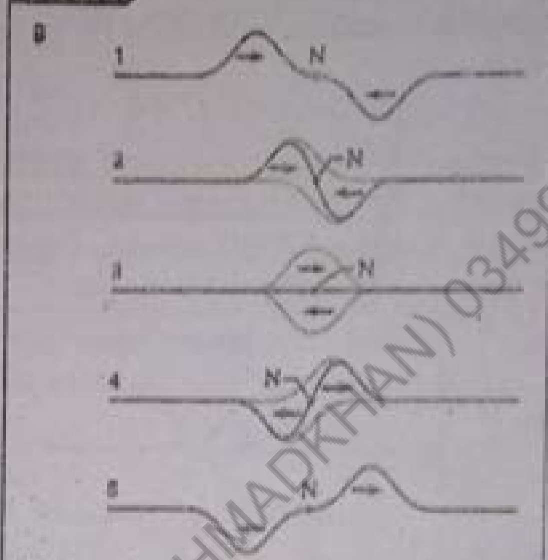
(iii) Two waves of same frequency traveling in opposite direction produce **stationary waves**.

FIGURE



Constructive interference occurs when two equal, in-phase waves meet.

FIGURE



Destructive interference occurs when two equal, out-of-phase waves meet.

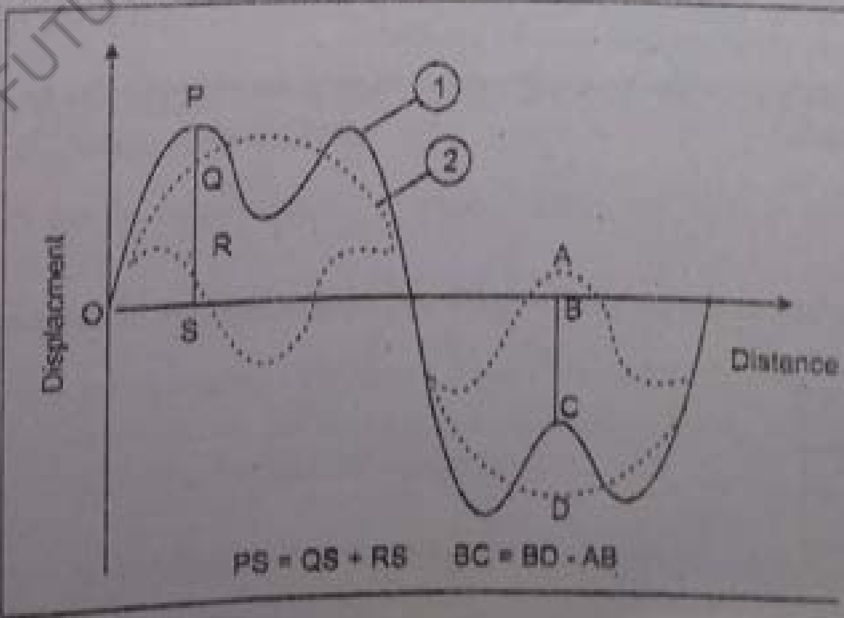
Q.10 What is interference of waves? Discuss types of interference.

Ans: **Interference of Waves**

"The effect produced by the superposition of waves from two coherent sources, passing through the same region is known as interference".

- The two sources are said to be **coherent** if the phase difference between the sources is constant.
- In a region where wave trains from coherent sources meet, superposition occurs, giving reinforcement of the waves at some points and cancellation at the others. The resulting effect is called an **interference** pattern.

Coherent sources have a constant phase difference which means that they must have the same frequency and amplitude. Interference is of two types.



i. **Constructive Interference:** "When two waves arrive at the same place at the same time in phase then they reinforce each other and constructive interference occurs".

- The resultant displacement at point of superposition is equal to the vector sum of the individual displacement due to each wave at that point.

"Hence in case of transverse waves constructive interference takes place when crest of one wave meets with the crest of the other wave while trough of one wave meet with trough of the other."

- The amplitude of the resultant wave in Fig (8.14) is shown
  (i) PS = QS + RS
  (ii) BC = BD − AB
- In case of longitudinal waves constructive interference occurs when compression of one wave meets with compression of the other wave and rarefaction of one wave with rarefaction of the other.

ii. **Destructive Interference:** "If two waves arrive at the same place at the same time but are out of phase (180°), then destructive interference takes place".

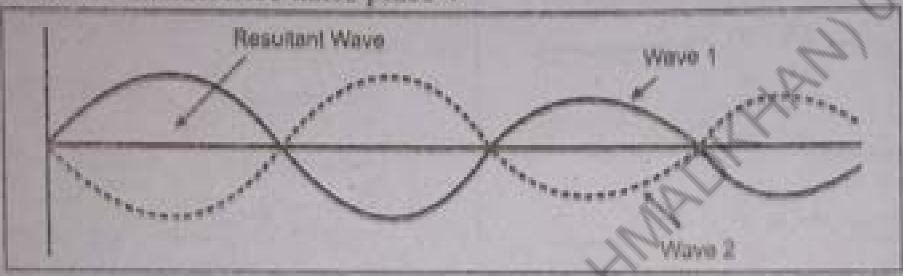


Fig : 8.15

The amplitude of the resultant wave is equal to the difference between the amplitudes of the individual waves. $y = y_1 - y_2$ (1)

- Destructive interference occurs when crest of one wave meets with trough of the other wave.
- "In case of sound waves (compressional waves) destructive interference takes place when compression of one wave meets with rarefaction of the other wave".

Conditions for Interference:

From the above discussion we conclude that the following conditions are necessary for constructive and destructive interference.

i. The two waves must be phase coherent.
ii. They must arrive at the same place at the same time.
iii. The two waves must be traveling in the same direction.
iv. The principle of linear super position must be satisfied.

(a) For constructive interference the path difference between the waves must be either zero or integral multiple of wavelength λ.

i.e. $d = 0, \lambda, 2\lambda, 3\lambda \ldots\ldots$

$\Rightarrow \boxed{d = m\lambda}$

(b) For destructive interference the path difference is odd integral multiple of half wavelength $\left(\frac{\lambda}{2}\right)$.

i.e. $d = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2} \ldots\ldots\ldots\ldots$

$\Rightarrow \boxed{d = \left(m + \frac{1}{2}\right)\lambda}$

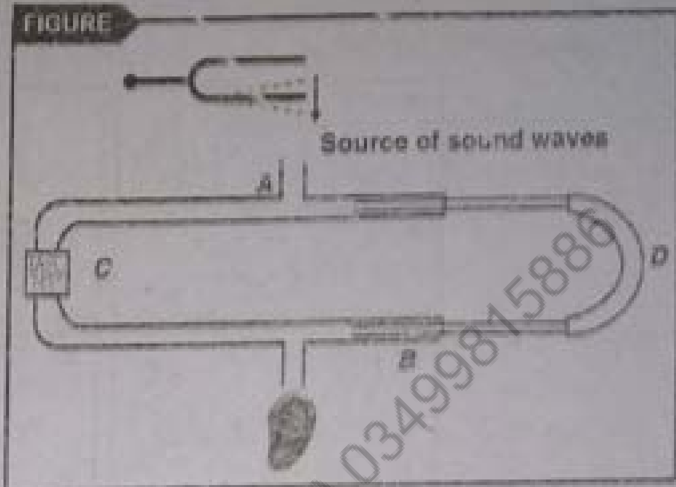
Where m = 0, 1, 2, ……

Q.11 Explain interference of the sound waves.

Ans: <u>**Interference of sound waves**</u>

Definition:

"The superposition of two sound waves of same frequency and travelling in same direction produce interference".



- The effect produced by the superposition of sound waves from two coherent sources, passing through the same region is known as interference of sound waves.
- When two coherent waves arrive at the same place at the same time, they reinforce each other and interference takes place.

Activity: The interference of sound waves can be demonstrated with the apparatus as shown in fig 8.16. A vibrating tuning fork is held above the tube. The sound waves entering the tube splits into two parts.

- Half the intensity goes through the path ACB to the point B and the remaining half through the path ADB to the point B.
- The two parts of sound waves re-unite at outlet B, which can be heard by ear.
- If the path ACB and ADB are equal, the two waves arrive at B are in phase and constructive interference takes place. As a result loud sound is heard.
- Now, if the sliding tube is drawn out and the path ADB becomes longer than the path ACB, then the sound waves arriving at B via D will be different coming from via C.
- When the path difference between the waves is half a wavelength $\left(\frac{\lambda}{2}\right)$, they interfere destructively and as a result no sound is heard at B.
- If the rubber portion of tube is pinched at C, so as to stop the sound waves through C, then the ear will again hear the sound.
- This proves that the silence is due to destructively interference of the two sound waves.

Q.12 Define and explain beats?

Ans: **Beats:**

"The periodic variations between maximum and minimum loudness of sound waves are called beats".

(OR)

<u>When two tuning forks of slightly different frequencies are sounded together, the rise and fall in the intensity of sound per second are called beats.</u>

<u>**Beat frequency:**</u>

The difference between the frequencies of two waves is called beat frequency.

It is denoted by N.

$$[N = f_1 - f_2]$$

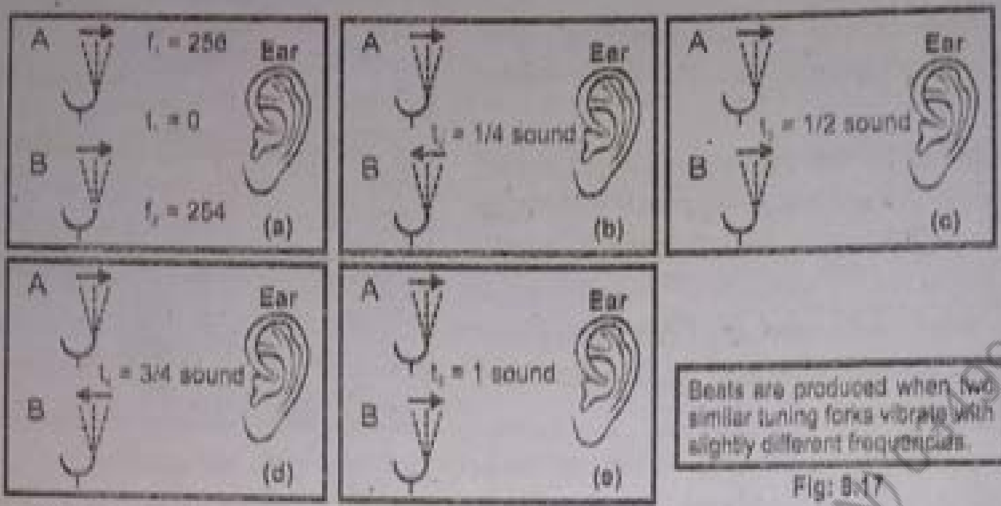
<u>**Explanation:**</u>

Take two audio-frequency generators say tuning forks of frequency 256 Hz. Slightly load the prong of fork B with a little wax, so that its frequency becomes 254 Hz. The two tuning forks are now placed at equal distance from the ear and sounded simultaneously.

i. <u>**At the time $t_1 = 0$**</u>

Both the forks are in phase and sending compressions, indicated by the right ward pointing arrows as shown in fig 8.17 (a).

- The two compressions will arrive at the ear together and interfere constructively, due to which loud sound is heard.
- As the time goes on the fork B vibrates with a slightly lower frequency than A so it will begin to fall behind.



Beats are produced when two similar tuning forks vibrate with slightly different frequencies.

Fig: 8.17

ii. **After time t_2 =1/4 seconds**

- Tuning fork A will complete 64 vibrations (compression)
- Tuning fork B will complete 63 vibrations (Rarefaction)) Fig 8.17 (b).
- The compression and rarefaction at the ear will cancel each other and faint sound is heard.

iii. **After time t_3 = 1/2 second**

- Tuning fork A will complete 128 vibrations (compression)
- Tuning fork B will complete 127 vibrations (compression) Fig 8.17 (c).
- The two tuning forks will be sending compressions together and thus again loud sound will be heard.

iv. **After time t_4= 3/4 second**

- Tuning fork A will complete 192 vibrations (compression)
- Tuning fork B will complete 190 vibrations (Rarefaction) fig 8.17 (d).
- When a compression from A and rarefaction from B reach the ear at the same time, they will cancel each other and faint sound is heard.

v. **After t_5 =1second**

- Tuning fork A will complete 256 vibrations (compression)
- Tuning fork B will complete 254 vibrations (compression) fig 8.17 (e).
- Both tuning forks will sending compression so again loud sound is heard by the ear.
- Thus in one second two beats are produced while the difference in frequencies of the forks is also two.
- Therefore we conclude that the number of beats per second is equal to the difference between the frequencies of the two forks.

$$N = f_1 - f_2$$

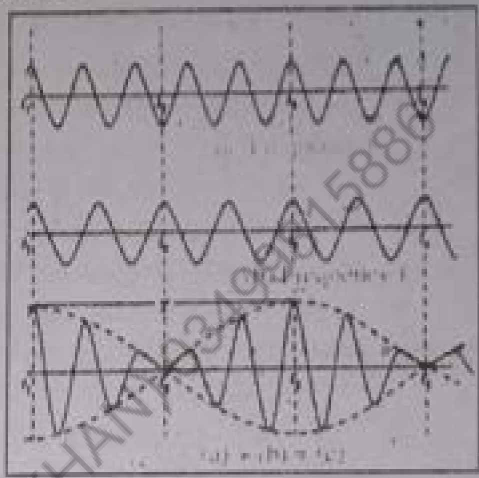
- We can understand the phenomenon of the beats by considering the displacement curves of the sound waves produced by the two forks .
- The displacements of the particles of the medium due to two waves are plotted separately as a function of time as shown in fig.8.18.
- If both the waves travel simultaneously along the same line then according to the superposition principle, the resultant displacement of any particle will be the vector sum of the displacements due to each of the two waves.
- The resultant wave which is produced is also shown.
- It is seen that amplitude varies with time that gives rise to variation of loudness which we call beats.

- The time interval between the two successive loud sound is $T = t_2 - t_1$.
- During this time interval the number of oscillation of 1st wave is f_1T and that of the 2[nd] wave is f_2T ,but the 1[st] wave should have made one oscillation more than 2[nd] one. Therefore

$$f_1T - f_2T = 1$$

$$f_1 - f_2 = \frac{1}{T}$$

$$N = \frac{1}{T} \quad OR \quad f = \frac{1}{T} \quad \text{where } N = f \text{(beat frequency)}$$



Where $f_1 - f_2 = f$ a beat frequency and T is the time period.

Uses of beats:

1. It is used to tune the musical instruments.
2. It is used to determine unknown frequency of tuning fork.
3. Beats are used to produce variety in music.

Example:

One can use beats to tune a string instrument such as piano or violin by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beat is heard.

Explanation:

Tuning is the process of adjusting the pitch of one or many tones from the musical instruments until they form a desired arrangement. Pitch is the perceived fundamental frequency of a sound.

Tuning a musical instrument

Tuning may be carried out by sounding two pitches and adjusting one of them to match or to relate the other.

In order to tune a musical instrument;

- Beat the instrument against a note of known frequency.
- If the two frequencies do not match, beats will be produced.
- Adjust the frequency of the un-tuned instrument by ***tightening or loosening*** the string.
- When ***no beats*** are heard, the instrument is said to be tuned.

NOTE

When the difference between the frequencies of two sounds is more than **10 Hz**, then it becomes difficult to recognize the beats.

Assignment 8.2:

How many beats per second are heard when two tuning forks of 256 Hz and 259 Hz are sounded together?

Given Data: $f_1 = 256\text{Hz}$

$f_2 = 259\text{Hz}$

Bat frequency $N = ?$

Solution:

$N = 259\text{ Hz} - 256\text{ Hz}$

$N = 3\text{Hz}$

Q.13 Define and explain reflection of waves.

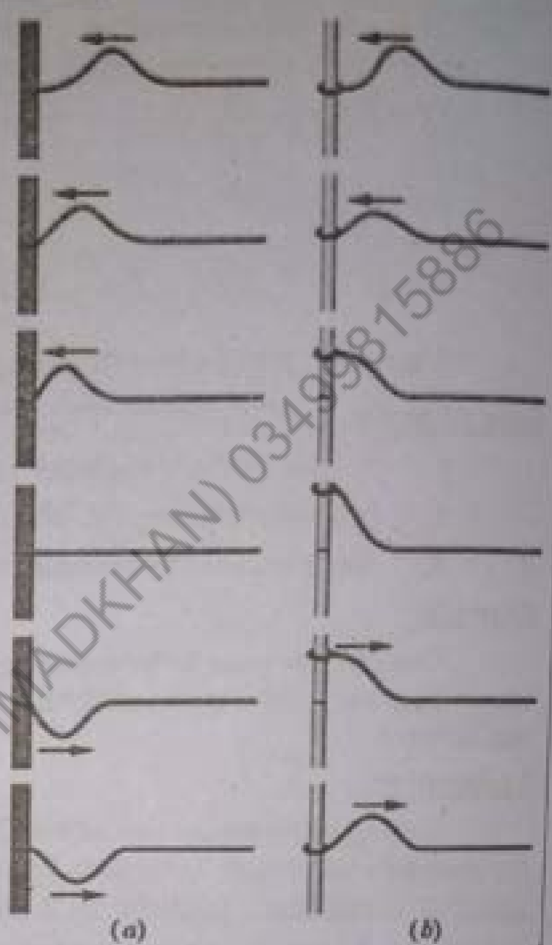
Ans: **REFLECTION OF WAVES**

"The bouncing back of waves from the boundary of a certain medium is called reflection of waves".

i. **Mechanical Waves:**

- In the Fig 8.19, the right hand end of string is fixed at the wall and a transverse upward pulse is set in it by hand traveling towards the wall.
- When this crest strikes the wall a part of its energy is absorbed and the rest is reflected.
- Since the wall does not move up with the crest in same way as it pulls the string upward, so the wall exert downward pull on the string.
- This pull accelerates the string downward to such an extent that its momentum carries it below the zero line.
- The result is that upward displacement pulse is reflected as a downward displacement.
- **A phase change of 180° or π-radians** has occurred which is equal to half a **wavelength** $\left(\frac{\lambda}{2}\right)$ **between** the incident and reflected pulses.
- On the other hand if the fixed end of the string is attached to a ring which can move freely up and down, as shown in fig.8.20.
- When the wave pulse arrives the end, the ring moves up and as the ring moves down, an upward pulse is produced.
- There is **no phase change** in this case.

FIGURE



(a) the left end of the string, which is tied to a wall. (b) left end of the string is tied to a ring

Electromagnetic Waves

- The phase change also occurs when electromagnetic waves such as light waves are reflected from the boundary of a denser medium.
- Various techniques have been developed for locating the position of objects by reflecting the waves of known speed from them, as radar (radio audio detection and ranging) waves.

Reflection of Sound Waves

- Sound waves obey the **laws of reflection** just like the other types of waves.
- The angle of incidence is equal to the angle of reflection.
- The regular reflection of sound waves occur at the surface.
- The sound waves of frequency greater than 20 kHz are used to determine the crowed of fishes in the depth of ocean.
- A sound pulse is sent out under water from a ship, after being reflected from the sea bottom, the sound is detected by an underwater receiver, mounted on the ship and the time interval is recorded by special device.
- Submarines are also detected by the underwater sound waves produced by their propellers.

Fig: 8.21 Guitar

Q.14 What is Echo and reverberation?

Ans: **Echo**

"The reflection of an original sound from a certain object is received at 0.1 s later than the direct sound is called echo".

Let the speed of sound is 340 m s^{-1}, then:

$$S = \text{effective distance for echo} = \frac{340 \times 0.1}{2} = 17 \text{ m.}$$

- The formation of echoes in public halls and auditoriums which annoying to ear can be remedied by selecting proper dimensions and by avoiding continuous flat smooth walls.

Reverberation:

- When the reflecting surface is at a distance less than 17m away from the source of sound, then the echo follows, so close upon the direct sound that they cannot be distinguished.
- This effect is known as reverberation; it causes the general confusion of the sound impression on the ear.

MCQ's

1. Beats can be heard when difference of frequency is not more man:
(A) 8 Hz (B) 10 Hz (C) 4Hz (D) 6 Hz
2. In order to produce beats, the two sound waves should have:
(A) same amplitude (B) slightly different amplitudes (C) same frequencies (D) slightly different frequencies
3. Two tuning forks of frequencies 240 Hz and 243Hz are sounded together, the number of beats per second will be:
(A) Zero (B) 4 (C) 3 (D) 2
4. Beats are used to find unknown:
(A) Frequency (B) Wavelength (C) Speed (D) Intensity
5. The periodic increase and decrease in loudness of sound are called:
(A) Resonance (B) Interference (C) Beats (D) Polarization
6. Which of the following changes due to interference of two sound waves of same frequency?
(A) Frequency (B) Time period (C) Wave length (D) Amplitude
7. Which of the following is the basic principle of beats?
(A) Interference (B) Diffraction (C) Reflection (D) Refraction
8. On loading the prong of a tuning fork with wax, the frequency of sound:
(A) Increases (B) Decreases (C) Increases (D) First increases then decreases
9. Which of the following is the path difference for constructive interference?
(A) $\frac{\lambda}{2}$ (B) $\frac{5\lambda}{2}$ (C) $m\lambda$ (D) $\frac{3\lambda}{2}$
10. The pitch of sound depends upon:
(A) Intensity of sound (B) Wavelength of sound (C) Frequency of sound (D) Loudness of sound

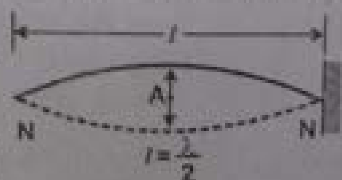
Answers Key

1. B	2. D	3. C	4. A	5. C	6. D	7. A	8. B	9. C	10. C

Q.15 What are stationary waves? Explain.

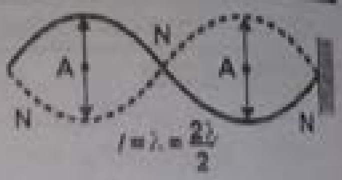
Ans: **Stationary waves (standing waves)**

Two identical waves traveling with same speed along same line but in opposite direction superpose each other and give rise the stationary wave.



- The sound produced by most of the string and wind type musical instrument is due to formation of stationary waves in these instruments.
- Consider a rubber string fixed at one end and held in hand by other end. If we wiggle the string from the end in hand then at a particular frequency f_1 the wave is reflected from fixed end. The incoming wave and reflected wave superpose each other and one **stationary wave** is set up.
- *When wiggling frequency is doubled $2f_1$ then two stationary waves are set up.*

- ***When wiggling frequency is $3f_1$ then three stationary waves are set up.***
- ***When wiggling frequency is nf_1 then string vibrates in n loops.***



Nodes: *"The points of zero displacement on stationary waves are called node".*

Antinodes

"The points of maximum displacement on stationary waves are called antinodes".

Properties of stationary waves

1. There are points of medium in stationary waves which permanently show zero displacement are called *nodes.*
2. The points between two successive nodes are *in phase* with each other.
3. Each point along the stationary wave vibrates with *different amplitudes.*
4. There are points of medium in stationary waves which have maximum displacement are called *antinodes.*
5. The distance between two consecutive nodes is **λ/2.** (where λ is wavelength)
6. The distance between two consecutive antinode is **λ/2.**
7. The distance between one node and next antinode is λ/4.
8. The energy remains standing in the medium between nodes because the nodes remain at rest, so energy cannot flow through these points. That is why stationary waves are also called **standing waves.**
9. Energy oscillates between P.E. and K.E. between nodes.

Note

- When antinodes are at their extreme positions the whole energy is P.E while at passing through equilibrium position, the whole energy is K.E.
- Commonly the standing waves are produced due to superposition of a wave traveling down a string *with its reflection traveling in opposite direction.*

Q.16 Show that frequencies of stationary waves in a stretched string are quantized.

OR Prove that for stationary waves in a stretched string $f_n = n f_1$

Ans: **Stationary Waves in Stretched String**

Speed of waves on string

Let us consider a string of length L stretched and is clamped at its two ends.

Tension in the string = T

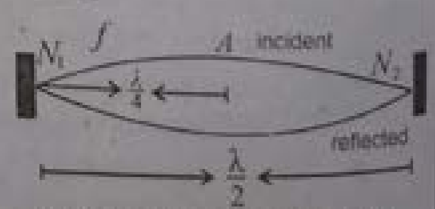
- When the string is plucked and then released, two waves are generated which moves in opposite directions along the string. Both of these are reflected back from the clamped ends.
- They meet at middle, superpose each other and stationary wave is setup.
- As the two ends are clamped with rigid supports, so these do not vibrate and we get nodes at these ends.
- The speed of wave depends upon tension T in the string and mass M of string of length L.

$$V = \sqrt{\frac{T \times L}{M}} \quad \text{---------------} (1)$$

First mode of vibration

When the string is plucked at the middle of its length then the string vibrates in a single loop as shown in figure. Such a mode is called fundamental mode of vibration.

FIGURE



String vibrating in one loop

Distance between two consecutive nodes $= \frac{\lambda}{2}$

If λ_1 be the wave length and f_1 be the frequency of vibration in this mode, then

$$L = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L$$

Thus, speed of wave v is

$$v = f_1 \lambda_1$$

OR

$$f_1 = \frac{v}{\lambda_1}$$

Putting value of $\lambda_1 = 2L$ We get

$$f_1 = \frac{v}{2L} \quad \text{———— (2)}$$

Putting value of v from equation (1) in equation (2), we get

$$f_1 = \frac{1}{2L}\sqrt{\frac{T \times L}{M}} \quad \text{———— (3)}$$

If m is the mass per unit length i.e $m = \frac{M}{L}$ then equation (3) can be written as

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

Second Mode of Vibration

- When the string is plucked from one quarter (1/4) of its length then the string vibrates into two loops as shown in figure.
- If λ_2 be the wave length and f_2 be the frequency of vibration in this mode, then

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

OR $L = \frac{2\lambda_2}{2}$

OR $\lambda_2 = \frac{2L}{2}$

Thus, speed of wave v is $v = f_2 \lambda_2$

OR $f_2 = \frac{v}{\lambda_2}$

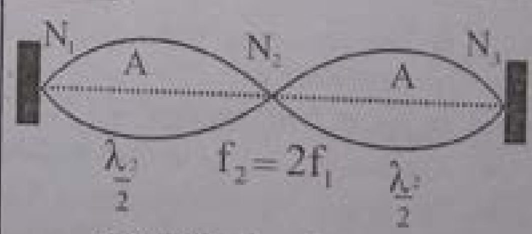
Putting value of λ_2, we get

$$f_2 = \frac{v}{2L/2}$$

or $f_2 = 2\left(\frac{v}{2L}\right)$

So $\boxed{f_2 = 2f_1}$ [since $\frac{v}{2L} = f_1$]

FIGURE



String vibrating in two loops

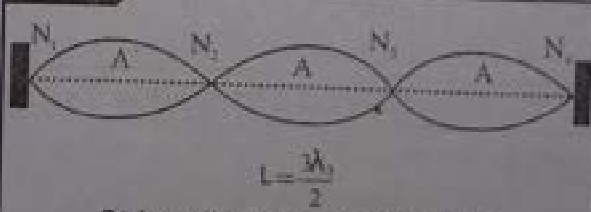
Thus when the string vibrates in two loops, its frequency is double than when it vibrates in one loop.

- f_2 is called second harmonic.

Third mode of vibration

- When the string is plucked from one sixth (1/6) of its length then the string vibrates into three loops as shown in figure.
- If λ_3 be the wave length and f_3 be the frequency of vibration in this mode, then

FIGURE



String vibrating in three loops

$$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$L = \frac{3\lambda_3}{2}$$

or $$\lambda_3 = \frac{2L}{3}$$

So the speed becomes,

$$v = f_3 \lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

Putting $$\lambda_3 = \frac{2L}{3}$$

$$f_3 = \frac{v}{2L/3}$$

$$f_3 = 3\left(\frac{v}{2L}\right)$$

$$\boxed{f_3 = 3f_1} \quad \left[\text{since } \frac{v}{2L} = f_1\right]$$

- The frequency f_3 is called third harmonic.

EXPLANATION: All wind instruments are provided with a column of air, called a resonator, which may be in the form of a pipe. The periodic movement is caused by mouth piece. The mouth-piece acts as a generator and supplies the energy necessary to maintain the vibrations in air column. The air from the mouth-piece strikes the upper lip and sets up vibrations in the resonating pipe.

nth mode of vibration

- If string vibrates in n loops then,

$$f_n = n\left(\frac{v}{2L}\right) = n f_1$$

$$f_n = n\left(\frac{1}{2L}\sqrt{\frac{T}{m}}\right)$$

And wavelength is

$$\lambda_n = \frac{2L}{n} \quad \text{where } n = 1,2,3,4,5 \ldots\ldots\ldots$$

So the stationary wave have a discrete set of frequencies f_1, $2f_1$, $3f_1$, …….. nf_1, which is known as harmonic series.

Quantization of frequencies

- The frequency f_1 is known as *fundamental frequency.*
- The other possible modes of vibration whose frequencies are integral multiple of lowest frequency i.e $2f_1$, $3f_1$, $4f_1$ …..nf_1 are called harmonic or *over tones.*

The stationary waves on the string can be set up only with discrete set of frequencies f_1, f_2, f_3 ……

i.e $\boxed{f_2 = 2f_1}$

$\boxed{f_3 = 3f_1}$

………

$f_n = nf_1$

- This phenomenon is known as quantization of frequency.

Q.17 How can we change the frequency of string on a musical instrument?

Ans: The frequency of a string on a musical instrument can be changed either by varying the tension in string and length of string.

For example

- The tension in guitar and violin strings is varied by tightening the pegs on the neck of the instrument.

- Once the instrument is tuned; the musicians vary the frequency by moving their fingers along the neck.
- By doing so the change the length of the vibrating portion of the string.

Assignment 8.3:

A 40-g string 2 m in length vibrates in three loops. The tension in the string is 270 N. What is the wavelength and frequency?

Given Data: Length of string = L = 2.0m

Mass of string = M = 40.0 g = 0.04 kg

Mass per unit length $m = \frac{M}{L} = \frac{0.04}{2} = 0.02$ kg/m

Tension T = 270 N

(a) Wavelength λ = ?

(b) Frequency f = ?

Solution: The speed; $v = \sqrt{\frac{T}{m}}$

$$v = \sqrt{\frac{270}{0.02}} = 116 \text{ m/s}$$

(a) Wavelength= λ_3 = ?

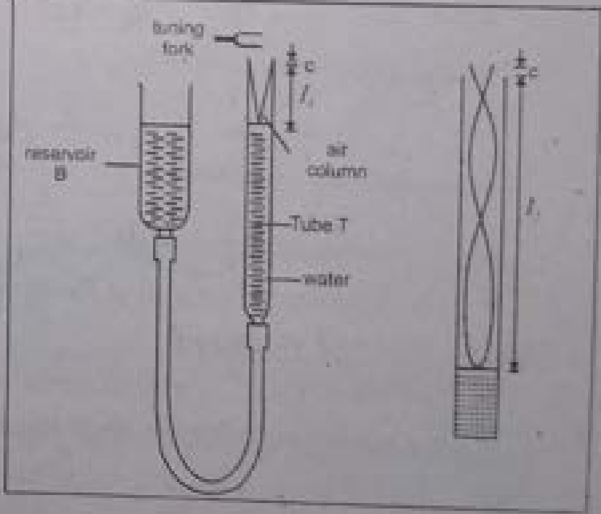
$$\lambda_n = \frac{2L}{n}$$

$$\lambda_3 = \frac{2L}{3} = \frac{2\times2.0}{3} = 1.33\text{m}$$

(b) frequency = $f_3 = v/\lambda_3 = 116/1.33 = 87.3$Hz

Q.18 Explain resonance in air column.

Ans: Resonance of air Column and Organ pipes



- The stationary waves can be setup in the air column.
- If you hold a vibrating tuning fork over the open end of a glass tube filled with water then the sound of tuning fork can be greatly amplified under certain conditions.
- Holding the vibrating turning fork, slowly lower the water surface in the tube with the help of reservoir. At a certain height of the water level, the air column in the tube will resonate loudly to the sound being sent into it by the tuning fork as shown in Fig 8.28.
- The resonance occurs when the frequency f_e of the periodic force due to tuning fork becomes equal to the fundamental frequency f_o of the air column. In fact there are usually several heights at which the tube will resonate.

FIGURE

Resonating musical instruments

- The stationary longitudinal waves in the air column in a pipe or tube are the source of sound in wind musical instruments, such as flute, organ, trombone, clarinet etc.

Q.19 What are organ pipe? Derive expression for fundamental frequency and higher harmonics in closed pipe?

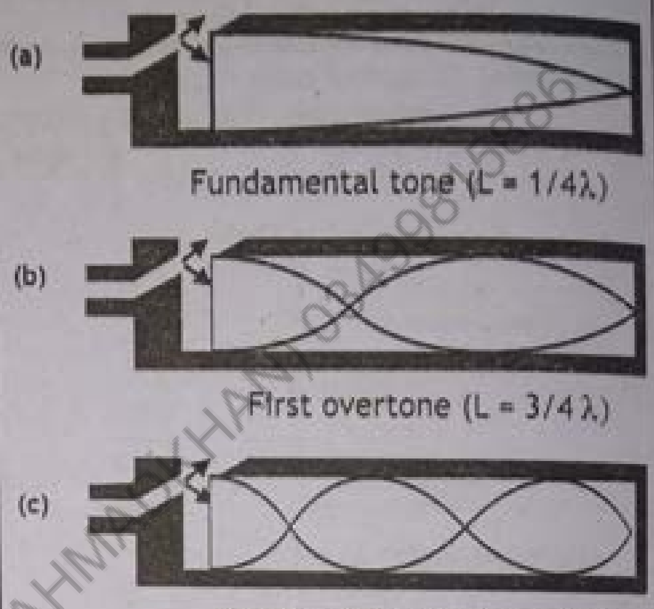
Ans: <u>**Organ Pipes**</u>

An organ pipe is the simplest example of an instrument which produces sound by means of vibrating air column. Organ pipes are of two types.

- Closed organ pipe
- Open organ pipe

In both these types an air column is made to vibrate by blowing in to the whistle end, which is simplest in construction but its action is very simple.

FIGURE



Fundamental tone ($L = 1/4\lambda$)

First overtone ($L = 3/4\lambda$)

Second overtone ($L = 5/4\lambda$)

The air column vibrating in the organ pipe

1. **Close organ pipe**

- Let us consider an organ pipe of length ℓ which is closed at one end.
- At the closed end we get node while at the open end we get anti-node.

Fundamental mode of vibration:

- Fundamental mode of vibration has one node and one antinode.
- If λ_1 is the wavelength of fundamental mode, then length of the pipe is.

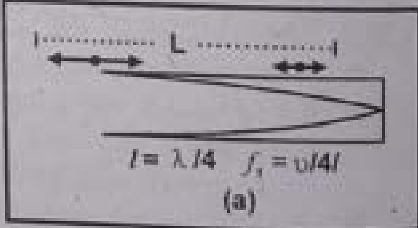
$$L = \frac{\lambda_1}{4}$$

Or $\boxed{\lambda_1 = 4L}$

So the speed becomes $v = f_1 \lambda_1$

Or $f_1 = \frac{v}{\lambda_1}$ Putting value of λ_1

$$f_1 = \frac{v}{4L} \quad \text{............ (1)}$$



- The frequency f_1 is called fundamental frequency.

Second Mode of vibration

- Second mode of vibration contains two nodes and two anti-nodes.
- If λ_2 is the wavelength, then length of the pipe is

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

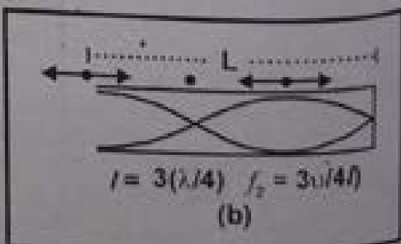
$$L = \frac{3\lambda_2}{4}$$

$$\lambda_2 = \frac{4L}{3}$$

If f_2 is the frequency of sound, then $f_2 = \frac{v}{\lambda_2}$

Putting value of $\lambda_2 = \frac{4L}{3}$ we get

$$f_2 = \frac{v}{4L/3}$$



or $$f_2 = 3\left(\frac{v}{4L}\right)$$

So $$\boxed{f_2 = 3 f_1} \qquad [\text{since } \frac{v}{4L} = f_1]$$

- This is called second harmonic.

$l = 5(\lambda/4)$ $f_5 = 5\upsilon/4l$
(c)

Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

Third Mode of vibration:

- Third mode of vibration contains there nodes and three anti-nodes. If λ_3 is the wavelength, the length of the pipe is.

$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$

$$L = \left(\frac{1+2+2}{4}\right)\lambda_3$$

$$L = \frac{5\lambda_3}{4}$$

OR $$\lambda_3 = \frac{4L}{5}$$

- If f_3 is the frequency of sound, then speed becomes, $v = f_3 \lambda_3$

OR $$f_3 = \frac{v}{\lambda_3}$$

Putting value of $\lambda_3 = \frac{4L}{5}$ we get

$$f_3 = \frac{v}{4L/5}$$

or $$f_3 = 5\left(\frac{v}{4L}\right)$$

So $$\boxed{f_3 = 5 f_1} \qquad [\text{since } \frac{v}{4L} = f_1]$$

Which is the frequency of third harmonic.

n^{th} mode of vibration

- If air column vibrates in n loops then,

$$f_n = (2n - 1)\frac{v}{4L}$$

Q.20 Derive expression for the fundamental frequency and higher harmonics in open pipe.

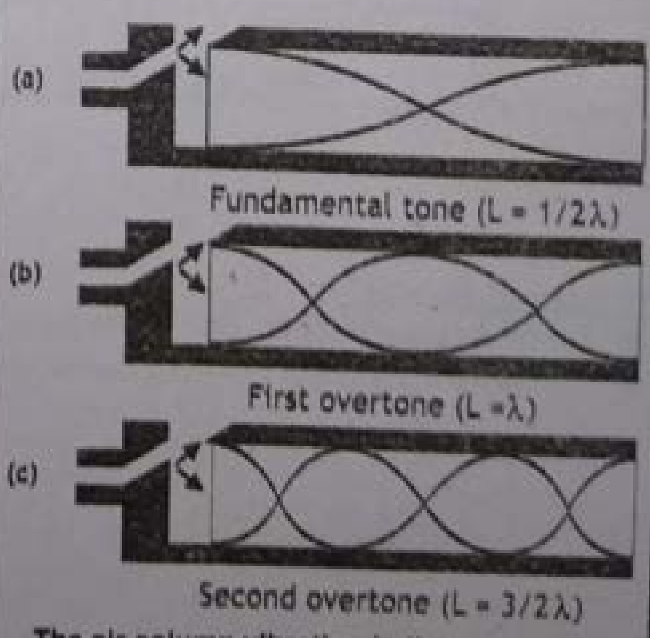
Ans. Open organ pipe (open at both ends)

- Let us consider an organ pipe of length L which is open at both ends.
- At the open ends air molecules have complete freedom of motion so it acts as antinode.
- Longitudinal waves set up inside, the pipe have been represented by transverse curves which represent the displacement and amplitude of variations of air at various points.

Fundamental Mode of Vibration

- In this case there is only one node at the middle of the pipe.

FIGURE



The air column vibrating in the organ pipe

- There are two antinodes at both the open ends.
- If λ_1 is the wavelength of sound,

$$L = \frac{\lambda_1}{4} + \frac{\lambda_1}{4}$$

$$L = \frac{\lambda_1}{2}$$

Or $\boxed{\lambda_1 = 2L}$

- If f_1 is the frequency of sound, then the velocity of sound is

$$v = f_1 \lambda_1$$

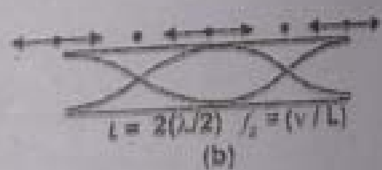
OR $f_1 = \frac{v}{\lambda_1}$ Putting value of $\lambda_1 = 2L$ we get

$$f_1 = \frac{v}{2L} \quad \ldots\ldots\ldots\ldots(1)$$

- This frequency is called fundamental frequency or first harmonic

Second mode of vibration:

- In this case, there are three antinodes and two nodes.
- If λ_2 is the wavelength of sound then



$$L = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

$$L = \left(\frac{1+2+1}{4}\right)\lambda_2$$

OR $\lambda_2 = L$

If f_2 is the frequency of sound, then speed becomes, $v = f_2 \lambda_2$

OR $f_2 = \frac{v}{\lambda_1}$ Putting value of $\lambda_2 = L$ we get

$$f_2 = \frac{v}{L}$$

(Multiplying and dividing R-H-S by 2)

or $f_2 = 2\left(\frac{v}{2L}\right)$

So $\boxed{f_2 = 2f_1}$ [since $\frac{v}{2L} = f_1$]

- f_2 is 2[nd] harmonic

Third mode of vibration

- For three loops, there are four antinodes and three nodes.
- If λ_3 is the wavelength of sound, then length of the pipe is

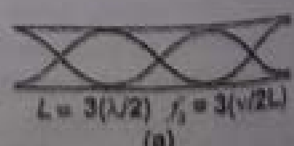


Fig. A
Stationary longitudinal waves in a pipe open at both ends.

$$L = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4}$$

$$L = \left(\frac{1+2+2+1}{4}\right)\lambda_3 \quad \text{(OR)} \quad L = \frac{3\lambda_3}{2}$$

$$\boxed{\lambda_3 = \frac{2L}{3}}$$

So the speed becomes, $v = f_3 \lambda_3$

$$f_3 = \frac{v}{\lambda_3}$$ Putting value of $\lambda_3 = \frac{2L}{3}$

$$f_3 = 3\left(\frac{v}{2L}\right)$$

$$\boxed{f_3 = 3f_1}$$ [since $\frac{v}{2L} = f_1$]

- The frequency f_3 is called third harmonic.

n^{th} mode of vibration

- If air column vibrates in n loops then,

$$f_n = n\left(\frac{v}{2L}\right) = n f_1$$

- The frequency f_1 is known as fundamental frequency, and the other are called over tones.
- So the longitudinal stationary waves have a discrete set of frequencies $f_1, 2f_1, 3f_1, \ldots\ldots\ldots nf_1$, are known as harmonic series.

And wavelength is $\lambda_n = \frac{2L}{n}$ where n = 1,2,3,4,5 ....

ACTIVITY

If you place your lips near the edge of a soft-drink bottle, as in figure, and blow softly across the opening, the sound wave reflected from the bottom of the bottle interferes with the incoming wave to produce a standing wave in the bottle. Since the bottle is closed at one end, there should be a displacement node at the bottom of the bottle.

Once you have heard one resonance, add varying amounts of water to raise the level within and listen for other resonances. The resonant sound is noticeably louder than the nonresonant sounds. Notice that the longer the air column within the bottle, the lower the pitch heard.

Assignment 8.5:

What length of closed pipe will produce a fundamental frequency of 256 Hz at 20° C?

Given Data: $f_1 = 256$ Hz

temp = t = 20° C

Length of closed pipe L = ?

Solution:

The speed of sound at 20°C is;

$$v = v_o + 0.61\,t$$

$$v = 332 + 0.61 \times 20 = 332 + 12.2 = 344.2 \text{ m/s}$$

$$f_1 = \frac{v}{4L}$$

$$L = \frac{v}{4f_1} = \frac{344.2}{4 \times 256} = 0.336 \text{ m}$$

$$L = 33.6 \text{ cm}$$

MCQ's

1. When two identical waves move in opposite direction superpose each other they give rise to:
(A) Stationary waves (B) Beats (C) Constructive interference (D) Destructive interference

2. Which of the following is the distance between two consecutive nodes or antinode in stationary waves?
(A) $\frac{\lambda}{2}$ (B) $\frac{\lambda}{4}$ (C) λ (D) 2λ

3. On increasing the tension, the frequency of the vibration of string:
(A) Decreases (B) Increases (C) Remains constant (D) First increases then decreases

4. Phase angle of 180° is equivalent to a path difference of:
(A) $\frac{\lambda}{4}$ (B) $\frac{\lambda}{2}$ (C) λ (D) 2λ

5. If a string vibrates in n loops, then the wavelength of stationary wave will be:
(A) $\frac{2L}{n}$ (B) $\frac{n\ell}{2}$ (C) $\frac{2n}{\ell}$ (D) $\frac{\ell}{2n}$

6. When transverse wave is reflected from boundary of denser medium then the phase of wave changes by:
(A) 0° (B) 90° (C) 180° (D) 270°

7. In stationary waves, the velocity of particle at the node is:
(A) Maximum (B) Infinite (C) Variable (D) Zero

8. Which of the following is the wavelength of a wave for closed pipe having the length L in fundamental mode?
(A) 2L (B) 4L (C) L / 2 (D) L

9. Which one is true for organ pipe closed at one end?
(A) $\lambda_n = \frac{2L}{n}$ (B) $\lambda_n = \frac{4L}{n}$ (C) $\lambda_n = \frac{nL}{2}$ (D) $\lambda_n = \frac{nL}{4}$

10. Stationary waves are generated on a string of length ℓ its fundamental frequency is:
(A) $f_1 = \frac{v}{\ell}$ (B) $f_1 = \frac{v}{2L}$ (C) $f_1 = \frac{2v}{L}$ (D) $f_1 = \frac{v}{4L}$

11. A stationary wave is established in a string which vibrates in four segments at a frequency of 120 Hz .Its fundamental frequency will be:
(A) 15Hz (B) 20 Hz (C) 30 Hz (D) 480 Hz

12. A set of frequencies, which is the multiple of its fundamental frequency are called:
(A) Beat frequency (B) Harmonics (C) Nodal frequency (D) Doppler frequency

13. The string of length L fixed at the both ends is vibrating in two segments. Which of the following is wavelength of wave?
(A) $\frac{L}{4}$ (B) $\frac{L}{2}$ (C) L (D) 2L

14. The fixed ends of vibrating string act as:
(A) Antinodes (B) Nodes (C) Overtones (D) Neither node nor

15. For same mass per unit length and lengths of string if tension of a vibrating string is increased by 4 times then frequency:
(A) 2 times (B) 4 times (C) 8 times (D) $\sqrt{2}$ times

16. A stretched string of length 2m vibrates in 2 segments. The distance between consecutive nodes is:
(A) 1m (B) 2m (C) 0.5m (D) 4m

17. The distance between node and anti node is equal to:
(A) λ (B) $\frac{\lambda}{2}$ (C) $\frac{\lambda}{3}$ (D) $\frac{\lambda}{4}$

18. If the organ pipe is closed at one end, the frequency of fundamental harmonic is|:
(A) $f_1 = \frac{v}{2}$ (B) $f_1 = \frac{v}{4L}$ (C) $f_1 = \frac{4}{v}$ (D) $f_1 = \frac{2}{v}$

19. The number of node between two consecutive antinodes is:
(A) One (B) Two (C) Three (D) Four

20. A stretched string 4m long and it has 4 loops of stationary waves, then the wave length is:
(A) 4m (B) 3m (C) 2m (D) 1m

21. The distance between 1st node and 4th antinode is:
(A) $\frac{7}{4}\lambda$ (B) $5\frac{\lambda}{4}$ (C) $7\frac{\lambda}{4}$ (D) $11\frac{\lambda}{4}$

22. When a transverse wave travelling through denser medium is reflected from rarer medium the phase change will be:
(A) 90° (B) 60° (C) 180° (D) 0°

Answers Key

1. A	2. A	3. B	4. B	5. A	6. C	7. D	8. B	9. B	10. B	11. C	12. B
13. C	14. B	15. A	16. A	17. D	18. B	19. A	20. C	21. C	22. D		

Q.21 Define Doppler effect and discuss its different cases.

Ans: Doppler Effect

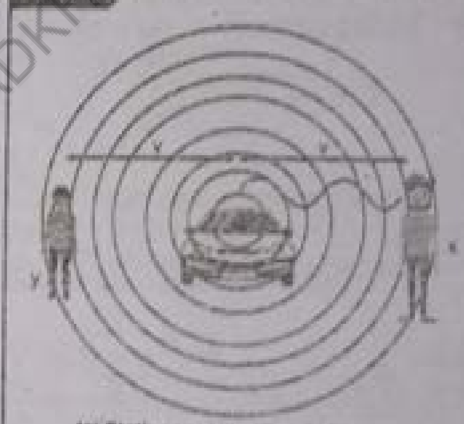
"The apparent change in the frequency or pitch of sound, caused by the relative motion of either the source of sound or listener or both, is called Doppler Effect".

- Doppler Effect inter relates the measured frequency of the wave to the relative velocity of the source of sound and listener.
- This phenomenon is called Doppler Effect after **Christian Johann Doppler** who showed in **1842** that frequency shift should be observed for sound and light waves due to relative motion between source and observer.
- To further describe Doppler Effect consider a source of sound "S" emitting sound waves of velocity 'v', original frequency "f" and original wavelength "λ".
- When the source "S" and listener L are at rest then the listener will receive "λ" number of waves in one second.
- The distance between source "S" and listener "L" is 'v'.
- Since "f" number of waves are compressed in distance 'v', so the wavelength λ is

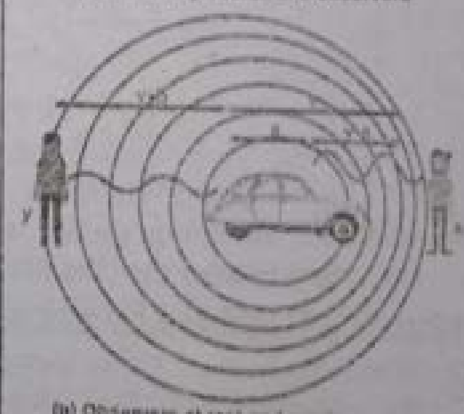
$$\lambda = \frac{v}{f}$$

$$f = \frac{v}{\lambda}$$

FIGURE



(a) Stationary source and observers



(b) Observers at rest and moving source

Sounds emitted by a source spread out in spherical waves.

The source is moving and listener is at rest:

- In this case, either the sounding source moves towards the stationary listener or away from the stationary listener.

(I-A) **Source moves towards a stationary listener:**

- Let the sounding source "S" is moving with speed "a" towards the stationary listener "L" shown in the Fig 8.32.
- The first wave emitted by the source covers distance 'v' after 1 second and reaches the listener "L".
- At the end of one second, the source covers distance "a" where it gives the last wave.
- In this case "f" number of waves are compressed in a distance (v – a) as shown in Fig. 8.31

So, the apparent wavelength λ′ **decreases than original wavelength λ:**

$$\lambda' = \frac{v-a}{f}$$

The changed frequency f′ is $f' = \frac{v}{\lambda'}$

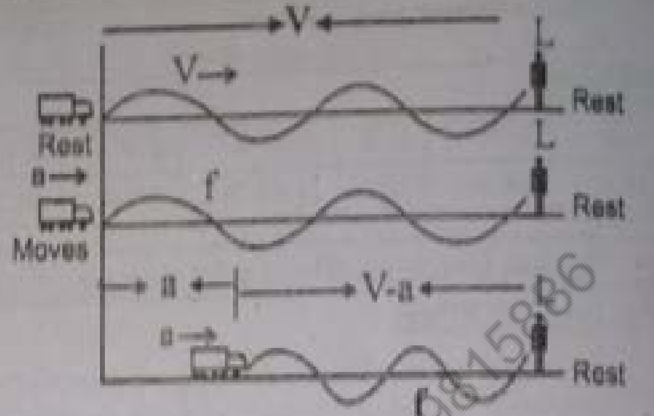
Putting value of λ'

$$f' = \frac{v}{\frac{v-a}{f}}$$

$$f' = \left(\frac{v}{v-a}\right) f \longrightarrow (1)$$

$$\left(\frac{v}{v-a}\right) > 1$$

Therefore f ' > f



Source Moving Towards Stationary Listener

- If the sounding source is approaching a stationary listener then frequency and pitch of sound increases.

(1-B) <u>The source of sound moves away from stationary listener:</u>

- If the sounding source "S" is moving away from the stationary listener then "f" number of waves are contained in distance (v + a),
- So the apparent wavelength λ' **increases than original wavelength λ.**

$$\lambda' = \frac{v+a}{f}$$

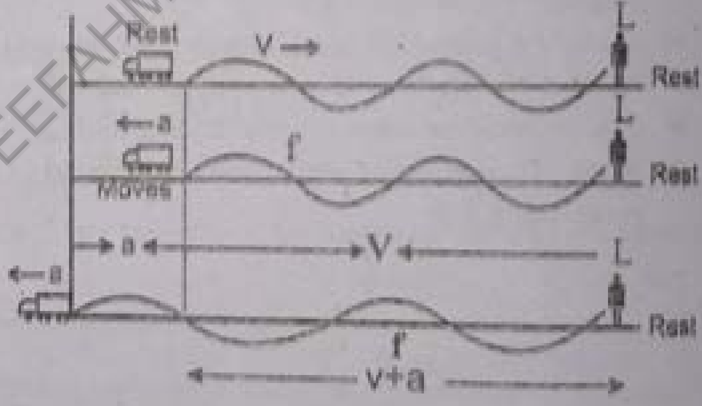
The apparent frequency f' is $f' = \frac{v}{\lambda'}$

Putting value of λ'

$$f' = \frac{v}{\frac{v+a}{f}}$$

$$f' = \left(\frac{v}{v+a}\right) f \longrightarrow (2)$$

$$\left(\frac{v}{v+a}\right) < 1$$



Source Moving Away from Stationary Listener

As f' < f,

- when the sounding source is moving away from the stationary listener then the pitch of sound decrease.

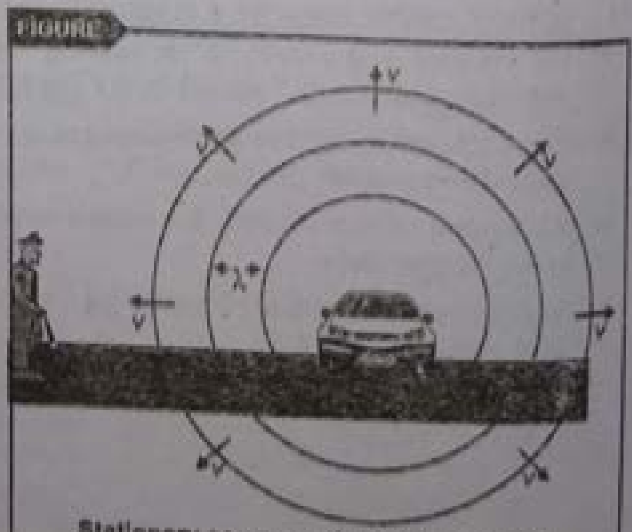
<u>The source is at rest and listener is moving</u>

When the listener either moves, towards or away from the stationary sounding sources, the pitch of sound changes.

(II-A) <u>The Listener moves towards a stationary sounding source</u>

- Let the listener "L" is moving with speed 'b' towards a stationary sounding source "S" as shown in the Fig 8.32.
- So the speed of sound relative to the listener is (v + b) and wavelength remains the same.

Therefore, the apparent frequency f" is



Stationery source and moving observers

$$f'' = \frac{v+b}{\lambda}$$

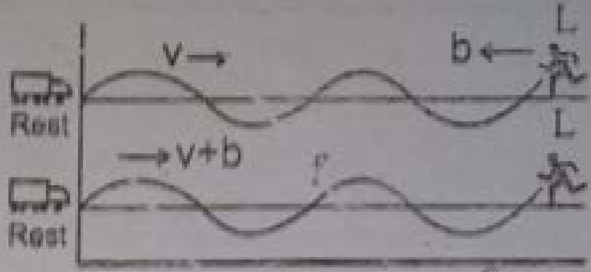
But $\lambda = \frac{v}{f}$

Putting $f'' = \frac{v+b}{v/f}$

Therefore $f'' = \left(\frac{v+b}{v}\right)f \longrightarrow (3)$

$$\left[\frac{v+b}{v}\right] > 1$$

$$f'' > f$$



Listener Moving Towards Stationary Source

- When the listener moves towards a stationary sounding source then pitch of sound increases.

(II-B) Listener moves away from stationary sounding source:

- When the listener moves away with speed "b" from a stationary sounding source, the speed of sound relative to the listener becomes (v – b). As the wavelength remains the same so the observed frequency is

$$f'' = \frac{v-b}{\lambda}$$

Putting $\lambda = \frac{v}{f}$

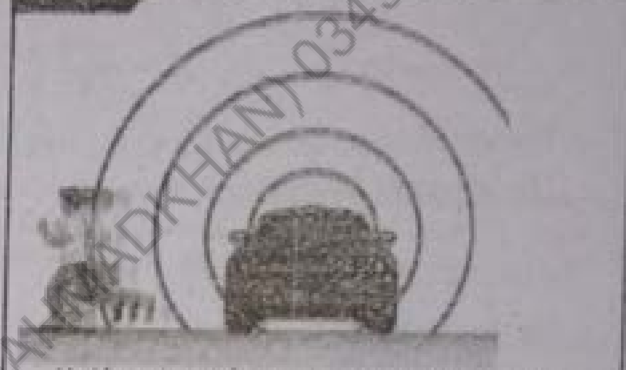
or $f'' = \frac{v-b}{v/f}$

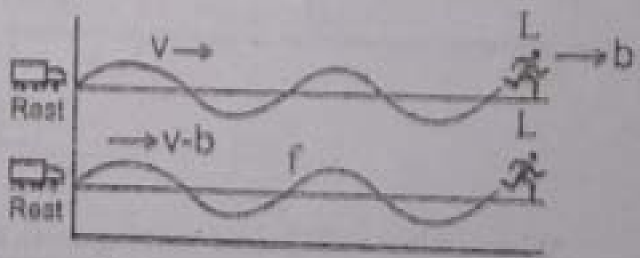
$$f'' = \left(\frac{v-b}{v}\right)f \longrightarrow (4)$$

Where $\left(\frac{v-b}{v}\right) < 1$

$$f'' < f$$

FIGURE


Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests



Listener Moving Away from Stationary Source

- When the listener moves away from the stationary sounding source then pitch of sound decreases.

III. When Source and listener both moves

It is also possible that when source and listener both moves, the pitch of sound also changes.

(C.I) Source and listener both moves towards each other:

- If the source and listener are approaching each other with velocities 'a' and 'b' respectively, then the apparent wave length λ' is given by:

$$\lambda' = \frac{v-a}{f}$$

The speed of sound relative to the listener is $v' = (v + b)$

The apparent frequency f' is $f' = \frac{v'}{\lambda'}$

Putting value of v' and λ' we get

$$f' = \frac{v+b}{\frac{v-a}{f}}$$

$$f' = \left(\frac{v+b}{v-a}\right)f \longrightarrow (5)$$

$$\left[\frac{v+b}{v-a}\right] > 1$$

$$f' > f,$$

► When source and listener are approaching to each other then pitch of sound increases.

When source and listener move away from each other:

When the source of sound and listener are moving away from each other, then apparent wave length λ' is

$$\lambda' = \frac{v+a}{f}$$

The speed of sound relative to the listener is $v' = (v - b)$

The apparent frequency f' is

$$f' = \frac{v'}{\lambda'}$$

Putting value of v' and λ' we get

$$f' = \frac{(v-b)}{\frac{v+a}{f}}$$

$$f' = \left(\frac{v-b}{v+a}\right)f \longrightarrow (6)$$

$$\left[\frac{v-b}{v+a}\right] < 1$$

$$f' < f$$

When source and listener moving away from each other then pitch of sound decreases.

Q.22 Discuss various applications of Doppler effect.

Ans: Applications of Doppler Effect:

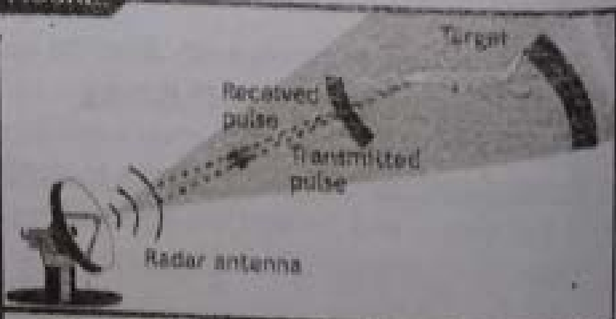
Following are the some important applications of Doppler frequency shift.

i. Doppler Effect is also applicable to light waves.

► The frequency of light being received from certain stars which are moving towards or away from the earth is found to be slightly different than the frequency of the same light emitted by a source on the earth.

ii. Another interesting application of Doppler Effect is the reflection of radar waves from an aero plane.

FIGURE



Radar detects the aeroplane due to the reflection of waves from aeroplane

► The frequency of reflected waves **decreases** if the plane is moving away from the source.

► The frequency of reflected **waves increases** if the plane is moving towards the source.

► From this frequency shift the speed and direction of the plane can be determined.

iii. When sound waves are reflected from a moving submarine, their frequency is changed.

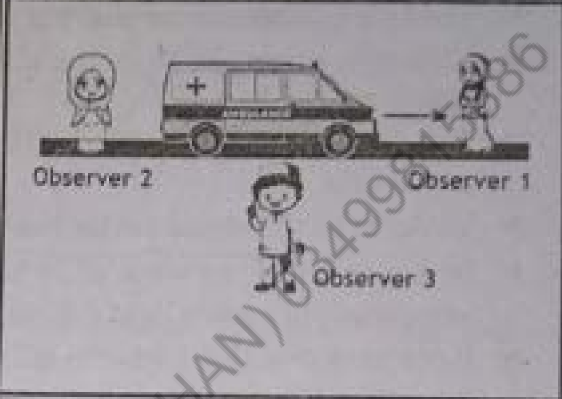
- By this change in frequency we can calculate the speed and direction of the submarine.
- The velocities of the earth satellites are also determined from the Doppler Shift in the frequency of radio waves which they transmit.

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

QUIZ

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Three stationary observers observe the Doppler shift from an ambulance moving at a constant velocity. The observers are stationed as shown below. Which observer will observe the highest frequency? Which observer will observe the lowest frequency? What can be said about the frequency observed by observer 3?



Ans: If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and clear the road for ambulance.

Assignment 8.6:

(a) What frequency is received by a woman watching an oncoming ambulance moving at 110 km/ h and emitting a steady 800 Hz sound from its siren? The speed of sound on this day is 345m/s. (b) What frequency does she receive after the ambulance has passed?

Given Data: Speed of source = a = 110 km/hr = 110x1000 m/3600s = 30.6 m/s

F = 800 Hz

v = 345 m/s

Required:

a) The apparent frequency when the ambulance approaches = $f' = ?$

b) The apparent frequency when ambulance recedes past the listener = $f'' = ?$

Solution:

a) When the sounding source approaches the stationary listener then apparent Frequency heard is;

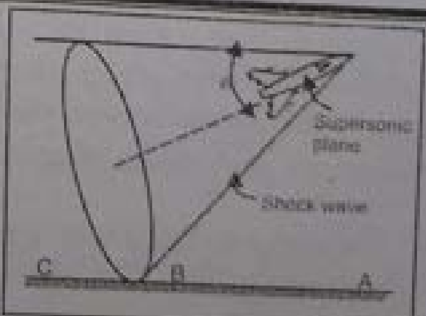
$$f' = \left(\frac{v}{v-a}\right)f = \left(\frac{345}{345-30.6}\right) \times 800 = 878\text{Hz}$$

b) When the car receding:

$$f'' = \left(\frac{v}{v+a}\right)f = \left(\frac{345}{345-30.6}\right) \times 800 = 735\text{Hz}$$

For Your Information

An interesting situation arises when the speed of sounding source equals the speed of sound, then the entire wave crests in front of the source lie upon one another. These wave crests together with the source itself, passing a given point at the same time. All the energy of the sound waves is compressed into a very small region in front of the source. This very concentrated region of sound builds up into a shock wave which causes an extremely loud sound called sonic boom.



Q.23 What are ultrasonic waves? Discuss the uses generation and detection of ultrasonic waves.

Ans: **Ultrasonic Waves**

"A normal human ear can hear a sound if its frequency lies between 20 Hz and 20,000 Hz. If the frequency of a sound is higher than 20000 Hz, it cannot be heard. The sounds of frequencies higher than 20,000 Hz are called ultrasonic".

- The term ultrasonic means **above or beyond sound.**
- Ultrasonic sound can be produced by an object vibrating at a frequency higher than the frequency which human ear can hear.
- This frequency is greater than 20,000 Hz to any desired frequency, but normally with a range of 20 kHz to 100 kHz.
- However an ultrasonic device has been developed that vibrates at 25 billion Hz.
- In fact an ultrasonic wave of 25 billion hertz has wavelength of about 10^{-8}m. Which is smaller than the wavelength of visible light (10^{-6}m) and comparable to the wavelength of x-rays (10^{-10}m).
- It can be shown that a wave is affected only by an object which is larger than its wavelength.
- Therefore there, is a direct relationship between the depth of penetration and the wavelength of the wave falling on an object.

Uses of ultrasonic waves

Ultrasonic waves carry much more energy than the sound waves of equal amplitude but low frequency. Following are the some important uses of ultrasonic waves.

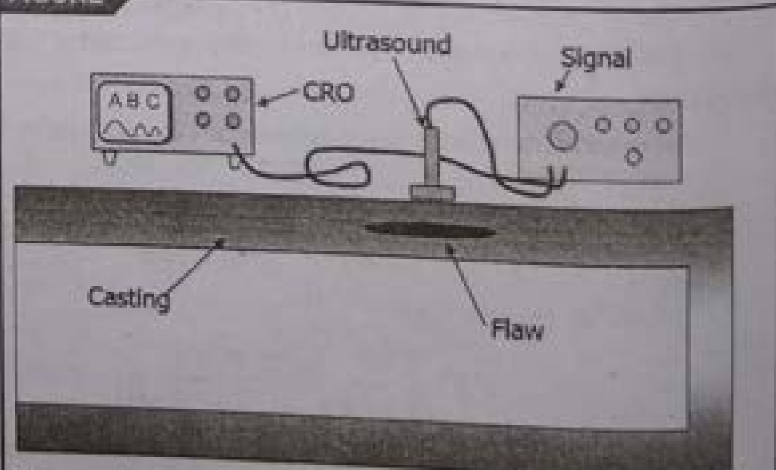
i. The penetrating power of ultrasonic waves makes them valuable in medicine for diagnostic work and bloodless surgery.

- In diagnostic work an ultrasonic signal is transmitted through a patient. By the analysis of reflected or refracted signals, the cysts and tumor in the body can be located.
- The use of ultrasonic waves in surgery has shown great importance.

ii. One technical application of ultrasonic wave is non- destructive testing of metals.

- Ultrasonic waves of high frequency are made to travel in a beam with a little spreading.
- An ultrasonic pulse penetrating a metal when strike a flaw, which has different acoustical properties from the surrounding material, is reflected.
- In the Fig 8.35, three pulses are obtained on the display CRO, they are due to the transmitted pulse A, the pulse reflected B from the flaw and pulse C reflected from the boundary of the specimen.
- The detection of this reflected pulse reveals the presence of defects such as an internal crake or cavity in the metal.

FIGURE



Three pulses A, B & C displayed on CRO showing flaws in metal.

iii. Ultrasonic waves in liquids can be used for cleaning metal parts by removing all the traces of foreign matter sticking to the metal in otherwise inaccessible places.

- Bacteria and micro-organisms in liquids and air can be killed by ultrasonic waves of sufficient intensity.
- The ability to focus very intense ultrasonic waves in a small region without disturbing the surrounding tissues provides a very effective tool in neurosurgery.

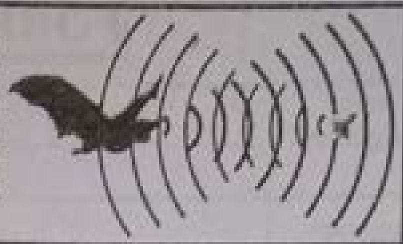
- Special types of ultrasonic equipment are in use for the treatment of arthritics, muscular rheumatism and sciatica.

iv. The ultrasonic waves are used in a process, such as cavitation, which helps in degassing.

- When ultrasonic waves are focused on a small space in liquid, very high intensity can be produced. The liquid is rapidly volatilized and a large number of bubbles are formed. This process is called cavitation.
- The collapse of these bubbles produces violently destructive forces near the solid surface in a liquid.

Do You Know?

A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance. A use of the speed of sound by a bat to sense distances.



Generation of ultrasonic sound waves

- Ultrasonic waves can be generated by any object which is capable of oscillation at a frequency higher than 20 kHz.

(1) In most of application, ultrasonic waves are generated by applying an electric current to a specialized kind of crystal known as Piezoelectric Crystal. This crystal converts electrical energy in to mechanical energy, as a result the crystal is set to vibrate at high frequency, generating ultrasonic waves

$$v = f\lambda \quad \text{..........} (1)$$

(2) In another Technique a magnetic field is applied to a special crystal which causing it to oscillate at a frequency higher than 20 K Hz, emitting ultrasonic waves.

Detection of Ultrasonic Waves

- There are so many methods by which ultrasonic waves can be detected, but here we shall consider only two methods.

(a) **Piezoelectric Detection method:**

- As ultrasonic waves consisting of compressions and rarefaction so when they are allowed to fall on a quartz crystal, a certain Potential difference is produced across the crystal faces.

This potential difference is amplified by an amplifier and the ultrasonic waves are detected.

$$V = V_0 \sin\omega t$$

$$V = V_0 \sin(2\pi f t)$$

(b) **Kundt's tube method:**

- Kundt's tube is a long glass tube supported horizontally with an air column in it.
- The lycopodium powder is sprinkled in thy tube.
- When ultrasonic waves are allowed to pass through this Kundt's tube, the lycopodium powder in the tube collects at the nodes and blown off at the antinodes.
- This method is used when the wave length is not very short.

MCQ's

1. **Which of the following is used in sonar?**
(A) sound waves (B) ultrasonic waves (C) radio waves (D) microwaves

2. **Radar is the application of:**
(A) Chemical effect (B) electric effect (C) Doppler's effect (D) magnetic effect

3. **Stars moving towards the earth show:**
(A) Blue shift (B) Red shift (C) No shift (D) Longer wavelength

4. **Car A has a siren sounding a note of 540 Hz. A listener in car B hears a note of 544 Hz . Both move in the same direction, one concludes that:**
(A) B leads A and move faster (B) B is behind A and moves faster
(C) B leads A and move slower (D) B is behind A and moves slower

5. The location of submarines can be detected by:
(A) Doppler effect (B) Temperature effect (C) Diffraction (D) Compton's effect

6. Star moving away from earth shows:
(A) Blue Shift (B) Red Shift (C) Doppler's Shift (D) Frequency Shift

Answers Key

1. B	2. C	3. A	4. C	5. A	6. B

FORMULAE

Relation between velocity, frequency and wavelength of waves	$v = f\lambda$	
Relation between frequency and time period	$f = \frac{1}{T}$	
Relation between phase difference and path difference	$\phi = \frac{2\pi}{\lambda} x$	
Speed of sound	$v = \sqrt{\frac{E}{\rho}}$	
Boyle's law for isothermal process	PV = constant	
Newton's formula for speed	$v = \sqrt{\frac{P}{\rho}}$	
Boyle's law for adiabatic process	PV^{γ} = constant	
Speed of sound in air	$v = \sqrt{\frac{\gamma P}{\rho}}$	
Relation between speed of sound and temperature	$\frac{v_t}{v_o} = \sqrt{\frac{T}{T_o}}$	$v_t = v_0 + 0.61t$
Condition for constructive interference in sound waves	$\Delta S = n\lambda$ where $n = 0, \pm1, \pm2, \pm3, \ldots$	
Condition for destructive interference in sound waves	$\Delta S = (2n+1)\frac{\lambda}{2}$ where $n = 0, \pm1 \pm2, \pm3\ldots$	$\Delta S = (n + \frac{1}{2})\lambda$ where $n = 0, \pm1 \pm2, \pm3\ldots$
Beat frequency	$f_A - f_B = n$	
Velocity of transverse waves on string	$v = \sqrt{\frac{F}{m}}$	
Fundamental frequency of stationary waves on string	$f = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$	
Frequency of stationary waves on string(nth mode)	$f_n = n\left(\frac{v}{2\ell}\right)$	$f_n = n f_1$ where $n = 1, 2, 3, 4, \ldots$

Wave length of stationary waves on string (nth mode)	$\lambda_n = \frac{2\ell}{n}$ where n = 1,2,3, ...	
Frequency of longitudinal stationary waves in organ pipe open at both ends (nth mode)	$f_n = n\left(\frac{v}{2\ell}\right)$	$f_n = n f_1$ where n = 1, 2, 3, 4, ----
Wave length of longitudinal stationary waves in organ pipe open at both ends (nth mode)	$\lambda_n = \frac{2\ell}{n}$ where n = 1,2,3, ...	
Apparent frequency when observer(A) moves towards stationary source	$f_A = \left[\frac{v+u_o}{v}\right]f$	
Apparent frequency when observer(B) moves away from stationary source	$f_B = \left[\frac{v-u_o}{v}\right]f$	
Doppler shift	$\Delta\lambda = \frac{u_s}{f}$	
Apparent frequency when source moves towards stationary observer(C)	$f' = \left[\frac{v}{v-u_s}\right]f$	
Apparent frequency when source(s) moves away from stationary observer(D)	$f_D = \left(\frac{v}{v+u_s}\right)f$	

Key Points

- **Wave:** A disturbance of some kind by means of which energy is transmitted from one place to another place, is called wave.
- **Mechanical Waves:** Those waves which require medium for their propagation are called mechanical waves.
- **Electromagnetic Waves:** Those waves which do not require medium for their propagation are known as electromagnetic waves.
- **Transverse Waves:** A wave in which the particles of the medium vibrate along a line perpendicular to the direction of propagation of the wave is called transverse wave.
- **Longitudinal Wave:** A wave in which the particles of the medium vibrate along a line parallel to the direction of propagation of the wave is known as longitudinal wave.
- **Sound:** The stimulus due to a sounding body capable of producing sensation of hearing is known as sound.
- **Interference of Waves:** The effect produced due to the superposition of waves from two coherent sources is known as interference.
- **Beats:** The periodic variation in the loudness of sound which is heard when two notes of nearly the same frequency are played simultaneously is known as beats.
- **Stationary Waves:** The superposition of two plane waves having the same amplitude and frequency, travelling with the same speed in opposite direction along a line, produces a wave known as stationary wave.
- **Resonance:** The vibration of a body or the air column under the influence of periodic force which has the same frequency is called resonance.

- **Organ Pipes:** Organ pipes are of two types. A pipe, whose end opposite to the whistle end is closed, is called a closed organ pipe. While a pipe whose end opposite to the blowing end is open, is known as open organ pipe.
- **Doppler Effect:** The apparent change in the frequency of sound caused by the relative motion of either the source of sound or listener or both is called Doppler Effect.
- **Ultrasonic Waves:** The sound waves of frequency higher than 20000 Hz are called ultrasonic waves.

Solved Examples

Example 8.1: **A wave generator produces 500 pulses in 10s. Find the period and frequency of the pulses it produces.**

Solution:

Number of pulses produced = 500 pulses

Time taken = 10 s

(a) Frequency = f = ?

(b) T = ?

$$f = \frac{\text{No. of waves}}{\text{total time}}$$

$$f = \frac{500}{10}$$

$$f = 50 \text{ Hz}$$

$$T = \frac{1}{f}$$

$$T = \frac{1}{50} \text{ s}$$

Example 8.2: In a ripple tank, 40 waves passes through a certain point in one second. If the wavelength of the waves is 5 cm, then find the speed of the waves.

Solution:

Frequency of the waves = $f = 40 \text{ waves s}^{-1}$

Wavelength = $\lambda = 5 \text{ cm}$

Speed of waves = $v = ?$

Formula:

$$v = f\lambda$$

$$v = 40 \text{ waves s}^{-1} \times 5 \text{ cm} = 200 \text{ cm s}^{-1}$$

$$v = 2 \text{ m s}^{-1}$$

Example 8.3: **Find the speed of sound in a steel railway track, if the density of steel is 7800 kg m^{-3} and Elastic modulus is 2.0×10^{11} Nm^{-2}.**

Solution: Density of steel = $\rho = 7800 \text{ kg m}^{-3}$

Elastic modulus = $E = 2.0 \times 10^{11} \text{ Nm}^{-2}$

Speed of sound = $v = ?$

Formula

$$v = \sqrt{\frac{E}{\rho}}$$

$$v = \sqrt{\frac{2.0 \times 10^{11} \text{ N/m}^2}{7800 \text{ kg/m}^3}}$$

$$v = 5.06 \times 10^3 \text{ m s}^{-1}$$

Example 8.4: **Find the speed of sound in a Neon gas at 0°C (m = 29 g/mol and γ for monoatomic gas = 1.66).**

Given: Temperature of Neon gas T = 0°C,
Mass of the gas m = 29 g/mol,
Nature of the gas γ = 1.66

Required: speed of sound = ?

Solution: The speed of sound in gas is $v = \sqrt{\frac{\gamma RT}{m}}$

$$v = \sqrt{\frac{1.66 \times 8314\ J/K\ .\ mol\ .\ K \times 273\ K}{20.18\ Kg/K\ .\ mol}} = 432\ ms^{-1}$$

Hence $\boxed{v = 432\ ms^{-1}}$

Example 8.5: **Two pianos sound the same note. If the vibration from one is 221.60Hz and that of the other is 221.40 Hz. What is the beat frequency between the two notes?**

Solution:

Beat frequency = $f_1 - f_2$

$f = 221.60\ Hz - 221.40\ Hz$

$f = 0.20\ Hz$

Example 8.6: **The speed of a wave on a particular string is 24 m s⁻¹. If the string is 6.0 m long to what driving frequency will it resonate.**

Solution:

The possible resonance wavelength are given by

Length of string = L = 6 m

Speed of the wave = v = 24 m s^{-1}

$$\lambda_n = \frac{2L}{n}$$

$\lambda_1 = 12\ m,\ \lambda_2 = 6\ m,\ \lambda_3 = 4\ m$

Thus the possible frequencies are

(i) $f_1 = \frac{v}{\lambda_1} = \frac{24}{12} = 2\ Hz$

(ii) $f_2 = \frac{v}{\lambda_2} = \frac{24}{6} = 4\ Hz$

(iii) $f_3 = \frac{v}{\lambda_3} = \frac{24}{4} = 6\ Hz$

Example 8.7: **A string 4.0 m long has a mass of 3.0 g. One end of the string is fastened to a staff and the other end hangs over a pulley with a 20 kg mass attached. What is the speed of a transverse wave in this string?**

Solution:

Length of string = L = 4.0 m

Mass of string M = 3.0 g = 0.003 kg

Mass per unit length $m = \frac{M}{L} = \frac{0.003}{4.0} = 7.5 \times 10^{-4}\ kg\ m^{-1}$

Mass suspended with wire = m' = 20 kg

Tension T = m'g = 20 × 9.8 = 196 N

Speed of transverse wave = v = ?

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{196N}{7.5\times10^{-4}\,kgm^{-1}}}$$

$v = 511.2\ m\ s^{-1}$

Example 8.9: **A pipe open at one end and close at the other end is 82 cm long. What are the three lowest frequencies to which it will resonate? Take the speed of sound as 340 m s⁻¹.**

Solution:

Length of the pipe L = 82 cm = 0.82 m

Speed of sound $v = 340\ m\ s^{-1}$

Frequency $f_1 = ?$ $f_2 = ?$ $f_3 = ?$

For the lowest frequency the wavelength $\lambda_1 = 4L$

And $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{340}{4\times0.82} = 104Hz$

As in closed pipe only odd harmonics are present. Therefore the frequency of 2nd harmonics is

$f_2 = 3f_1 = 3 \times 104 = 312\ Hz$

And $f_3 = 5f_1 = 5 \times 104 = 520\ Hz$

Example 8.10: **A car is moving at 20 m s⁻¹ along a straight road with its 500 Hz horn sounding. You are standing at the road side. What frequency do you hear as the car is (a) approaching you and (b) receding from you at 20 m s⁻¹? Take the speed of sound as 340 m s⁻¹.**

Solution:

Frequency of sound = f = 500 Hz

Speed of sound = $v = 340\ m\ s^{-1}$

(a) The apparent frequency when the car approaches = f' = ?

(b) The apparent frequency when receding the car = f'' = ?

(a) When the sounding source approaches the stationary listener, the apparent frequency heard is

$$f' = \left(\frac{v}{v-a}\right)f$$

$$f' = \frac{340}{340-20}\times500 = 531Hz$$

(b) When the car receding $f'' = \left(\frac{v}{v+a}\right)f$

$$f'' = \left(\frac{340}{340+20}\right)500$$

$$f'' = \frac{340}{360}\times500$$

$f'' = 472\ Hz$

Text Book Exercises

Q.1 Select the correct answer of the following questions.

(1) **When a wave goes from one medium to another medium, which one of the following characteristics of the wave remains constant?**

(a) Velocity (b) Frequency (c) Wavelength (d) Phase

(2) **When a transverse wave traveling through a rarer medium is reflected from denser medium then it undergoes a phase change of:**

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

(3) **If the tension in the string is doubled and its mass per unit length is reduced to half. Then the speed of transverse wave on it is**

(a) Doubled (b) Halved (c) Constant (d) One fourth

(4) **Which one of the following properties is not exhibited by the longitudinal waves?**

(a) Reflection (b) Interference (c) Diffraction (d) Polarization

(5) **A sounding source and a listener are both at rest relative to each other. If wind blows from the listener towards the source, then which one of the following of sound will change?**

(a) Frequency (b) Speed
(c) Both Frequency and Speed (d) Wavelength

(6) **Which one of the following factors has no effect on the speed of sound in a gas?**

(a) Humidity (b) Pressure (c) Temperature (d) Density

(7) **There is no net transfer of energy by particles of medium in:**

(a) Longitudinal wave (b) Transverse wave (c) Progressive wave (d) Stationary wave

(8) **Which one of the following could be the frequency of ultraviolet radiation?**

(a) $1.0 \times 10^{6} H_z$ (b) $1.0 \times 10^{9} H_z$ (c) $1.0 \times 10^{12} H_z$ (d) $1.0 \times 10^{15} H_z$

(9) **When a stationary wave is formed then its frequency is:**

(a) Same as that of the individual waves (b) Twice that of the individual waves
(c) Half that of the individual waves (d) $\sqrt{2}$ that of the individual waves
(e) Triple that of the individual waves

(10) **The fundamental frequency of a closed organ pipe is f. If both the ends of pipe are opened then its fundamental frequency will be**

(a) F (b) 0.5 f (c) 2f (d) 4f

(11) **If the amplitude of wave is doubled, then its intensity is**

(a) Doubled (b) halved (c) quadrupled (d) one fourth

(12) **A sound source is moving towards stationary listener with 1/10th of the speed of sound. The ratio of apparent to real frequency is**

(a) $\frac{11}{10}$ (b) $\left[\frac{11}{10}\right]^2$ (c) $\left[\frac{9}{10}\right]^2$ (d) $\frac{10}{9}$

No.	Option	ANSWER	EXPLANATION
1	(b)	Frequency	It depends upon source which produce it and it does not depend on medium.
2	(c)	π	
3	(a)	Doubled	$v = \sqrt{\frac{T}{m}} \longrightarrow (1)$ When T doubled $T' = 2T$ and $m' = \frac{m}{2}$ Now speed of wave is $v' = \sqrt{\frac{T'}{m'}}$ Putting value of T' and m' $v' = \sqrt{\frac{2T}{m/2}} = \sqrt{\frac{4T}{m}}$ $v' = 2\sqrt{\frac{T}{m}} \longrightarrow (2)$

			Putting value of (1) and (2) $v' = 2v$
4	(d)	Polarization	Longitudinal sound waves cannot be polarized
5	(b)	Speed	Because velocity of wind is opposite to the velocity of sound waves.
6	(b)	Pressure	Speed of sound is independent of pressure.
7	(d)	Stationary wave	
8	(d)	$1.0 \times 10^{11} H_z$	
9	(a)	Same as that of the individual waves	
10	(c)	2f	$f_{1(open)} = \frac{v}{2L}$ ..... (1) $f_{1(close)} = \frac{v}{4L}$ $f_{1(close)} = \frac{1}{2}\left(\frac{v}{2L}\right)$ ..... (2) Putting value from equation (1) in (2) $f_{1(open)} = 2 f_{1(close)}$
11	(c)	quadrupled	Intensity is directly proportional to square of the amplitude of wave. Intensity ∝ (Amplitude)^2
12	(d)	$\frac{10}{9}$	$f' = \frac{fv}{v-a}$ $\frac{f'}{f} = \frac{v}{v-a}$ Putting $a = \frac{v}{10}$ $\frac{f'}{f} = \frac{v}{v - \frac{v}{10}}$ (or) $\frac{f'}{f} = \frac{v}{\frac{10v - v}{10}}$ $\frac{f'}{f} = \frac{v}{\frac{9v}{10}}$ (or) $\frac{f'}{f} = \frac{10v}{9v}$ $\frac{f'}{f} = \frac{10}{9}$

Short Answers of the Exercise

Q.2 **Write short answers of the following questions.**

Q.1 What is the difference between progressive and stationary waves?

Ans: Difference between progressive wave and stationary waves

	Progressive wave		Stationary wave
1.	Wave profile move.	1.	Wave profile does not move.
2.	All particles vibrate with the same amplitude.	2.	Particles between two adjacent nodes vibrate with different amplitudes.
3.	Neighboring particles vibrate with different phases.	3.	Particles between two adjacent nodes vibrate in phase.
4.	All particles vibrate.	4.	Particles at nodes do not vibrate at all.

5.	Produced by a disturbance in a medium.	5.	Produced by the superposition of two waves moving in opposite direction.
6.	Transmits the energy.	6.	Does not transmit the energy.

Q.2 Clearly explain the difference between longitudinal and transverse waves.

	Transverse waves		Longitudinal waves
1.	Transverse waves consist of crests and troughs.	1.	Longitudinal waves consist of compressions and rarefactions.
2.	There are no pressure variations.	2.	There is a pressure variation throughout the medium.
3.	In transverse waves, the particles of the medium vibrate at right angles to the direction of wave propagation.	3.	In longitudinal waves, the particles of the medium vibrate parallel to the direction of wave propagation.
4.	There is no change in the density of medium.	4.	There is a change in the density throughout the medium.
5.	Light wave is an example of transverse wave.	5.	Sound wave is an example of longitudinal wave.
6.	Transverse waves can be polarized	6.	Longitudinal waves cannot be polarized

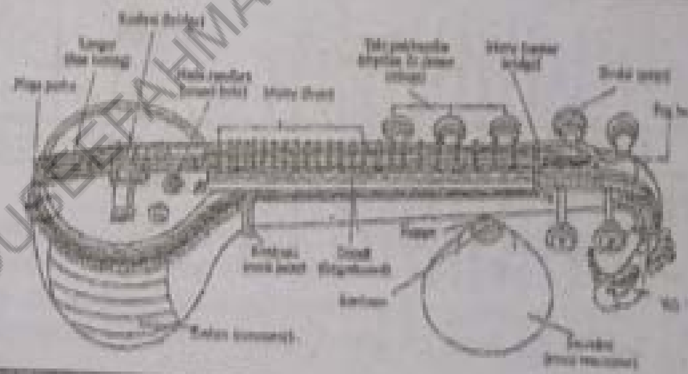
Q.3 How are beat useful in tuning musical instrument?

Ans: Tuning of musical instruments

In order to tune a musical instrument;

- Beat the instrument against a note of known frequency.
- If the two frequencies do not match, beats will be produced.
- Adjust the frequency of the untuned instrument by ***tightening or loosening*** the string.
- When ***no beats*** are heard, the musical instrument will produce the note of desired frequency

And it is said to be tuned.



Q.4 Two wave pulses traveling in opposite direction completely cancel each other as they pass. What happens to the energy possessed by the waves?

Ans: When two similar wave pulses are traveling in opposite direction and they superpose each other, then standing waves are produced.

- The standing waves consists of nodes and anti- nodes. At node the amplitude of vibration is zero i-e the particles of the medium remain at rest at the nodes. So there is no transfer of energy through the medium due to nodes.
- However only the inter-conversion of kinetic energy and potential energy takes place.
- At extreme position the P.E is maximum while at mean position K.E is maximum.

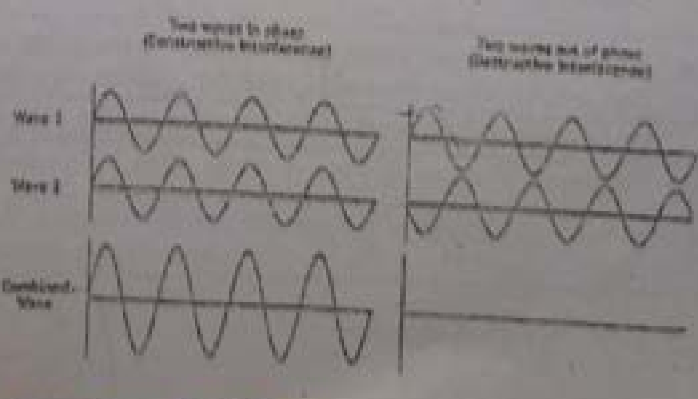
Q.5 What are the conditions of constructive and destructive interference?

Ans: The conditions for interference of waves

- The sources of the waves must be *coherent*, which means they emit identical waves with a constant phase difference.
- The waves should be of a single wavelength.
- The waves must travel in same direction and in same medium.

(i) Constructive Interference

- If crest of one wave falls on the crest of another wave or trough of one wave falls on the trough of other wave then they support each other. Such a interference in known as constructive interference.
- ***For constructive Interference path difference is zero or integral multiple of wave length.***



Path difference = o, λ, 2λ, 3λ, 4λ

Path difference = mλ

where m = 0,1,2,3..........

(ii) Destructive interference

- If crest of one wave falls on the trough of the other wave, then they cancel each other. Such a interference is known as destructive interference.
- For destructive interference the path difference is odd integral multiple of half of wavelength.

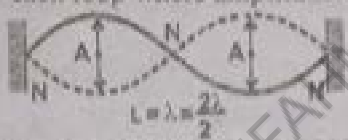
Path difference = λ/2, 3λ/2 , 5λ/2 , 7λ/2.......

Path difference = $\left(m+\frac{1}{2}\right)\lambda$, where m = 0,1,2,3,.....

Q.6 How might one can locate the position of nodes and anti-nodes in a vibrating string?

Ans:

- In stationary waves the nodes point the displacement of vibration is zero and tension is maximum.
- The nodes are at rest and located at the ends of each loop as shown in fig.
- At the antinode points the **amplitude of vibration is maximum** and tension is minimum.
- The antinodes are located at the mid of each loop where amplitude of vibration is maximum as shown in fig.



Q.7 Is it possible for an object which is vibrating transversely to produce sound wave?

Ans: Yes

Reason: It is possible for an object which is vibrating transversely to produce sound wave.

- All the string type musical instruments when played, produces transverse waves.
- And the energy from these vibrating instruments travels through air in the form of sound waves which are longitudinal waves in nature.
- For example: When the string of violin and guitar are plucked then vibrating wire produces transverse waves.
- But the disturbances of these waves in the wire vibrate air particles and produce longitudinal sound waves.

Q.8 Why does a sound wave travel faster in solid than in gases?

Ans:

- If E be the elastic modulus and ρ be the density of the medium then speed of sound waves is

$$v = \sqrt{\frac{E}{\rho}}$$

- Although the density of solids is greater than that of gases but the modulus of elasticity of solids is *much greater* than that of gases ($E_{Solid} >> E_{gases}$).

Therefore $\left[\frac{E}{\rho}\right]$ solids > $\left[\frac{E}{\rho}\right]$ gases

- Also molecules are closer in solids than in the gases, so they respond more **quickly** to a disturbance. That is why speed of sound is greater in solids than the gases.

Q.9 Why does the speed of a sound wave in a gas changes with temperature?

Ans: The speed of sound in a gas at temperature T (in kelvin) is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

As $\sqrt{\frac{\gamma R}{M}}$ = constant

Therefore, $v \propto \sqrt{T}$

- The speed of sound is directly proportional to square root of temperature in kelvin.
- When temperature of medium increases the speed of sound increases.
- When temperature of medium decreases the speed of sound decreases.
- Therefore speed of sound changes with change in temperature of gas medium.
- The speed of sound increases 0.61 m/s (61cm/s) with 1^0C rise in temperature according to the equation $v_t = v_0 + 0.61\,t$

Q.10 Is it possible for two astronauts to talk directly to one another even if they remove their helmets?

Ans: No.

- It is not possible for two astronauts to talk directly to one another if they remove their helmet.
- It is because the sound waves are longitudinal mechanical waves which require a material medium for their propagation.
- As there is no material medium between the two astronauts, so they cannot talk directly to one another through vacuum.

Q.11 Estimate the frequencies at which a test tube 15 cm long resonates when you blow across its lips?

Ans: Length of tube = L = 15 cm = 0.15 m

Speed of sound at $0C^0$ = v = 332 m/s

Fundamental frequency for closed tube is

$$f_1 = \frac{V}{4L}$$

$$f_1 = \frac{332}{4 \times 0.15}$$

$$f_1 = 553 \text{ Hz}$$

$$f_2 = 3\,f_1 = 3 \times 553 = 1660 \text{ Hz}$$

$$f_3 = 5\,f_1 = 5 \times 553 = 2765 \text{ Hz}$$

Comprehensive Questions

Q3. Give a short response to the following questions.

1. What is meant by wave motion? Define the terms wavelength and frequency and derive the relationship between them.

Ans: See Q.1 and Q.4 from book.

2. Describe longitudinal and transverse wave with examples and clearly explain the difference between them.

Ans: See Q.4 and Q.5 from book.

3. Explain the following terms:

(a) Crest **(b) Trough** **(c) Compression**
(d) Rarefaction **(e) Node** **(f) Anti-node**

Ans: See Q.4 and Q.15 from book.

4. What do you means by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decreases.

Ans: See Q.16 from book.

5. Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace?

Ans: See Q.7 from book.

6. Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

Ans: See Q.8 from book.

7. How the speeds of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61 m s^{-1}?

Ans: See Q.8 from book.

8. What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources.

Ans: See Q.12 from book.

9. What is Doppler's Effect? Derive expression for the frequencies heard.
(a) When the sounding source approaches a stationary listener.
(b) When listener move towards a stationary sounding source.

Ans: See Q.21 from book.

10. What are organ Pipes? Show that an open organ pipe is richer in harmonics than a closed organ pipe?

Ans: See Q.19 and Q20 from book.

11. Explain the vibrations in a closed organ pipe and show that the frequency of third harmonic is $\frac{5v}{4L}$.

Ans: See Q18 from book.

Numerical Problems

1. What are the wavelengths of a television station which transmits vision on 500 MHz and sound on 505MHz respectively? Take speed of electromagnetic wave as 3×10^8 m s^{-1}.

Given Data: 1st frequency = f_1 = 500MHz = 500×10^6Hz
2nd frequency = f_2 = 505 MHz = 505×10^6 Hz
Speed of e.m. waves = C = 3×10^8 m/s
Wavelength = λ_1 = ?
λ_2 = ?

Solution:

Formula:

$$C = f\lambda$$

$$C = f_1 \lambda_1$$

$$\lambda_1 = \frac{C}{f_1}$$

$$\lambda_1 = \frac{3 \times 10^8}{500 \times 10^6}$$

$$\boxed{\lambda_1 = 0.6m} \quad \text{or} \quad \boxed{\lambda_1 = 60cm}$$

Also

$$C = f_2 \lambda_2$$

$$\lambda_2 = \frac{C}{f_2}$$

$$\lambda_1 = \frac{3\times10^8}{505\times10^8}$$

$\boxed{\lambda_2 = 0.594\text{m}}$ or $\boxed{\lambda_2 = 59.4\text{cm}}$

2. A person on the sea shore observes that 48 waves reach the shore in one minute. If the wavelength of the waves is 10 metre then find the velocity of the waves.

Given Data:

frequency = f = 48 waves / min

$f = \frac{48}{60}\ \frac{\text{waves}}{\text{s}}$

$f = 0.8\ \text{Hz}$

Wavelength = λ = 10m

Velocity of wave = v = ?

Solution:

Formula:

$v = f\lambda$

$v = 0.8\times10$

$\boxed{v = 8\ \text{ms}^{-1}}$

3. In a ripple tank 500 waves passes through a certain point in 10 second, if the speed of the wave is 3.5ms^{-1}, then find the wavelength of the waves.

Given Data:

No. of waves = 500

Time = t = 10s

Speed of wave = v = 3.5 m s^{-1}

Wavelength of wave = λ = ?

Solution:

$\text{Frequency} = \frac{\text{No. of waves}}{\text{time}}$

$f = \frac{500}{10} = 50\ \text{Hz}$

As we know that $v = f\lambda$

$\lambda = \frac{v}{f}$

$\lambda = \frac{3.5}{50}$

$\boxed{\lambda = 0.07\text{m}}$ or $\boxed{\lambda = 7\text{cm}}$

4. A string of a guitar 1.3 m long vibrates with 4 nodes, 2 of them at the two ends. Find the wavelength & speed of the wave in the string if it vibrates at 500 Hz.

Given Data:

Length = L = 1.3 m

No. of loops = n = 3

Frequency = f_3 = 500Hz

(a) Wavelength λ_3 = ?

(b) Speed = v = ?

Solution:

Formula:

$\lambda_n = \frac{2L}{n}$

(a) $\lambda_3 = \frac{2L}{3}$

$\lambda_3 = \frac{2\times1.3}{3}$

$\boxed{\lambda = 0.866\ m}$

(b) $v = f_3 \lambda_3$

$v = 500 \times 0.866$

$\boxed{v = 433\ ms^{-1}}$

5. A tension of 400 N causes a 300 g wire of length 1.6 m to vibrate with a frequency of 40 Hz. What is wavelength of transverse waves?

Find the speed of sound in Helium gas at 27°C, (γ = 1.66 and R = 8334 J/k mol).

Data:

Tension = T = 400 N

Length = L = 1.6 m

Total mass = M = 300 g = 0.30 kg

Frequency = f = 40 Hz

Wavelength = λ = ?

Solution: speed of wave in stretched string is

$$v = \sqrt{\frac{T \times L}{M}}$$ Putting values

$$v = \sqrt{\frac{400 \times 1.6}{0.3}}$$

$$v = 46.2\ m/s$$

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{46.2}{40}$$

$$\lambda = 1.15\ m$$

6. Compare the theoretical speed of sound in hydrogen (M_H = 2.0 g/mol, γ_H = 1.4) with helium (M_{He} = 4.0 g/mol, γ_{He} = 1.66 & R = 8334jk^{-1}mol) at 0°C.

Given Data:

$M_H = 2$ g/mol

$\gamma_H = 1.4$

$M_{He} = 4$ g/mol

$\gamma_{He} = 1.66$ J mol K^{-1}

R = 8334 J mol K^{-1}

Temp = T = 0°C = 273 K

Solution:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{v_{He}}{v_H} = \frac{\sqrt{\frac{\gamma_{He} RT}{M_{He}}}}{\sqrt{\frac{\gamma_H RT}{M_H}}}$$

$$\frac{v_{He}}{v_H} = \sqrt{\frac{\gamma_{He} M_H}{\gamma_H M_{He}}} = \sqrt{\frac{1.66 \times 2}{1.4 \times 4}} = \sqrt{\frac{3.32}{5.6}}$$

$$\frac{v_{He}}{v_H} = \sqrt{0.593}$$

$$\frac{v_{He}}{v_H} = 0.77$$

$$v_{He} = 0.77\ v_H$$

7. The speed of sound in air at 0°C is 332 m s^{-1}. What will be the speed of sound at 22°C?

Data:

Speed of sound in air at 0°C = v_o = 332 m s^{-1}

T_o = 0°C = 0 + 273 = 273K

T = 22°C = 22 + 273 = 295K

Speed of sound at 22°C = v = ?

Solution:

Formula:

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}}$$

$$v = v_o\sqrt{\frac{T}{T_o}}$$

$$v = 332\sqrt{\frac{295}{273}}$$

$$v = 332\,(1.0395)$$

$$\boxed{v = 345\ ms^{-1}}$$

ALTERNATE METHOD

$v = v + 0.61\,t$ (where t = 22°C)

$v = 332 + 0.61\,(22)$

$v = 332 + 13.2$

$$\boxed{v = 345.2\ m/s}$$

8. Two tuning forks P and Q give 4 beats per second. On loading Q lightly with wax, we get 3 beats per second. What is the frequency of Q before and after loading if the frequency of P is 512 Hz?

Given Data: Frequency of tuning fork p = f_P = 512 Hz

Frequency of tuning fork Q = f_Q = ?

No. of beats before loading tuning fork Q = 4 beats/s

No. of beats after loading tuning fork Q = 3 beats/s

Before loading

$f_Q = f_P \pm 4$

$f_Q = f_P + 4$

$f_Q = 512 + 4 = 516$ Hz

f_Q may be

$f_Q = f_P - 4$

$f_Q = 512 - 4 = 508$ Hz

f_Q = 516 Hz or 508 Hz

Suppose frequency of tuning fork Q is 516Hz when tuning fork Q is loaded with wax its frequency decreases. After loading if frequency of tuning fork Q is 515Hz then it can produce 3 beats per second when it is vibrated with tuning fork P of frequency 512.

515 − 512 = 3 beats/s

Therefore, frequency of tuning fork Q before loading = 516Hz

After loading = 515Hz

9. On a sunny day, the speed of sound in the air is 340 m s^{-1}, 2 tuning forks A & B are sounded simultaneously. The wave length of the sounds emitted are 1.5 m and 1.68 m respectively. How many beats will produce per second?

Given Data:

Speed of sound = v = 340 m/s

Wavelength of wave produced by tuning fork A = λ_A = 1.5m

Wavelength of wave produced by tuning fork B = λ_B = 1.68m

No. of beats/s = beat frequency = $f_A - f_B$ = ?

Solution:

Frequency of sound waves produced by tuning fork A is

$$f_A = \frac{v}{\lambda_A} \qquad (v = f_A \lambda_A)$$

$$f_A = \frac{340}{1.5}$$

$$\boxed{f_A = 226.66 \text{ Hz}}$$

Frequency of sound waves produced by tuning fork B

$$f_B = \frac{v}{\lambda_B} \qquad (v = f_B \lambda_B)$$

$$f_B = \frac{340}{1.68}$$

$$\boxed{f_B = 202.38 \text{ Hz}}$$

No-of beat per second $= f_A - f_B$

$= 226.66 - 202.38$

$= 24.28$ beats/s

Therefore 24 beats per second produce approximately.

10. A sound source vibrates at 200 Hz and is receding from a stationary observer at 18 ms^{-1}. If the speed of sound is 331 m s^{-1} then what frequency does the observer hear?

Given Data:

Original frequency of sound = f = 200 Hz

Speed of source of sound = a = 18 m s^{-1}

Speed of sound = v = 331 m s^{-1}

Apparent frequency = f′ = ?

Solution:

As the source of sound is receding from stationary observer then apparent frequency is given by

$$f' = \left(\frac{v}{v + a}\right) f$$

Putting values

$$f' = \left(\frac{3.31}{331 + 18}\right) 200$$

$$f' = 189.68 \text{ Hz}$$

11. Suppose a train that has a 150 Hz horn is moving at 35 m/s in still air on the day when the speed of sound is 340 m/s.(a) what frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes and moves away from him ? (b) what frequency is observed by the train's engineer travelling on the train?

Given Data:

Speed of source of sound = a = 35 m/s

Frequency of sound = f = 150Hz

Speed of sound = v = 340 m/s

To Find:

(a (i)) The apparent frequency when the train approaches = f' = ?

(a (ii)) The apparent frequency when the train recedes past the listener = f' = ?

(b) Frequency observed by the train's engineer travelling on the train = ?

Solution:

(a (i)) When the sounding source approaches the stationary listener the apparent frequency heard is:

$$f' = \left(\frac{v}{v-a}\right) f'$$

$$= \left(\frac{340}{340-35}\right) \times 150$$

$$f' = 167 \text{ Hz}$$

(a (ii)) When the train is moving away from stationary observer the apparent frequency heard is

$$f' = \left(\frac{v}{v+a}\right) f$$

$$= \left(\frac{340}{340+35}\right) \times 150$$

$$f' = 136\text{Hz}$$

(b) There is no change in frequency observed by the train's engineer travelling on the train because his speed is equal to speed of train therefore, he observes the original frequency i-e 150 Hz

12. The first overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe 3.6 m in length. What is the length of the open organ pipe.

Given Data: Length of closed organ pipe = L = 3.6m

Frequency of first overtone of open pipe = $f_2 = 2f_1$

Putting $f_1 = \frac{v}{2L'}$

$$f_2 = 2\left(\frac{v}{2L'}\right) \quad \text{.......(1)}$$

Frequency of first overtone of close pipe = $f_2 = 3f_1$

Putting $f_1 = \frac{v}{4L}$

$$f_2 = 3 \times \frac{v}{4L} \quad \text{....... (2)}$$

According to given condition

First overtone of open pipe = First overtone of close pipe

Putting values from Equation (1) and (2)

$$2(f_1)_{open} = 3(f_1)_{close}$$

$$\cancel{2}\left(\frac{\cancel{v}}{\cancel{2}L'}\right) = 3 \times \frac{\cancel{v}}{4L}$$

$$L' = \frac{4L}{3}$$

$$L' = \frac{4 \times 3.6}{3}$$

$$L' = 4.8 \text{ m}$$

N/Q.(For book answer) The first harmonic of an open organ pipe has the same frequency as the first harmonic of a closed pipe 3.6 m in length. What is the length of the open organ pipe. (

Given Data: Frequency of first harmonic of closed pipe is

$$f_1 = \frac{v}{4L}$$

Frequency of first harmonic of open pipe is

$$f_1' = \frac{v}{2L'}$$

Length of open pipe = L' = ?

Solution:

According to given condition frequency of 1[st] harmonic of closed pipe = frequency of 1[st] harmonic of open pipe

$$f_1 = f_1'$$

Putting values of f_1 and f_1'

$$\frac{v}{4L} = \frac{v}{2L'} \quad \text{(v same)}$$

Rearranging

$$L' = 2L$$

$$L' = 2(3.6)$$

$$L' = 7.2\text{m}$$

13. What length of open organ pipe will produce a frequency of 1200 Hz as its first overtone on a day when speed of sound is 340m/s?

Given Data: frequency of 1st overtone of open organ pipe = f_2 = 1200 Hz
Speed of sound = v = 340 m/s

To Find: Length of open organ pipe = L = ?

Solution:

Since $f_2 = 2f_1$

Putting $f_1 = \frac{v}{2L}$

$$f_2 = 2\left(\frac{v}{2L}\right)$$

$$L = \frac{v}{f_2}$$

$$L = \frac{340}{1200}$$

$$L = 0.283 \text{ m} = 28.3\text{cm}$$

Additional Conceptual Short Questions With Answers

1. Why does the flash of lighting seen earlier than hearing the thunder?

Ans: The speed of light is 3×10^8 m/s

Speed of sound is 332 m/s

Speed of light is much larger than speed of sound, so the flash is seen earlier than thunder.

2. How much greater the speed of sound in hydrogen to that of oxygen?

Ans: As the density of oxygen is 16 times greater to that of hydrogen. And we know that

$$v \propto \frac{1}{\sqrt{\rho}}$$

So speed of sound in hydrogen is 4 times greater than the oxygen.

3. Is the speed of transverse wave on a string the same as the speed at which a particle on the string moves?

Ans: The speed V_{wave} of a transverse wave on a string specifies how fast the disturbance moves along the string. The wave speed depends upon the properties of the string as it is clear from the relation:

$$V = \sqrt{\frac{F \times L}{M}}$$

If F(Tension), ℓ(string length), m(mass of string) remains same then wave speed remains constant.

But particle speed $V_{particle}$ specifies how fast the particle is moving as it oscillates up and down and for periodic transverse wave, particles of string perform simple harmonic motion *i.e.*, speed of particles does not remain same, like V_{wave}, maximum at mean position and minimum at extreme position. Moreover, speed of particle $V_{particles}$ do not depend upon properties of the string.

So speed of wave V_{wave} is different from speed of particles of the string $V_{particles}$.

4. At which temperature will the velocity of sound in air become double than the velocity in air at 0°C.

Ans: Speed at 0°C = V_o

Absolute temp. at 0°C = T_o = 0 + 273 = 273

Speed at high temp. = V_t = ?

T = ?

$$\frac{V_t}{V_o} = \sqrt{\frac{T}{T_o}} \quad \ldots\ldots(1)$$

Putting $V_1 = 2V_o$ and $T_o = 273K$

Therefore, eq. (1) becomes

$$\frac{2V_o}{V_o} = \sqrt{\frac{T}{273}}$$

$$2 = \sqrt{\frac{T}{273}}$$

Squaring both sides

$$4 = \frac{T}{273}$$

$$T = 4 \times 273$$

$$T = 1092\ K$$

$$T = 1092 - 273$$

$$T = 819^\circ C$$

A boy moves away with a half the speed of sound emitted by a speaker of a mosque. What frequency of sound is heard by him?

Ans: When an observer moves away from the source of sound with velocity V_o then frequency of sound observed by him is given by

$$f' = \left(\frac{V - V_o}{V}\right)f$$

where V is velocity of sound

As $V_o = \frac{V}{2}$

$$f' = \left(\frac{V - V/2}{V}\right)f$$

$$f' = \left(\frac{2V - V}{2V}\right)f$$

$$f' = \left(\frac{V}{2V}\right)f$$

$$f' = \frac{f}{2}$$

The body would receive only half of the frequency of sound.

As a result of distant explosion, an observer senses a ground tremor and then hears the explosion. Explain the time difference?

Ans: Explanation

The speed of sound is given by

$$v = \sqrt{\frac{E}{\rho}}$$

Since the speed of waves in solids (earth) is greater than the speed of wave in gases (air) due to *much greater* value of elastic modulus. That is why the observer senses the ground tremor first and then hears the explosion. The time difference is due to different speed of waves in two different mediums.

Why is a diver under water unable to hear the sound produced in air?

Ans: The water surface reflects the most of the sound waves back in to air. And only a little fraction of incident intensity (= 0.1%) is refracted through water. Due to this reason the diver under water cannot hear sound produced in air.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **If Stretching force T of wire increases, then its frequency:**
   (a) Decreases (b) Increases (c) Remains the same (d) All of these
2. **A stretched string of length 2m vibrates in four segments, its wave length:**
   (a) 0.25m (b) 0.5m (c) 1m (d) 0.75m
3. **In a stretched string, if tension in string is increased four times, then speed of waves increases..**
   (a) 2 times (b) 4 times (c) 8 times (d) 16 times
4. **If pressure of air is increased four times the speed of sound will be:**
   (a) 2 times (b) 4 times (c) constant (d) 16times
5. **When wave enters from one medium to another then which of following does not change:**
   (a) Wavelength (b) frequency (c) speed (d) none
6. **Velocity of sound at 0C° is 332m/s at which temperature it will become 664m/s**
   (a) $273\,^{0}c$ (b) $546\,^{0}c$ (c) $819\,^{0}c$ (d) $1090\,^{0}c$
7. **If temperature is $10^{0}C$ then speed of sound in air will:**
   (a) $332\ ms^{-1}$ (b) $340\ ms^{-1}$ (c) $338.1\ ms^{-1}$ (d) $339\ ms^{-1}$
8. **Speed of stationary waves in stretched strings are independent of**
   (a) Number of loops (b) Tension in the string (c) Point where it is plucked (d) Both a & c
9. **Length of a pipe is 10cm (closed at one End), maximum wavelength can be ...**
   (a) 5cm (b) 10cm (c) 20cm (d) 40cm
10. **Waves which propagate by oscillations of material particle are known as**
    (a) Magnetic waves (b) Material waves (c) E.M waves (d) Mechanical waves
11. **To monitor blood flow, ultrasonic waves _______ are used**
    (a) 5 MHz – 10 MHz (b) 25 MHz – 30 MHz (c) 9 MHz – 90 MHz (d) 20 MHz – 200 MHz
12. **An organ pipe 50cm long with one end closed its fundamental frequency is _____(v = 332 m/s)**
    (a) 166 Hz (b) 200 Hz (c) 332 Hz (d) 400 Hz
13. **Fundamental frequency of pipe closed at one end is 85Hz next two harmonics are**
    (a) 170Hz, 255Hz (b) 255Hz, 340Hz (c) 170Hz, 340Hz (d) 255Hz, 425Hz
14. **Temperature at which velocity of sound is two times its velocity at $10^{o}C$**
    (a) 1321K (b) 1213K (c) 1132K (d) 1231K
15. **Frequency of n^{th} mode of vibration for stationary waves in a pipe open at both end is**
    (a) $f_n = \frac{nv}{4l}$ (b) $f_n = \frac{4l}{nv}$ (c) $f_n = \frac{2l}{nv}$ (d) $f_n = \frac{nv}{2l}$
16. **Distance between two consecutive nodes is:**
    (a) $\frac{\lambda}{2}$ (b) λ (c) $\frac{\lambda}{4}$ (d) 4λ
17. **Stars moving towards earth show a**
    (a) Red shift (b) Blue shift (c) White shift (d) yellow shift
18. **According to Laplace's point of view, sound waves travel in air under the conditions of:** (FBISE 2017)
    (a) Isothermal (b) Isobaric (c) Isochoric (d) Adiabatic
19. **If a transverse wave, travelling in a rarer medium, is reflected from a denser medium, it undergoes a path difference of:** (FBISE 2018)
    (a) $\frac{\lambda}{2}$ (b) λ (c) $\frac{\lambda}{8}$ (d) $\frac{\lambda}{4}$

20. **For 1°C rise in temperature, the speed of sound increases by:** [FBISE (ON) 2018]
(a) $0.61ms^{-1}$ (b) $0.061ms^{-1}$ (c) $61ms^{-1}$ (d) $6.1ms^{-1}$

21. **Which of the following factors has no effect on the speed of sound in a gas?** [FBISE 2019]
(a) Pressure (b) Temperature (c) Density (d) Humidity

22. **There is no net transfer of energy by particles of medium in:** [FBISE (ON) 2019]
(a) Longitudinal wave (b) Transverse wave (c) Progressive wave (d) Stationary wave

Answers Key

1.	b	2.	c	3.	a	4.	c	5.	b
6.	c	7.	c	8.	d	9.	d	10.	d
11.	a	12.	a	13.	d	14.	c	15.	d
16.	a	17.	b	18	d	19	a	20	a
21	a	22	d						

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 Choose the correct answer from the given options. **(1 x 6 = 6)**

SECTION – A

1. **The waves in which particles of the medium have displacement along the direction of propagation of waves are called:**
   (A) transverse waves (B) longitudinal waves
   (C) electromagnetic waves (D) stationary wave
2. **A taut wire is clamped at two points 1 m apart. It is plucked near one end. Which are the three longest wavelengths present on the vibrating wire?**
   (A) 1.0 m, 0.50 m and 0.25 m (B) 1.0 m, 0.67 m and 0.50 m
   (C) 2.0 m, 0.67 m and 0.40 m (D) 2.0 m, 1.0 m and 0.67 m
3. **When the amplitude of a wave becomes double, it energy becomes:**
   (A) one half (B) double (C) four times (D) six times
4. **Speed of sound in vacuum is:**
   (A) $280ms^{-1}$ (B) $332ms^{-1}$ (C) $333ms^{-1}$ (D) zero
5. **If a train of waves moving along a rope has a velocity of $100\ m\ s^{-1}$ and a wavelength of 20 m, then the time period is:**
   (A) 5 second (B) 2000 second (C) 0.2 second (D) 666 second
6. **A sound source is moving towards stationary listener with $1/10^{th}$ of the speed of sound. The ratio of apparent to real frequency is:**
   (A) $\frac{11}{10}$ (B) $\left[\frac{11}{10}\right]^2$ (C) $\left[\frac{9}{10}\right]^2$ (D) $\frac{10}{9}$

Question.No.2 Give short answers of followings: **(3 x 7 =21)**

SECTION – B

(i) Why sound produced due to sudden explosion going on in the sun cannot be heard on the earth?
(ii) When tension produced in increased by 16 times then what will be the effect on speed of wave in the stretched string?
(ii) Why does sound travel faster in solids than in gases?
(iv) Clearly explain the difference between longitudinal and transverse waves.
(v) Estimate the frequencies at which a test tube 15 cm long resonates when you blow across its lips?
(vi) A 40 g string 2 m in length vibrates in three loops. The tension in the string is 270 N. What is the wavelength and frequency?
(vii) Give some properties of stationary waves.

Question.No.3 Extensive Questions. **(13)**

SECTION – C

(a) Define Doppler effect and discuss its different cases. (8)
(b) Suppose a train that has a 150 Hz horn is moving at 35 m/s in still air on the day when the speed of sound is 340 m/s.
(i) what frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes and moves away from him ?
(ii) what frequency is observed by the train's engineer travelling on the train? (5)

The End

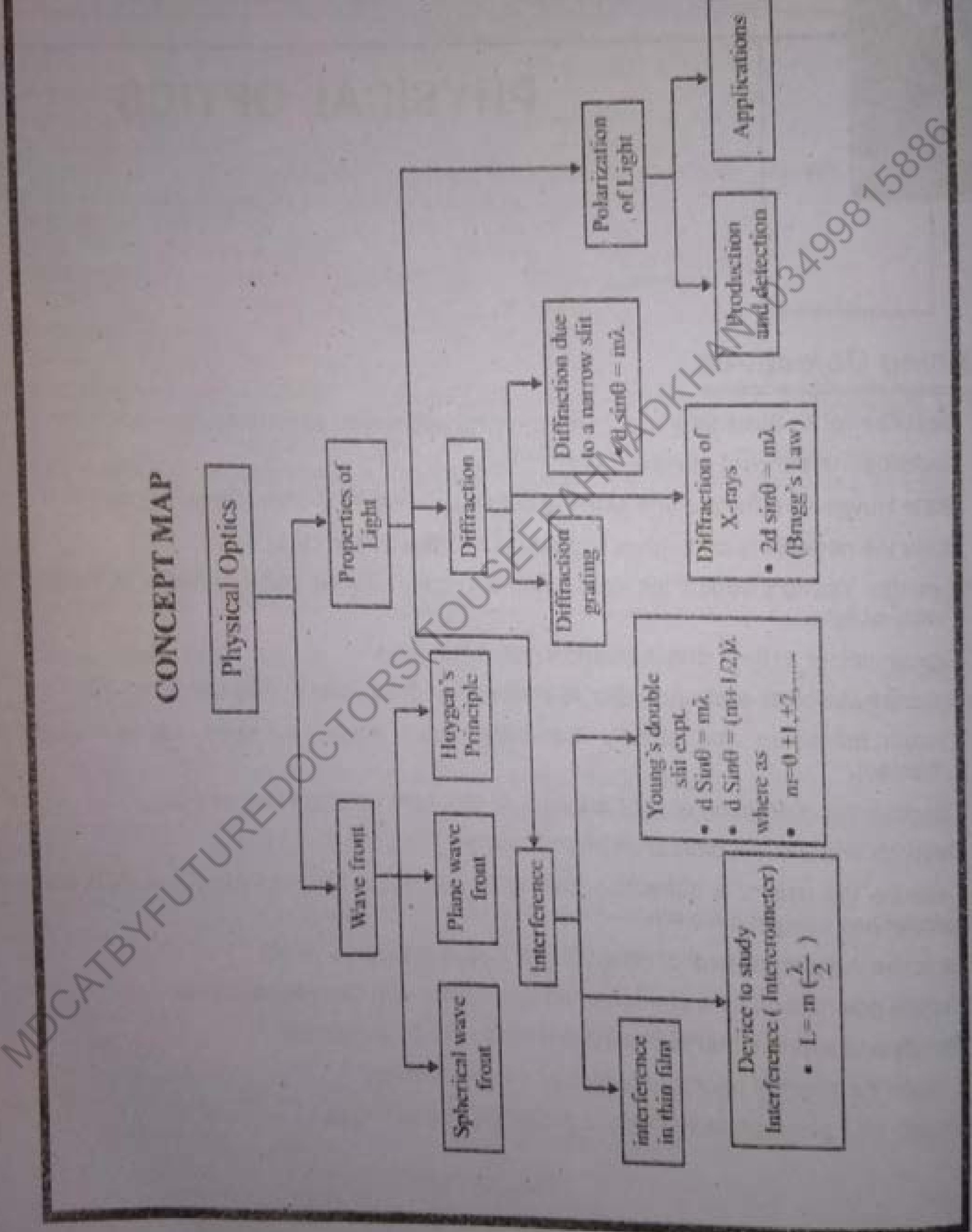
CHAPTER 9

PHYSICAL OPTICS

Learning Objectives

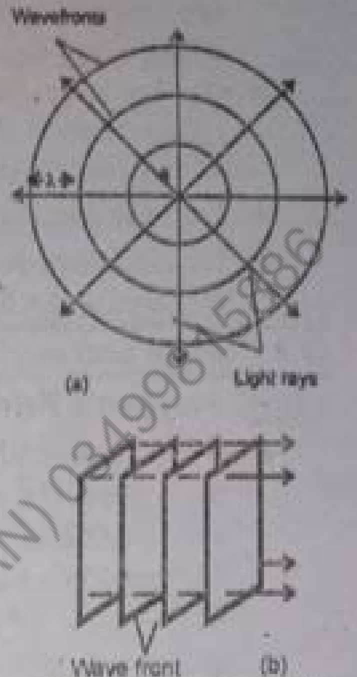
- Describe light waves as a part of electromagnetic waves spectrum.
- Describe the concept of wave front.
- State Huygen's principle and use it to construct wave front after a time interval.
- State the necessary conditions to observe interference of light.
- Describe Young's double slit experiment and the evidence that it provides to support the wave theory of light.
- Explain colour pattern due to interference in thin films.
- Describe the parts and working of Michelson interferometer and its uses.
- Explain diffraction and identify that interference occurs between waves that have been diffracted.
- Describe that diffraction of light is evidence that light behaves like waves.
- Describe and explain diffraction at a narrow slit.
- Describe the use of a diffraction grating to determine the wavelength of light and carry out calculations using $d \sin \theta = n \lambda$.
- Describe the phenomena of diffraction of X-rays through crystals.
- Explain polarization as a phenomenon associated with transverse waves.
- Identify and express that polarization is produced by a Polaroid.
- Explain the effect of rotation of polaroid on polarization.
- Explain how plane polarized light is produced and detected.

Chapter No. 9


CONCEPT MAP
Physical Optics
Properties of Light
Wave front
Plane wave front
Spherical wave front
Huygen's Principle
Interference
Diffraction
Polarization of Light
Production and detection
Applications
Diffraction due to a narrow slit
d sinθ = mλ
Diffraction grating
Diffraction of X-rays
2d sinθ = mλ (Bragg's Law)
Young's double slit expt.
d Sinθ = mλ
d Sinθ = (m+1/2)λ
where as
m=0,±1,±2,......
interference in thin film
Device to study Interference (Interferometer)
L = m(λ/2)

"The branch of physics which deals with the nature of light and its different phenomenon is called physical optics".

- Light is form of energy which produce the sensation of vision.
- Light is a transverse, electromagnetic wave that can be seen by humans.
- The wave nature of light was first illustrated through experiments on diffraction and interference. Like all electromagnetic waves, light can travel through a vacuum.
- Maxwell described light as a propagating wave of electric and magnetic fields. More generally, he predicted the existence of electromagnetic radiation: coupled electric and magnetic fields traveling as waves at a speed 3×10^8 m/s
- In *1678*, *Huygens's*, an eminent *Dutch scientist*, proposed that light is the form of energy which travels in form of waves.



Spherical wave fronts (a) and plane wavefronts (b) spaced a wavelength apart. The arrows represent rays.

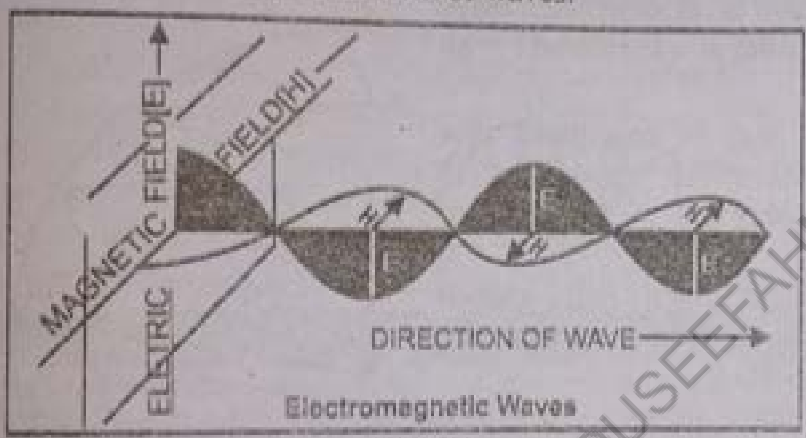


Fig: 9.1

Do You Know?

Small segments of large spherical wavefronts approximate a plane wavefront.

Q.1 What is a wave front?

Ans: Wave Fronts

Such a surface on which all the points of waves have same phase of vibration is knows as wave front

EXPLANATION:
The wave fronts far away from the source are very large spheres. A small portion of the sphere will be the plane wave front.

Explanation

- Suppose the light emitted from a point source propagates outward in all direction with speed c. After time t, the waves reach the surface of an imaginary sphere with center as S and radius as "ct".
- As the distance of all these points from the source is same so all the points on the surface of the sphere have the same phase of vibration. Such as surface is known as wave front.

FIGURE

Note

- The wave front from a point source are *spherical*.
- Thus wave propagates in space by the motion of wave fronts.
- The distance between two consecutive wave fronts is *one wave length.*

Ray of Light

- The line normal to the wave front which shows the direction of propagation of light is called a ray of light.

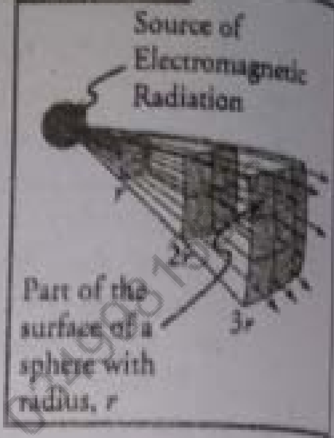
(1) **Spherical wave front:**
The wave front in which the light waves are propagated in spherical form with the source is called spherical wave front.

- For a point source of light in a homogenous medium, the wave fronts are the concentric sphere of increasing radii.

(2) Plane wave front

- At very large distance (i.e. at infinity) from the source, a small portion of spherical wave front will become very nearly plane. Such a wave front is known plane wave front as shown in figure 9.3.
- For example, *the sun light reaches the earth in plane wave fronts.*
- On laboratory scale, in order to obtain plane wave from a point source, it is placed at the focus of convex lens.

FIGURE 9.3



Q.2 State and explain the Huygen's principle?

Ans: Huygen's Principle:

- If the location of the wave front at any instant t is known then Huygen's principle enables us to determine shape and location of the new wave front at a later time $t + \Delta t$. This principle has two parts;

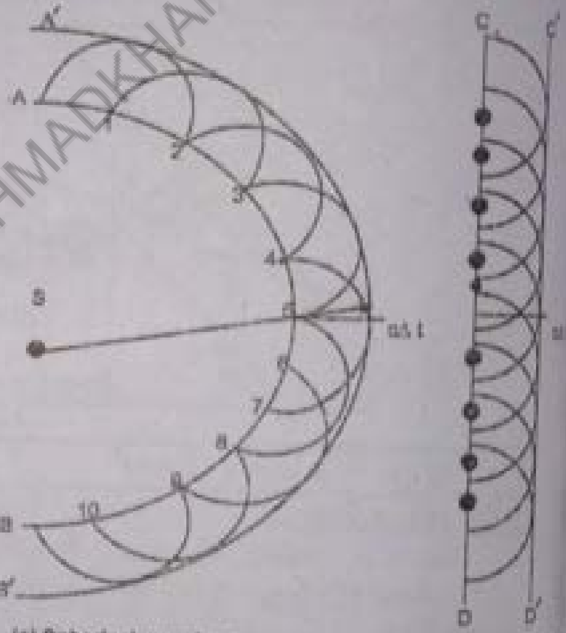
i. Every point of a wave front may be considered as a source of secondary wavelets which spread out in forward direction with a speed equal to the speed of propagation of the wave.

ii. The new position of the wave front after a certain interval of time can be found by constructing a surface that touches all the secondary wavelets.

Explanation:

Let AB is the wave front at time t.

To determine the wave front at time $t + \Delta t$, draw secondary wavelets with center at various points on the wave front A'B' and radius as $c\Delta t$, where c is the speed of propagation of wave. The new wave front at time $t + \Delta t$ is A'B' which is a tangent envelope to all the secondary wavelets.



(a) Spherical wavefront (b) Plane wavefront

Fig:9.4

MCQ's

1. Which of the following is the angle between the ray of light and the wave front?
   (A) 0° (B) 60° (C) 90° (D) 120°
2. In case of point source the shape of wave front is:
   (A) Plane (B) Spherical (C) Circular (D) Elliptical
3. Which of the following is the phase difference between two points on wave front?
   (A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) π
4. The wave nature of light was proposed by:
   (A) Young (B) Galileo (C) Huygens (D) Newton
5. Light from sun reaches the earth in the form of:
   (A) Spherical wave front (B) Plane wave front (C) Elliptical wave front (D) Hyperbolic wave front
6. The distance between the two consecutive wave fronts is called:
   (A) Time period (B) Frequency (C) Wavelength (D) Displacement
7. When the sunlight passes through atmosphere, the total energy is reduced due to
   (A) Reflection by dust particles (B) Scattering by dust particles (c) Absorption by dust particles (D) All of these
8. The blue colour of sky is due to:
   (A) Diffraction (B) Reflection (C) Polarization (D) Scattering

9. When a ray of light enters from denser into a rare medium, wavelength of light ray will:
(A) Increase (B) Decrease (C) Unchanged (D) Cannot be determined

10. According to Huygen's principle, each point on a wave front acts as a source of:
(A) Secondary wavelet (B) Primary wavelet (C) New wave front (D) Sound

Answers Key

1. C	2. B	3. A	4. C	5. B	6. C	7. D	8. D	9. A	10. A

Q.3 What is Coherent Source?

Ans: **Coherent Sources:**

- The sources which produce waves having the same frequency, equal or comparable amplitude and a constant phase difference are called coherent sources.
- Our most important source of light and life sustaining radiation is the sun.
- The most artificial sources of light are the hot bodies which radiate light and infrared radiation.
- Thus each source of light emits a very large number of waves with random phases. Because in a light source the phase constantly changes, as light is emitted in short bursts when electrons in individual atom suffer energy changes that occur very quickly and randomly.
- Phase changes occur abruptly when different atoms come in to action. This is true for light coming from different parts of the same source except laser. To get two coherent waves from a point source, one of the following two methods is adopted.
  i. Division of wave length, as in Young's double slits, Fresnel's by prism and Lioyld's mirror.
  ii. Division of amplitude by partial reflection and transmission at a boundary as in Newton's rings.

Do You Know?

Monochromatic or single colour light is specified by a single wavelength. It is very difficult to get a truly monochromatic source of light. However, using filters one can get a source which gives light within a narrow band of wavelength.

Q.4 Define the interference of light, discuss its different types and conditions for detectable interference.

Ans: **Interference of Light Waves**

"The effect produced by the superposition of light waves from two coherent sources passing through same region in same direction is called interference of light".

Types of interference

There are two types of interference

(i) **Constructive interference**

When two light waves superpose with each other in such a way that the crest of one wave falls on the crest of the second wave, and trough of one wave falls on the trough of the second wave, then the resultant wave has larger amplitude and it is called **constructive interference**.

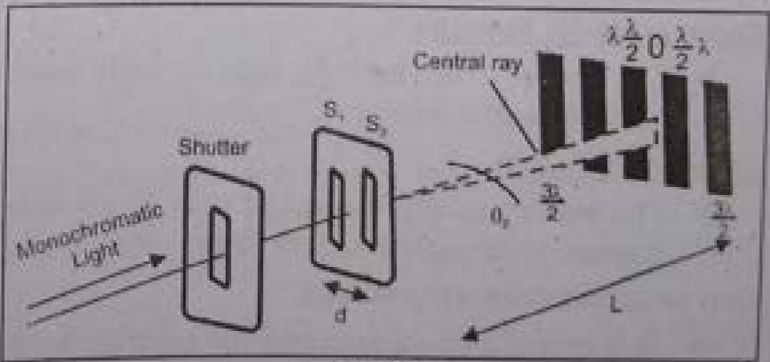


Fig:9.5

- In constructive interference maximum light appears.

For constructive interference
Path difference = d = 0, λ, 2λ, 3λ, 4λ, 5λ ....
Path difference = $d = m\lambda$ where m = 0, 1, 2, 3 ....
Phase differences are 0, 2π, 4π, 6π, ...

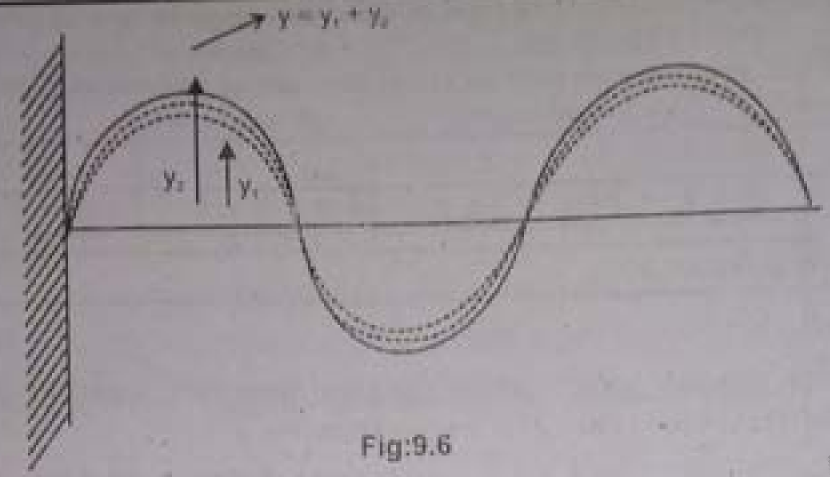


Fig:9.6

(ii) Destructive interference

- If crest of one wave falls on the trough of the other wave, then they cancel each other. Such a interference in known as destructive interference.
- For destructive interference

Path difference $= d = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, 7\frac{\lambda}{2}, 9\frac{\lambda}{2}, \ldots\ldots\ldots\ldots$

Path difference $= d = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0, 1, 2, 3, \ldots$

$$= (2m+1)\frac{\lambda}{2}$$

Phase differences are $\pi, 3\pi, 5\pi, 7\pi, \ldots\ldots\ldots$

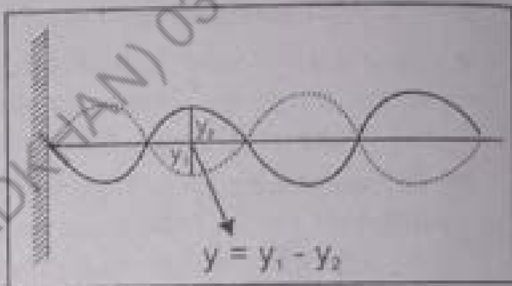


Fig:9.7

ϕ or $s = \frac{2\pi}{\lambda} \times$ (path difference)

Conditions for detectable interference pattern

The following condition must be met, in order to observe the interference phenomenon;

1. The interfering beams must be ***monochromatic***.
2. The interfering beams of light must be ***coherent***.
3. The sources should be ***narrow*** and very ***close*** to each other.
4. The intensity of the two sources be ***comparable***.

Monochromatic Sources

- The sources which emit the light of single wave length are called monochromatic sources.

Coherent Sources

- The monochromatic sources of light which emit waves, having a constant phase difference, are called coherent sources.

How to obtain coherent sources

- A common method to obtain the coherent light beam is to use a monochromatic source to illuminate a screen containing two narrow slits.
- The light emerging from the two slits is coherent because a single source produces the original beam and two slits serve only to split it into two parts.
- The points on a Huygens's wave front which sent out secondary wavelength are also coherent sources of light.

For Your Information

FOR YOUR INFORMATION:

Distortion of the picture on a television receiver

This occurs when an aircraft passes low overhead. The waves traveling directly from the transmitting antenna the receiving aerial, interferes with the waves reflected from the aircraft. Usually the reflected wave is much weaker and so the interference is never completely destructive the picture on television screen takes place.

Q.5 Describe the Young's double slit experiment for demonstration of interference of light. Derive an expression for fringe spacing.

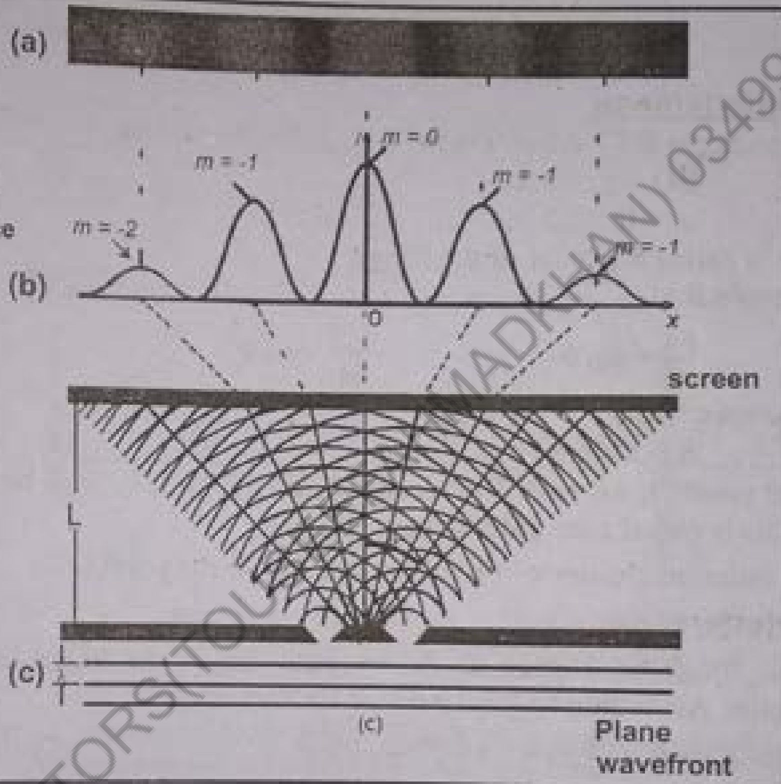
Ans: **Young's Double Slit Experiment**

In 1801, Thomas Young performed the interference experiment to prove the wave nature of light. A screen having two narrow slits is illuminated by a beam of monochromatic light.

- The principle of this experiment is based on division of wave front.
- The monochromatic light falls on the two slits as shown in figure.

FIGURE

Double-slit interference pattern using red light. (a) Photo of the interference pattern on the screen. Constructive interference produces a high intensity of red light on the screen while destructive interference leaves the screen dark.



- The portion of wave front incident on the slit behaves like the source of secondary wavelets.
- The wavelets leaving the slits are coherent. Superposition of these wavelets result into the series of bright and dark bands which are observed on the second screen placed at some distance parallel to the first screen.

Conditions for Maxima and Minima

Consider two monochromatic light rays I and II are passing through slits A and B and fall at point P on the screen.

Distance covered by ray-I = AP

Distance covered by ray-II = BP

BP > AP

- The line AQ is drawn such that AP = QP

Path difference between two rays = BQ = s

- The angle between EP and EO is θ, it can be proved that the angle BAQ=θ by assuming that AQ is nearly normal BP.
- The path difference between the wavelets, leaving slits and arriving at P, is BQ.

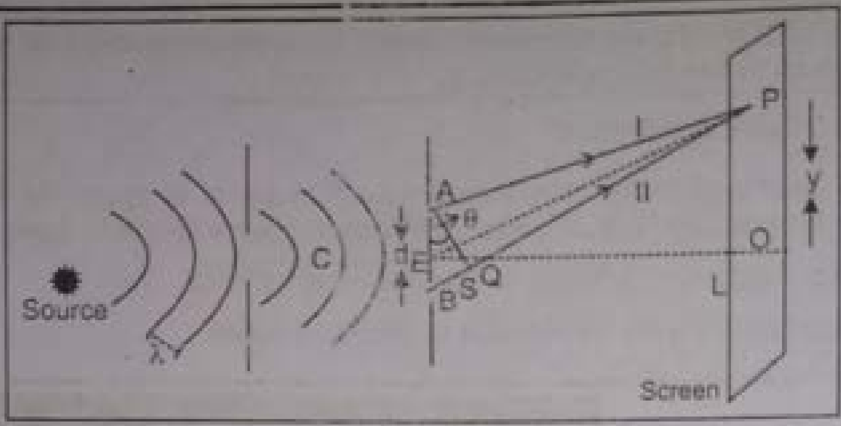


Fig: 9.9

Constructive Interference

Path-difference = BP – AP = S = BQ = 0, λ, 2λ, 3λ, 4λ, 5λ ______ (1)

BQ = s = m λ

Where m = 0,1,2,3 ,.......

Where m is called the order of the fringe.

From triangle BAQ

$$\frac{BQ}{AB} = \sin\theta \quad (or) \quad \frac{BQ}{d} = \sin\theta$$

Path difference = BQ = s = d sinθ

$$S = d \sin\theta = m\lambda \qquad \text{\_\_\_\_\_\_ (2)}$$

- At central point O, the path difference BP-AP = 0, so there will be a bright fringe at O corresponding to m = 0. This is called zero order fringe. For m = ± 1,
- The first order maximum will be above and below the point O.

Destructive Interference:

- If a dark fringe is formed at P, the path difference BQ must contain half integral multiple of wavelengths. According to the condition for dark fringe.

$$S = d\sin\theta = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, 7\frac{\lambda}{2}, 9\frac{\lambda}{2}, \text{\_\_\_\_\_\_ (3)}$$

$$s = [m + \frac{1}{2}]\lambda$$

Thus $s = d\,\text{Sin}\theta = [m + \frac{1}{2}]\lambda$ ______ (4)

Where m = 0, 1, 2, 3, ...

Position of Fringe on screen:

Let y is the vertical distance of point P from the central point O. From triangle ABQ and PEO

$$\frac{s}{y} = \frac{AB}{EP} \quad (OR) \quad \frac{s}{y} = \frac{d}{EP}$$

But $Y << L$ and $PE \approx EO = L$ so above equation becomes

$$\frac{s}{y} = \frac{d}{L}$$

$$Y = L\frac{s}{d} \qquad \text{\_\_\_\_\_\_ (5)}$$

For mth Bright Fringe:

Putting value of s From equation (1) in equation (5)

$$(y)_{bright} = 0, L\frac{\lambda}{d}, 2L\frac{\lambda}{d}, 3L\frac{\lambda}{d} \ldots\ldots$$

Putting s = m λ in equation 5

$$(y)_{bright} = m\frac{\lambda L}{d} \quad \text{.......................... (6)}$$

- This is the expression for the distance of m th bright fringe from the center of screen.

For m[th] Dark Fringe:

Putting value of s from equation (3) in equation 5

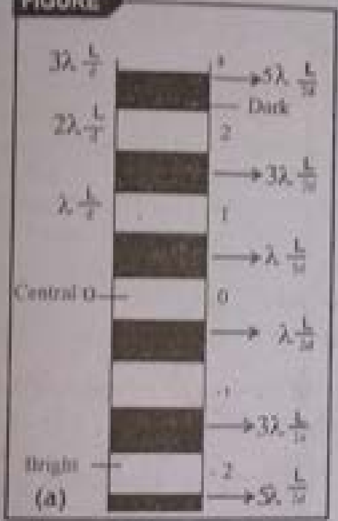
$$(y)_{dark} = L\frac{\lambda}{2d}, 3L\frac{\lambda}{2d}, 5L\frac{\lambda}{2d}, 7L\frac{\lambda}{2d} \ldots\ldots$$

Putting $s = [m + \frac{1}{2}]\lambda$ in equation (5)

$$(y)_{dark} = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \quad \text{.......................... (7)}$$

- This is the expression for the distance of m th dark fringe from the centre of screen.

FIGURE



FIGURE

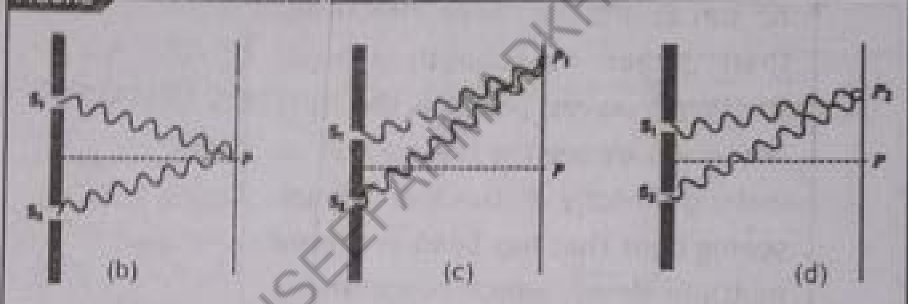


Figure shows ways in which the waves could combine to interfere constructively or destructively. Constructive interference (b) at P, and (c) at P_1. (d) destructive interference at P_2

Fringe Spacing:

- The distance between the centers of two consecutive bright or dark fringes is called fringe spacing.

For Two Bright Fringes:

In order to find the distance between two adjacent bright fringes on the screen 1[st] and 2[nd] fringes are considered.

- Distance of the 1st bright fringe from center of screen $= y_1 = \left(\frac{\lambda L}{d}\right)$
- Distance of the 2nd bright fringe from center of screen $= y_2 = 2\left(\frac{\lambda L}{d}\right)$

Fringe spacing is

$$\Delta y = y_2 - y_1$$

Putting values of y_1 and y_2

$$\Delta y = 2\left(\frac{\lambda L}{d}\right) - \left(\frac{\lambda L}{d}\right)$$

$$\Delta y = \frac{\lambda L}{d}$$

For Two Dark Fringes:

In order to find the distance between two adjacent dark band on the screen, 1[st] and 2[nd] fringes are considered

- Distance of the 1st dark fringe from center of screen $= y_1 = L\frac{\lambda}{2d}$
- Distance of the 2nd dark fringe from center of screen $= y_2 = 3L\frac{\lambda}{2d}$

$\Delta y = y_2 - y_1$ putting values of y_1 and y_2

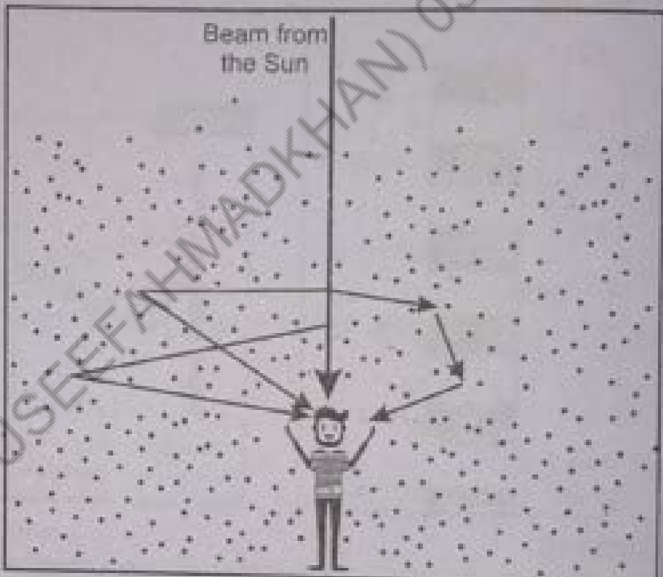
$$\Delta y = 3L\frac{\lambda}{2d} - L\frac{\lambda}{2d}$$

$$\Delta y = \frac{\lambda L}{d}$$

Hence the bright and dark fringes have equal width and equally spaced.

- The fringe spacing varies directly with distance 'L' between the slits and screen.($\Delta y \propto L$)
- The fringe spacing varies directly with wavelength of light λ . .($\Delta y \propto \lambda$)
- The fringe spacing varies inversely with the separation 'd' of the slits. .($\Delta y \propto \frac{1}{d}$)

For Your Information



Short wavelengths are scattered from the direct beam from the sun at mid day more effectively than longer wavelengths. These scattered waves produce the blue sky. When we see the sky, we are not looking directly at the sun. We are seeing light that has been scattered multiple times, which concentrates the shorter blue and violet wavelengths in the light that reaches our eyes. Since the spectrum of sunlight contains more blue than violet, and our eyes respond more strongly to blue wavelengths than to violet, the color we identify is blue.

QUIZ

Q. Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distance car?

A. No

Explanation

Because the light of two independence sources have different wave length which cannot produce interference.

Assignment 9.1:

The 3rd bright fringe in a double slit experiment makes a 2.4° angle with respect to the central line. The wavelength of the monochromatic light used is 480nm. Find the distance d between the two sources.

Given Data:

Wave length of light = λ = 480 nm

Order of fringe = m = 3

Angle = θ = 2.4°

Distance between the slits = d = ?

Solution:

For m^{th} bright fringe, $d \sin\theta = m\lambda$

$$d = \frac{m\lambda}{Sin\theta} = \frac{3\times480\times10^{-9}}{Sin\ 2.4^{o}}$$

$d = 0.000034$

$d = 0.034 \times 10^{-3}\ m$

$d = 0.034\ mm$

MCQ's

1. **For which colour of light the fringe spacing will be maximum:**
(A) Red light (B) Blue light (C) Yellow (D) Green light

2. **In Young's double slit experiment, the position for bright fringe is:**
(A) $Y_m = m\frac{\lambda d}{L}$ (B) $Y_m = \frac{m\lambda}{Ld}$ (C) $Y_m = \frac{m\lambda L}{d}$ (D) $Y_m = \frac{mLd}{\lambda}$

3. **Brilliant and beautiful colors in soap bubble are due to:**
(A) Diffraction of light (B) Polarization of light (C) Refraction of light (D) Interference of light

4. **The distance between the two adjacent bright fringes is:**
(A) $2\lambda L/d$ (B) $3\lambda L/d$ (C) $\lambda L/2d$ (D) $\lambda L/d$

5. **The appearance of colors in thin film is due to:**
(A) Diffraction (B) Dispersion (C) Interference (D) Polarization

6. **What happens to interference -pattern produced by double slit arrangement by doubling the slit spacing:**
(A) Fringe spacing is doubled (B) Fringe spacing is halved
(C) Fringe spacing is not changed (D) Intensity increases

7. **A maxima is produced at points where the path difference for two monochromatic waves is:**
(A) λ (B) $\lambda/4$ (C) $\lambda/2$ (D) $3\lambda/2$

8. **Formula for fringe spacing is:**
(A) $\frac{\lambda d}{L}$ (B) $\frac{\lambda L}{d}$ (C) $\frac{Ld}{\lambda}$ (D) $\frac{m\lambda L}{d}$

9. **The distance between two adjacent bright or dark fringes is:**
(A) $\Delta y = \frac{L\lambda}{d}$ (B) $\Delta y = \frac{\lambda}{d}$ (C) $\Delta y = \frac{\lambda}{Ld}$ (D) $\Delta y = Ld$

10. **Fringe spacing is inversely proportional to:**
(A) Wavelength (B) Slit separation
(C) Distance between the slits and screen (D) Frequency of light

Answers Key

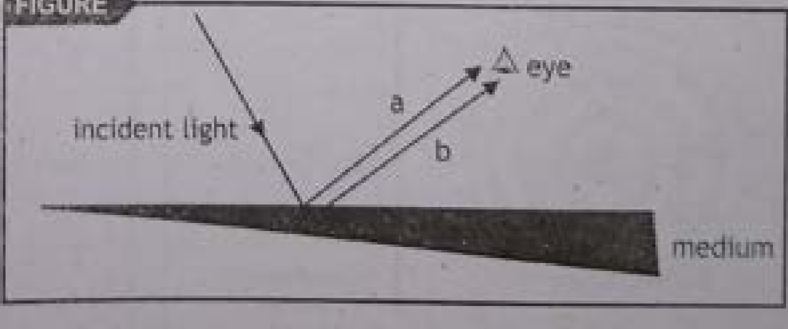
1. A	2. C	3. D	4. D	5. C	6. B	7. A	8. B	9. A	10. B

Q.6 Explain the phenomena of interference of light in a thin film?

Ans: **Interference in a Thin Film**

A transparent medium whose thickness is very small (Comparable with the wavelength of light), is called thin film.

FIGURE



Examples:

(i) oil film on the surface of water,
(ii) surface of soap bubble,
(iii) cracks in glass plate.

Explanation:

A beam of monochromatic light of wavelength λ is incident on its upper surface. It is partly reflected from upper surface as ray "a" again partly reflected from lower side of thin film as ray "b". The ray "b" covers greater distance than ray "a" .they will superpose and the result of their interference will be detected by the eye.

The path difference between two rays depends upon;

1. Thickness of the film
2. Nature of the film
3. Angle of incidence

- If the two reflected waves reinforce each other, then the film will look bright.
- However, if the thickness of the film and the angle of incidence are such that the two reflected waves cancel each other, the film will look dark.

(a) When a transverse wave travels from a medium of low refractive index to a medium of high refractive index, it under goes a phase change of 180^0 (π rad) after reflection.

(b) When a transverse wave travels from a medium of high refractive index to a medium of low refractive index, then there will no phase change after reflection.

Do You Know?

Different colours can be seen on the soap bubble whenever the light of the sun is falling on its surface.

Interference of White Light:

If white light is incident on a film of irregular thickness at all possible angles, we shall observe the interference pattern due to each spectral colour separately.

But if the thickness of the film and the angle of incidence are such that the destructive interference takes place from one colour. Then the remaining colour of the white light will appear on the film.

Q.7 Describe the principle, construction and working of Michelson's interferometer. How can you find the wave length of light used?

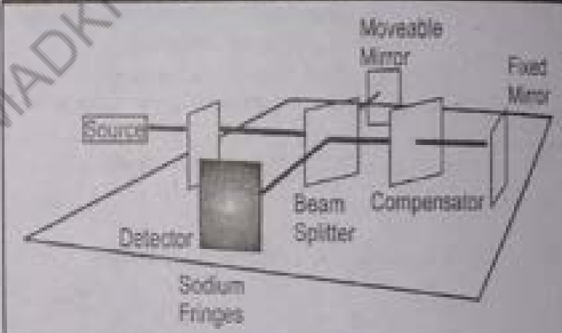
Ans: Michelson's Interferometer

Michelson Interferometer is an optical instrument.

It is used to study the interference of light waves and find its wavelength.

Principle- The principle of Michelson interferometer is based on the division of amplitude, usually by partial reflection and transmission of light at the boundary of the two medium.

For Your Information

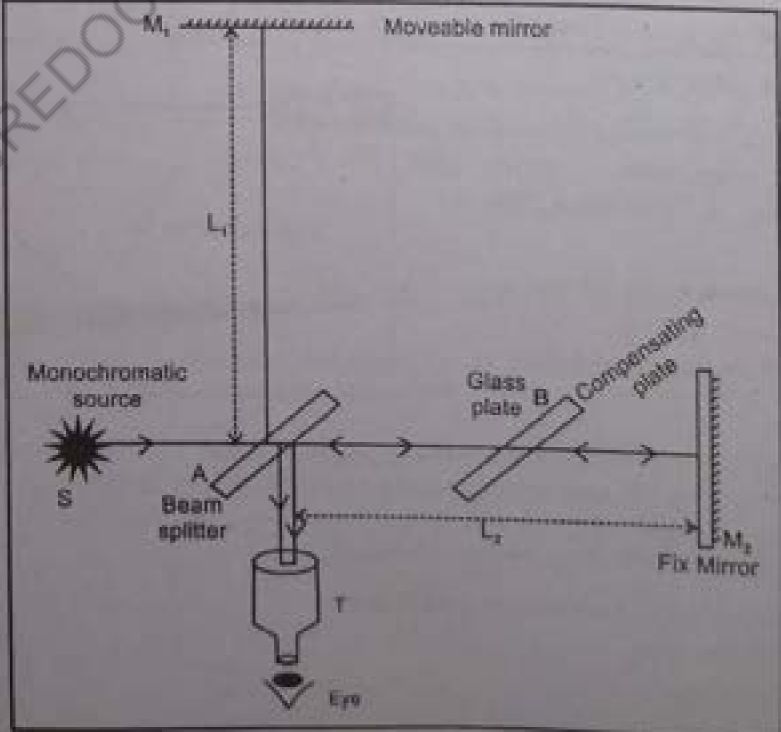


A photograph of Michelson Interferometer.

Construction:

The essential parts of the interferometer, devised by Michelson are

- Two plane mirrors M_1 & M_2
- Two glass plates "A & B" arranged as shown in the figure.



- The plate "A" is lightly silvered on the back, so that light which falls on that surface, half is reflected and half is transmitted.
- The mirror M_1 is moveable.
- The mirror M_2 is fixed.

Theory and working:

A beam of monochromatic light from the source "S" falls on the plate "A", where it is splitted in to two parts.

- The first part is reflected from plate "A" moves towards the mirror "M_1". After reflection from the mirror "M_1", it transmits through the plate "A" and enters the eye through the telescope.
- The second part of the light transmits through the plate "A" and moves towards the mirror M_2. After reflecting from there, this ray comes to the plate "A" from which it is again reflected and enters the eye.
- The plate B is introduced in the path of beam II as an compensator plate to equalize the path length of beam I and II in glass.

Conditions for constructive interference

- If the path difference between the two parts of light is either zero or integral multiple of wavelength "λ" then constructive interference will takes place and brightness will be observed.

$$d = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots$$

$$d = m\lambda$$

Where $m = 0, 1, 2, 3, \ldots\ldots$

Conditions for destructive interference

- But if the path difference between the two parts of light is odd integral multiple of half wavelength then destructive interference will occur and darkness will be seen.

$$d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots$$

$$d = \left(m + \frac{1}{2}\right)\lambda$$

Where $m = 0, 1, 2, 3, \ldots\ldots$

Let the two mirrors "M_1" and "M_2" are at equidistance from plate "A", so the two beams travel optically similar path.

- Here plate "B" is used to introduce the same retardation in beam "2" as introduced in beam "1" by its two passages through plate "A".
- So the path difference between the two beams of light is zero and bright fringe is observed, due to constructive interference.

Alternate Bright and Dark Fringes

- Now if the mirror "M_1" is moved through a distance λ/4 backward, then the path difference between the two beams will be equal to λ/2 and dark fringe will be seen.
- When the mirror "M_1" is further moved through distance λ/4 then the path difference will become "λ" and now bright band will be observed.
- Thus as the mirror "M_1" is moved slowly through distance λ/4 each time, bright and dark fringes will appear alternately.

FIGURE

Fringes produced with a Michelson interferometer.

Wavelength of light

- A fringe is shifted, each time the mirror is displaced through λ/2.
- By counting number m of fringes which are shifted by displacement P of mirror, we can write the equation.

$$P = \frac{m\lambda}{2}$$

$$\lambda = \frac{2P}{m} \longrightarrow (1)$$

Knowing the value of m and P we can find λ.

Uses:

Michelson's interferometer is used for the following purposes:

(1) It is used for the determination of wavelength of light

(2) It is ued for precise length measurement

(3) Michelson measured the length of the standard meter in terms of wavelength of red cadmium light and proved that Standard meter = 1553163.5 wavelength of light

(4) It is used to study the interference of light.

MCQ's

1. **When the mirror of Michelson Interferometer is moved a distance of 0.5 nm, 2000 fringes are observed, the wavelength of the light is used as:**
   (A) 5000 m (B) 500 cm (C) 5000 A° (D) None
2. **In Michelson interferometer to switch the fringe from bright to dark, the mirror should be displaced by:**
   (A) $\frac{\lambda}{4}$ (B) $\frac{\lambda}{3}$ (C) $\frac{\lambda}{2}$ (D) λ
3. **Constructive interference of two coherent beams is obtained if path difference is:**
   (A) $\frac{n\lambda}{2}$ (B) $\frac{n\lambda}{4}$ (C) $\frac{n(3\lambda)}{2}$ (D) $n\lambda$

Answers Key

1. C	2. A	3. D

Q.8 What is meant by diffraction of light?

Ans: **Diffraction of Light**

In some cases waves do not always cast sharp shadows, such as in Young's double slit experiment the light rays bend out of their straight line path and spreads into the region beyond the slits which would otherwise be shadow. This effect is common for all types of waves, such as sound waves are spread behind the obstacle or corner, which hides the source. Because the sound waves are bending around the obstacle or corner.

- ▶ Besides interference light also exhibits diffraction which provides experimental proof in favour of wave theory of light.

Def: *The spreading of light waves round the edges of a narrow opening or the spreading of light into the region behind an obstacle is called diffraction.*

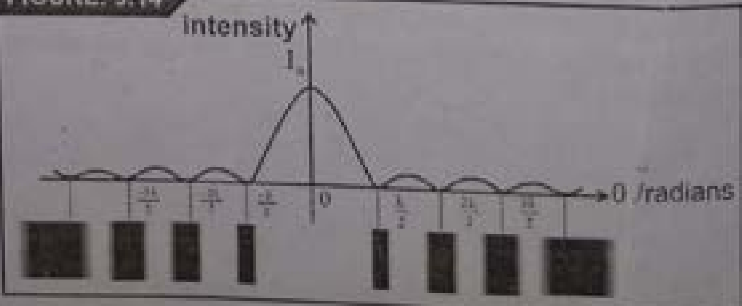
(OR) ***The property of bending of light around obstacles and spreading of light waves into the geometrical shadow of an obstacle is called diffraction.***

- ▶ The diffraction effects of light can only be observed, when the size of the opening or obstacle is so small that it is comparable with wavelength of light used.
- ▶ In young's double slit experiment the light from the slit C simultaneously illuminates the slits A and B. This is only possible if the light passing through the narrow-slit C, bends around the corners of C and hence spreads out in the region between C and the slits A & B.

FIGURE. 9.13

Narrow gap waves spread out beyond the gap. This spreading is called diffraction. Diffraction is most obvious when the gap is approximately the same size as the wavelength of the wave.

FIGURE. 9.14



(a) The diffraction effects are produced when a beam of monochromatic light passes through a narrow slit as in the Fig 9.13.
It should be noted that the central maximum is of a high intensity and very broad as compared to the other maximum.

(b) Similarly, the diffraction effects are exhibited when a knife edge is held up against a monochromatic light source. The diffraction of light waves round the knife edge produces the pattern as shown in Fig 9.14.

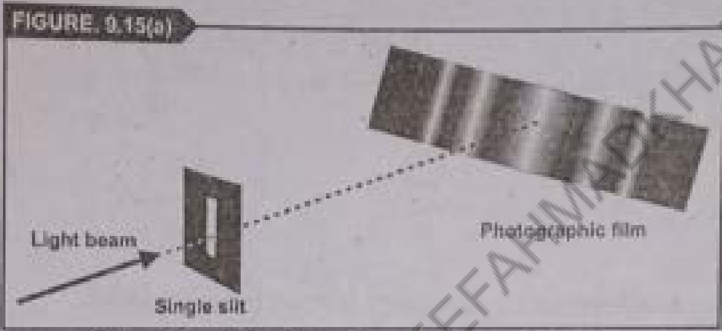
Q.9 Discuss Fraunhofer diffraction of light through a narrow slit?

Ans: **Fraunhofer diffraction of light**

"The diffraction of light produced by a narrow slit when plane light waves are incident normally on the slit and light waves emerging from the slit are also plane, is called Fraunhofer diffraction".

- Consider the light passes through a single slit and record the transmitted light on a photographic film as shown in Fig 9.15(a).

FIGURE. 9.15(a)



- Here the central bright is considerably wider than the slit.
- Moreover, some other bright bands occur on each side of the central image and these must result from some sort of interference effect.
- Consider plane wave fronts which are incident on a narrow-slit AB, as shown in fig 9.15(b).

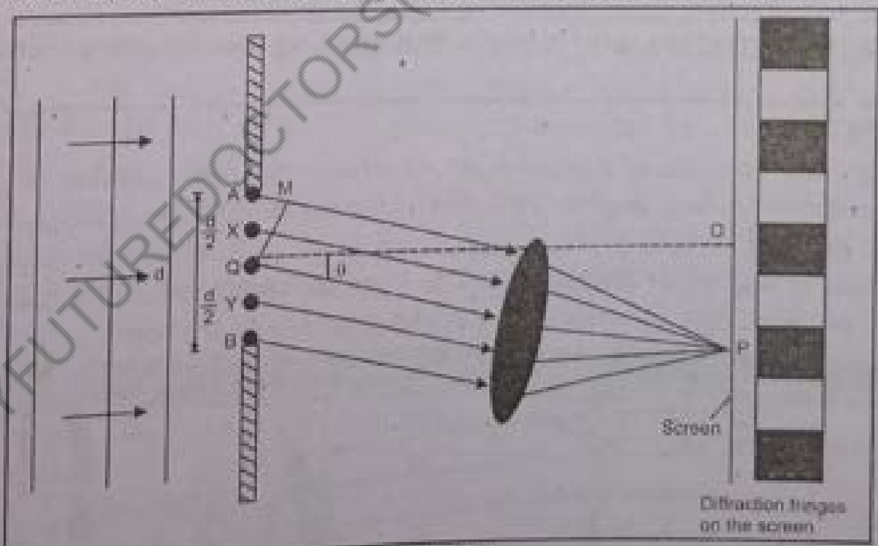


Fig 9.15(b)

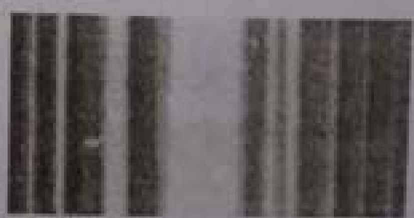


Fig. 9.10 (b)
Diffraction pattern produced by white light through a single slit.

According to Huygen's principle each point on the wave front at the slit AB acts (as source for the secondary wavelet).

- At the point O on the screen, wavelets from A & B arrive in phase, as these points are equidistance from O. Path difference between the waves coming from the points A and B as well as from X and Y is zero. Hence constructive interference will occur at O and bright fringe is observed.

- The path difference between the wavelets sent out by A and Q to the point P on the screen is approximately equal to half wavelength $\lambda/2$. Similarly the path difference for the pair of point X & Y and Q & B is also $\lambda/2$.
- Hence the wavelets reaching the point P will interfere destructively and dark fringe will be observed.
- If θ is the angle between QP with the axis of the slit as shown in the Fig 9.15(b) then the angle AQM is also θ, so the equation for the first dark fringe, considering the triangle AQM is

$$\text{Sin}\,\theta = \frac{perp}{Hyp} = \frac{AM}{AQ} \quad \text{putting} \quad AM = \frac{\lambda}{2} \text{ and } AQ = \frac{d}{2}$$

$$\text{Sin}\,\theta = \frac{\lambda/2}{d/2}$$

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

$$d\sin\theta = \lambda$$

$$d = \frac{\lambda}{\sin\theta}$$

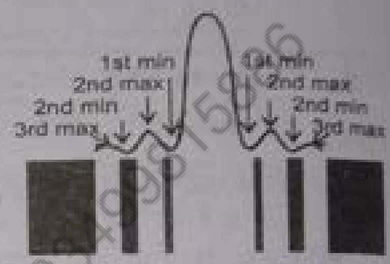


Fig. 9.10 (a)

Diffraction pattern of mono chromatic light produced due to a single slit; graphical representation and photograph of the pattern.

Similarly it can be shown that for m order minima, we have

$$d\sin\theta = m\lambda$$

$$\sin\theta_m = \frac{m\lambda}{d}$$

Where m = 1, 2, 3, ...

- The dark fringes are also formed on the other side of the axis QO.
- In between the dark fringes there are bright fringes.
- The central maximum (bright fringe) is of highest intensity and all the other maxima are of much lower intensities.
- Notice that if a narrow slit is used then the angle θ increases as $\theta = \frac{\lambda}{d}$ when θ is small.

Q.10 What is diffraction grating and obtain the grating equation to find the wavelength of light?

Ans: Diffraction Grating

"A diffraction grating consists of a glass plate on which very fine equidistant parallel lines (scratches) are drawn by mean of ruling engine with fine diamond point. The transparent spacing between the scratches on the glass plate acts as slits".

A grating is basically a glass or plastic plate 2 to 3 cm in length and 2 to 3 mm in thickness, on which a large number of parallel, equally spaced slits of the same width are ruled.

A typical diffraction grating has about 400 to 5000 lines per centimeter.

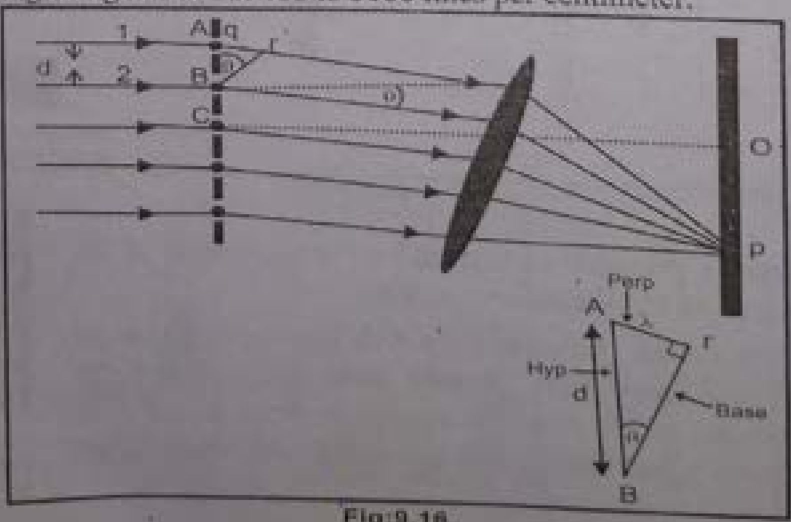


Fig:9.16

Grating element:

- "The distance between the centers of two adjacent lines is called grating element."
- *Its value is obtained by dividing the length L of the grating by the total number N of the lines ruled on it.*

  So the grating element = d = L / N.

If we consider the unit length of the diffraction grating then d = 1/N.

Principle-The principle of diffraction grating is based on the interference and diffraction of light waves.

- The light waves after diffraction through the grating are allowed to interfere.
- The schematic diagram of a common diffraction grating and its working is shown in the Fig 9.16

Working and Theory:

- A parallel beam of monochromatic light falling on the grating sends out waves from each slit.
- Along certain directions, waves from the adjacent slits are in phase and reinforce each other.
- The parallel rays after diffraction through the grating make an angle θ with the normal at the point of incidence.
- These rays are then brought to focus on the screen at the point P by a convex lens.
- If the path difference between the ray number 1 and number 2 is one wavelength, then rays interfere constructively and bright fringe will be observed at the point P.

Hence the condition for constructive interference is that the path difference rq between the two consecutive rays should be equal to λ.

$$Ar = \lambda$$

$$AB = d$$

But from the triangle ABr we know that

$$\text{Sin}\,\theta = \frac{perp}{Hyp} = \frac{Ar}{AB}$$

Putting values of Ar and AB

$$\text{Sin}\,\theta = \frac{\lambda}{d}$$

$$d\,\text{Sin}\,\theta = \lambda$$

- In general there will be other direction on each side of line $\overline{OC}$ for which the waves from adjacent slits differ in path by 2λ, 3λ etc and for which the corresponding bright images will be observed. These are called the second, third order images etc. The grating equation can thus be written in more general form as

$$d\sin\theta = m\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

- This is called the equation of diffraction grating for maxima.

Where m = 0, 1, 2, 3, ... and is called the order of the image.

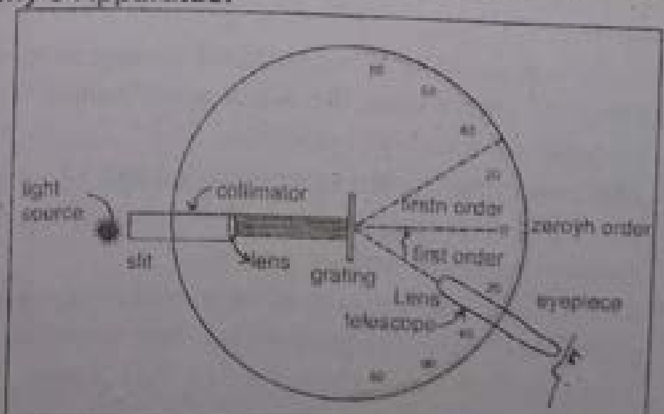
The diffraction grating is used to find wavelength of light.

For You Information

Grating spectrometer is used to calculate the wavelength "λ". Consider the diagram in which monochromatic light falls on the grating normally through the collimator. The diffracted light leaves the grating at angle "θ" and telescope is used to view the image as shown.

In this way angles for various orders on each side of the central maximum can be measured to calculate the wavelength.

Cauchy's Apparatus:



Q.11 Describe the diffraction of x-rays through crystals? Also describe the Braggs equation and its different uses?

Ans: **Diffraction of X-Rays by Crystals**

X-rays is type of electromagnetic radiation of much shorter wavelength, of the order of 10^{-10} m.

- In order to observe the effect of diffraction, the grating spacing must be of the order of the wavelength of the radiation used.
- The regular array of atoms in a crystal forms a natural diffraction grating because inter-planar spacing of crystal is also of the order of 10^{-10} m.

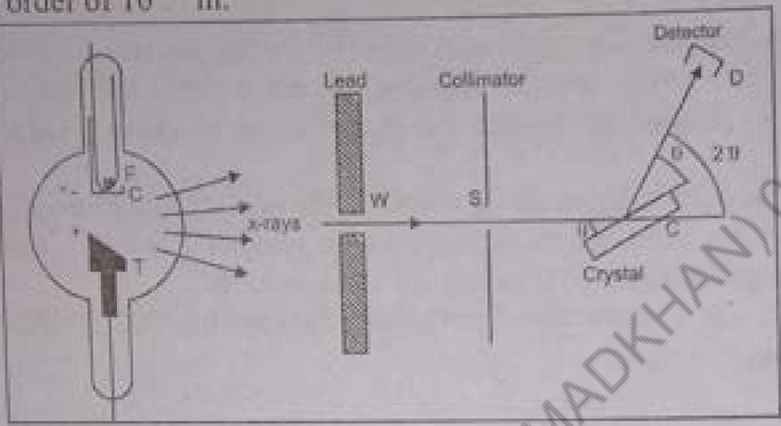


Fig 9.17

- It is not possible to produce interference fringes of X-rays by Young's double slit experiment or by thin film method. The reason is that the fringes space is given by $L\frac{\lambda}{d}$ and unless the slits are separated by a distance of the order of 10^{-10} m, the fringes obtained will be so closed together that they cannot be observed.

However, it is possible to obtain X-rays diffraction by using sodium chloride (NaCl) crystal which is common table salt. The atoms of crystals are uniformly spaced in planes and are distance 'd' apart.

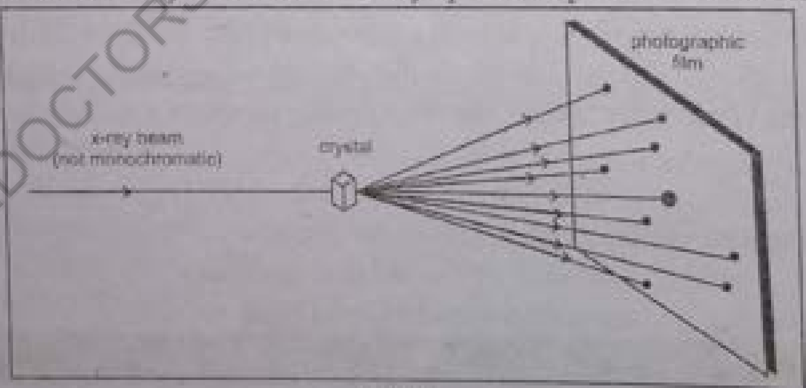


Fig 9.19

Experiment

Fig 9.18

- A typical *laue* type experiment arrangement is shown in the Fig 9.19. A narrow beam of X-rays from the X-ray is collimated through the slit and is allowed to fall on a crystal such as NaCl crystal.
- The transmitted beam enters the detector D.
- The Laue photograph obtained consists of the central spot surrounded by many other spots arranged in defined pattern.
- These spots are known as laue spots. The arrangement of laue spots for different crystal is different depending on their structure.
- These types of experiment prove that X-rays are electromagnetic waves and the atoms are arranged in three dimensional lattices.

Bragg's Law

- The study of atomic structure of crystals by X-rays was initiated in 1914 by W.H. Bragg and W.L Bragg with remarkable achievements.
- To get an expression for Bragg's law consider the diagram (9.20) in which two parallel x-rays I and II are incident at the first and second layer of the crystal.
- The separation between the two layers is d and θ is the glancing angle which is complementary angle to the angle of incidence.

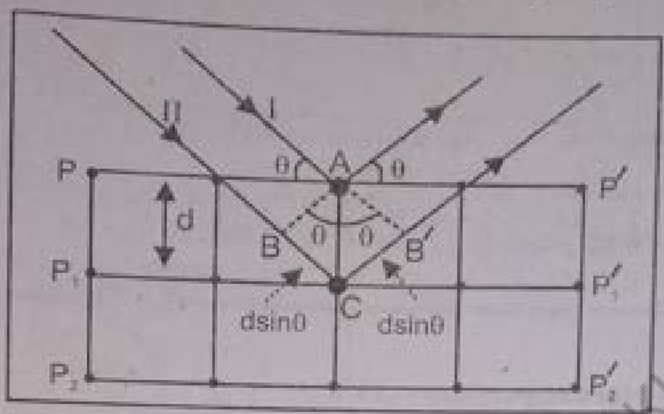


Fig:9.20 braggs law illustration

- Now the two reflected rays from the successive planes will reinforce each other if the path difference between them is an integral multiple of wavelength λ.
- Since ray II covers larger distance then the ray I so the path difference between the two rays is

Path difference = $BC + CB' = m\lambda$ ---------------- (1)

From triangle ABC, we have

$$\frac{BC}{AC} = \sin\theta$$

Or $BC = AC\sin\theta$

putting $AC = d$

Or $BC = d\sin\theta$ ---------------- (2)

Similarly, from triangle ACB′, we have

$$\frac{CB'}{AC} = \sin\theta$$

Or $CB' = AC\sin\theta$

Or $CB' = d\sin\theta$ ---------------- (3)

Putting values from equation (2) and (3) in equation (1)

Therefore, $d\sin\theta + d\sin\theta = m\lambda$ ---------------- (4)

$2d\sin\theta = m\lambda$

Where $m = 1, 2, 3, \ldots$

This equation is known as Bragg's law.

Uses:

1. Bragg's equation can be used to determine the inter-planar spacing between similar parallel planes of a crystal. $d = \frac{m\lambda}{2\sin\theta}$
2. X-rays diffraction is very useful in determining the structure of biologically important molecules such as hemoglobin, which is an important constituent of blood, and double helix structure of DNA.
3. Bragg's equation can be used to determine the wave length of X-rays. $\lambda = \frac{2d\sin\theta}{m}$

Assignment 9.2:

A beam of X-rays of wavelength 0.3 nm is incident on a crystal, and gives a first-order maximum when the glancing angle is 9 degrees. Find the atomic spacing

Given Data:

Wave length of light = λ = 0.3 nm = 0.3×10^{-9}m
Order of fringe = m = 1
Glancing Angle = $\theta = 9^o$
Atomic spacing = d = ?

Solution:

For Bragg's law $2d\sin\theta = m\lambda$

$$d = \frac{m\lambda}{2\sin\theta} = \frac{1\times0.3\times10^{-9}}{2\times\sin 9^0}$$

$$d = 9.58 \times 10^{-10}\text{ m}$$

$$d = 9.6\text{ A}^o$$

MCQ's

1. Bending of light around the edges of an obstacle called:
   (A) Refraction (B) Polarization (C) Interference (D) Diffraction
2. Diffraction is special type of:
   (A) Interference (B) Diffraction (C) Polarization (D) None
3. The wavelength of X-Rays is of the order:
   (A) 10m (B) 10^{-10}m (C) 10^{-2}m (D) 10cm
4. $2d\sin\theta = n\lambda$, is called:
   (A) Laplace's" equation (B) Reflection equation (C) Refraction equation (D) Bragg equation
5. When the light passes from the pinhole type opening it seems to spread out. This phenomena is called:
   (A) Dispersion (B) Reflection (C) Diffraction (D) Polarization
6. A diffraction grating has 5000 lines/cm .Its grating element will be:
   (A) 2.0×10^{-6}m (B) 2.0×10^{-4}m (C) 1.0×10^{-6}m (D) 1.0×10^{-4}m
7. The bending of beam of light when it passes from 1 medium to another known as:
   (A) Refraction (B) Reflection (C) Diffraction (D) Dispersion
8. Michelson interferometer can be used to find the:
   (A) Wavelength of light (B) Wavelength of sound (C) Velocity of sound (D) Velocity of light
9. If 'N' is number of lines ruled on the grating having length "L" then grating element "d" is given by:
   (A) $\frac{N}{L}$ (B) $\frac{2N}{L}$ (C) $\frac{L}{N}$ (D) $\frac{N}{2L}$

Answers Key

1. D	2. A	3. B	4. D	5. C	6. A	7. A	8. A	9. C

Q.12 Explain the phenomenon of polarization. How plane polarized light is produced and detected?

Ans: **Polarization**

The process of confining the light in to one plane of vibration is called polarization of light.

- The phenomenon of interference and diffraction have proved that light has wave nature, but these phenomena do not show whether light waves are longitudinal or transverse.
- The phenomenon of polarization shows that light waves are transverse.
- In transverse mechanical waves, the vibration can be oriented along vertical, horizontal or any other direction. In each of these cases, the wave is said to be polarized.
- **The plane of polarization is the plane containing the direction of vibration of the particles of the medium and the direction of propagation of wave.**
- A light wave produced by oscillating charge consists of a periodic variation of electric field vector along with magnetic field vector at right angle to each other. The direction of polarization in a plane polarized light wave is taken as the direction of electric field vector.

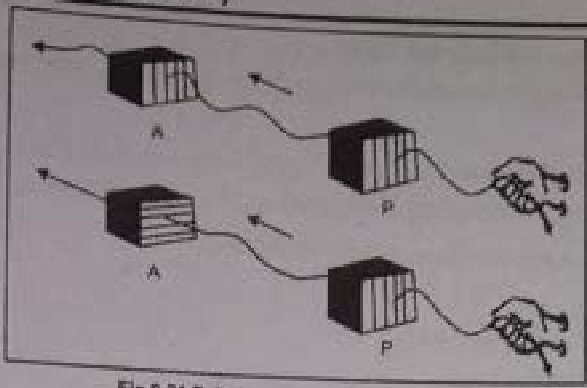
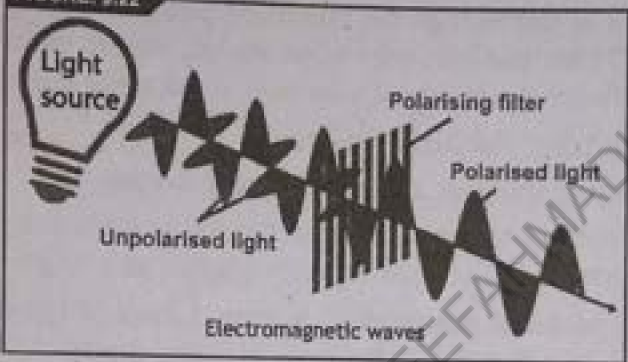


Fig 9.21 Polarization of transverse waves

Ordinary light:

Question arises that why ordinary light is not polarized?
To answer this question, we know that light wave is emitted when an electron orbiting around a nucleus jumps from higher energy level to a lower energy state.

FIGURE. 9.22



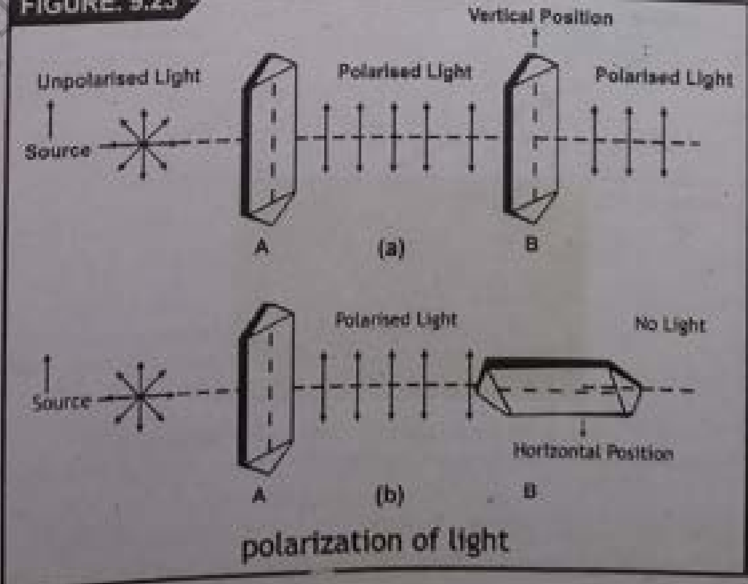
Do You Know?
It is very effective to use a polarized sun glasses than using ordinary sun glasses.

- If the transition is due to an electron orbiting in a vertical plane, the light wave emitting is polarized in the vertical plane.
- But if the transition is due to an electron orbiting in a horizontal orbit then the light wave emitted is polarized in the horizontal plane.
- The light waves emitted from a source such as candle flame, a filament of bulb or the sun is non polarized. This is because the light waves are from different atoms whose electrons experiencing the transition in different planes in all direction.
- Therefore light waves are electromagnetic waves which consist of periodic vibrations of electric field vectors "E" accompanied by the magnetic field vector "B" at right angled to each other as shown in Fig 9.22.

Polarized light:

The beam of light in which all vibrations are confined to a single plane of vibration is called polarized light.

FIGURE. 9.23



polarization of light

Production and Detection of plane polarized light

- The light emitted by an ordinary incandescent bulb is un-polarized, because its vibrations are randomly oriented in space.
- It is possible to obtain plane polarized beam of light from un-polarized light by removing all waves from the beam except those having vibrations along one particular direction.
- This can be achieved by various method as given below:
  - (1) Selective absorption
  - (2) Reflection from different surfaces
  - (3) Scattering by small particles
  - (4) Refraction through crystals

Selective Absorption Method:

- Selective abortion method is the most common method to obtain plane polarized light by using certain types of materials called ***dichroic substances***. It is made up from tiny crystals of quinine iodosulphate. Such crystal have property to transmit all the vibrations parallel to its crystallographic axis while absorb all remaining vibrations. These transmit only those waves, whose vibration are parallel to the particular direction and will absorb those waves whose vibration are in other directions.

  One such commercial polarizing material is Polaroid.
- The un-polarized light is made incident on the tourmaline crystal.
- The internal molecular structure of the tourmaline crystal is such that it allows only those electric and magnetic vibrations which are parallel to its crystallographic axis while absorb all remaining vibrations.
- The transmitted light will be plane polarized. If a second sheet of Polaroid is placed in such a way that the axes of the Polaroid, as shown by the straight lines drawn on them, are parallel, the light is transmitted through the second Polaroid also.
- If the second Polaroid is slowly rotated about the beam of light, as axis o rotation, the light emerging out of the second Polaroid gets dimmer and dimmer and disappears when the axes become mutually perpendicular.
- The light reappears on further rotation and become brightest when the axes are again parallel to each other.

Transverse Nature of Light:

- This experiment proves that light waves are transverse waves. If the light waves were longitudinal, they would never disappear even if the two Polaroids were mutually perpendicular.
- Sunlight also becomes partially polarized because of scattering by air molecules of the Earth's atmosphere or by reflection we can obtain the partially polarized light instead of glare of light.

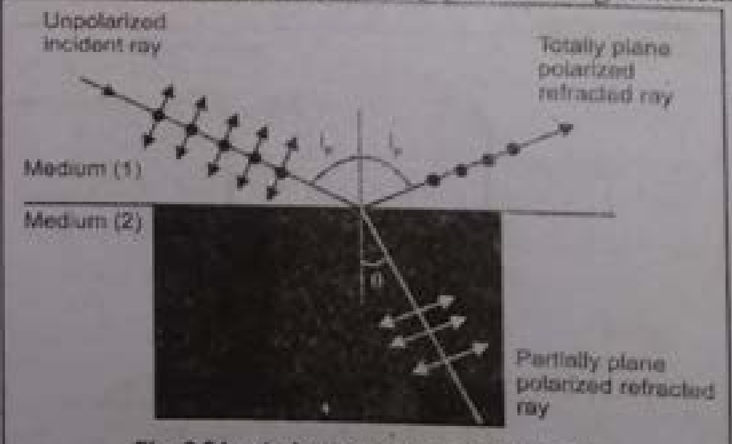


Fig: 9.24 polarization of light after reflection

Polarization by Reflection

- When un-polarized light falls on glass, water etc, the reflected light is in general partially plane polarized but at a certain angle of incidence called polarizing angle, the polarization is complete.

▶ At polarizing angle the reflected ray and the refracted ray in a transmitted medium are found to be at right angle to each other. The vibrations in the reflected ray are parallel to the surface, as shown. Applying Snell's law we have

$$n_1 \sin i_p = n_2 \sin r \qquad \text{--------------- (1)}$$

Where n_1 and n_2 are the absolute refractive indexes of the medium 1 and 2. Since from the Fig 9.24 we have

$$i_p + 90^\circ + r = 180^\circ$$
$$r = 90^\circ - i_p$$

Hence, putting this value in Eq. (1)

$$n_1 \sin i_p = n_2 \sin (90 - i_p) \qquad \text{--------------- (2)}$$
$$n_1 \sin i_p = n_2 \cos i_p$$
$$\frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$
$$\frac{n_2}{n_1} = \tan i_p \qquad \text{--------------- (3)}$$

This is known as Brewster's law. For glass of refractive index 1.55, the angle of incidence $i_p = 57^\circ$

Q.13 Discuss the applications of polarized light?

Ans: **Applications of Polarized Light**

i. **Reducing glare**

Glare caused by light reflected from a smooth surface such as roads and lakes can be reduced by using polarizing materials as the reflected light is partially or completely polarized. Thus, Polaroid discs, suitably oriented are used in sun glasses to avoid the polarized light.

▶ Similarly in photography, the Polaroid discs are placed in front of the camera lens which enabling us to see the detail which would be otherwise hidden by glare.

ii. **Optical Activity**

When a beam of light is made to pass through certain crystals (Quartz), or liquids, (Sugar solution), the directions of vibration of the transmitted polarized light is found to be rotated. This phenomenon is called optical activity. For a solution the angle of rotation depends on its concentration, and an instrument known as a Polari meter is used to measure the concentration of the given solution. In sugar mills, Polari meter is used to measure the sugar concentration in the solution obtained from sugar cane.

iii. **Certianless window**

Two polarizing sheets are fixed in a window, one inside and the other outside. The inner one is rotated in such a way to adjust the amount of light to be admitted.

iv. **Control of head light glare**

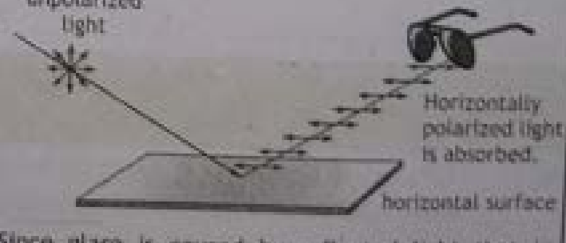
During night safe driving is possible if each car having polarizing head lights and polarized light viewer, Polaroid glasses eliminate the glare of light as it is partly polarized by reflection from water and road.

v. **Stress analysis**

When glass, Polythene and some other plastics are under stress e.g. by bending, twisting or uneven heating) they become doubly refracting and if viewed in white light between two 'crossed' Polaroid's coloured fringes are seen round the regions of strain'. This effect is called photo elasticity and is used to analyze stresses in plastic model of various structures.

Do You Know?

When light is reflecting from the glass then it is partially polarized.



Since glare is caused by reflected light that is horizontally polarized, sunglasses with polarized lenses can eliminate glare by allowing only vertically polarized light to pass through.

MCQ's

1. **Which phenomena shows that light is transverse in nature:**
   (A) Refraction (B) Polarization (C) Interference (D) Reflection
2. **The process of confining the beam of light to vibrate in one plane is called:**
   (A) Interference (B) Diffraction (C) Polarization (D) Total internal reflection
3. **If the un- polarized light is made incident on sheet of polaroid, the transmitted beam of light will be:**
   (A) Non plane Polaroid (B) Plane polarized (C) Un polarized (D) Diffraction
4. **Optical rotation, a property of optically active substances can be used to determine their:**
   (A) Density (B) Viscosity (C) Concentration in solutions (D) Elasticity
5. **Which characteristic of light is evidence from polarization of light?**
   (A) Wave nature (B) Particle nature (C) Dual nature (D) Light waves in transverse
6. **Optically active crystals rotates the:**
   (A) Vibrating plane (B) Polarization plane (C) Diffraction plane (D) Interference plane
7. **Light is polarized by using:**
   (A) NaCl (B) Dichroic substances (C) Optical fiber (D) Plane glass

Answers Key

1. B	2. C	3. B	4. C	5. A	6. B	7. B

FORMULAE

Path difference for constructive interference in light	$d\sin\theta = m\lambda$, where m= 0,1,2,3,...	
Path difference for destructive interference in light	$d\sin\theta = [m+\frac{1}{2}]\lambda$, where m= 0,1,2,3,...	$d\sin\theta = [2m+1]\frac{\lambda}{2}$ where m= 0,1,2,3,...
Position of mth bright fringe	$y = m\frac{\lambda L}{d}$	
Position of mth dark fringe	$y = (m+\frac{1}{2})\frac{\lambda L}{d}$	
Fringe width /spacing	$\Delta y = \frac{\lambda L}{d}$	
Grating element	$d = \frac{L}{N}$	$d = \frac{1}{N}$ [when L = 1m]
Displacement of mirror in Michelson interferometer	$L = m\frac{\lambda}{2}$	$\lambda = \frac{2L}{m}$
Bragg's Law	$2d\sin\theta = m\lambda$ Where m = 1,2,3,...	

Key Points

- **Nature of light:** According to modern concept the nature of light is dual. Some time it behaves as a wave such as in reflection, refraction, particle, such as photo-electric effect and Compton Effect.
- **Wave front:** Diffraction, interference and polarization. But some time it behaves as the locus of all the points in a medium which have the same phase, is called a wave front.
- **Ray:** The arrows to indicate the direction of wave fronts are known as the rays. The rays are always perpendicular to the wave fronts.

- **Phase coherent sources:** The two sources of light which maintains a constant phase relationship between their waves during emission are called phase coherent sources.
- **Interference of light waves:** The effect produced by the superposition of two coherent waves is known as interference. The interference may be constructive or destructive.
- The principle of Young's double slits experiment and diffraction grating is based on the division of a wave front. While the principle of Michelson's interference, thin film and Newton's ring is based on the division of amplitude.
- Michelson's interferometer is used for the precise measurement of wavelength.
- **Diffraction:** The bending of light waves around an obstacle and spreading into its geometrical shadow is called diffraction. For a diffraction grating with grating element d and m th order bright image, we have $d \sin\theta = m\lambda$.

  The diffraction of X-rays can be obtained by means of crystals. For diffraction of X-rays the Bragg's Law is $2d \sin\theta = m\lambda$.
- Polarization of light: The process by which the electric and magnetic vibrations of light waves are restricted to a single plane of vibration is called polarization of light.

Solved Examples

Example 9.1: **Yellow light from a sodium vapour lamp of wavelength 5893×10^{-10} m is directed upon two narrow slits 0.10 cm apart of the Young's experiment. Find the position of the first bright and dark fringes on a screen 100 cm away.**

Solution:

Wavelength of light = $\lambda = 5893 \times 10^{-10}$ m

Separation of slits = d = 0.10 cm = 0.001 m

Distance of slits from the screen = L = 100 cm = 1 m

Order of the fringe = m = 1

Position of the 1st bright fringe = y_{bright} = ?

Position of the 1st dark fringe = y_{dark} = ?

(a) As for the bright fringe is

$$y_{bright} = L\frac{\lambda}{d}$$

$$y_{bright} = \frac{1 \times 5893 \times 10^{-10}}{0.001} = 5.893 \times 10^{-4}\,\text{m}$$

(b) And

$$y_{dark} = L\frac{\lambda}{2d} = \frac{1 \times 5893 \times 10^{-10}}{2 \times 0.001}$$

$$y_{dark} = 2.953 \times 10^{-4}\,\text{m}$$

Example 9.2: **Light of wavelength 546 nm produces Young's interference pattern. The second order dark fringe is along the direction that makes an angle of 18 min, related to the direction to the central maximum. What is the distance between the slits?**

Solution:

Wavelength of light $= \lambda = 546\,\text{nm} = 546 \times 10^{-9}$ m

Order of dark fringe = m = 2

Angle $= \theta = 18\,\text{min} = \frac{18}{60} = 0.30^\circ$

(As 1° = 60 minutes)

For m^th dark fringe, we know that $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$

$$d = \left(m + \frac{1}{2}\right)\frac{\lambda}{\sin\theta}$$

$$d = \left(1 + \frac{1}{2}\right)\times\frac{546\times10^{-9}}{\sin 0.3^\circ}$$

$$d = \frac{1.5\times546\times10^{-9}}{5.236\times10^{-3}}$$

$$d = 1.564\times10^{-4}\ \text{m}$$

Example 9.3: **A red laser light of wavelength 630 nm is used in a Michelson interferometer. While keeping the mirror M_1 fixed, mirror M_2 is moved. The fringes are found to move past a fixed cross-hair in the view. Find the distance the mirror M_2 is moved for a single fringe to move past the reference line.**

Given Data: wavelength = 630 nm,
For single fringe crossing m = 1

Required: The distance traveled by M_2 = ?

Solution: For a 630-nm red laser light, and for each fringe crossing (m = 1), the distance traveled by M_2 if you keep M_1 fixed is

$$P = \frac{m\lambda}{2} = \frac{1\times630}{2}\ \text{nm} = 315\ \text{nm}$$

$$\boxed{P = 315\ \text{nm}}$$

Example 9.4: **The deviation of the second order diffracted image formed by an optical grating having 5000 lines per centimeter is 32°. Calculate the wavelength of light used.**

Solution:

Number of lines per centimeter = 5000 lines/cm

Angle $\theta = 32^\circ$

Order of image = m = 2

Grating element $= d = \frac{1\ \text{cm}}{5000} = 0.00020\ \text{cm}$

Wavelength = λ = ?

Formula $\lambda = \frac{d\sin\theta}{m}$

$$\lambda = \frac{0.0020\ \text{cm}\times\sin 32}{2}$$

$$\lambda = \frac{0.0020\text{cm}\times0.53}{2} = 5.3\times10^{-5}\ \text{cm}$$

Example 9.5: **X-rays of wavelength 3nm are incident on a crystal for which the lattice spacing is 5nm. Calculate the angle at which the first Bragg diffraction is observed.**

Solution:

Wavelength $= \lambda = 3\times10^{-9}$ m

Lattice spacing $= d = 5\times10^{-9}$ m

Order of the image = m = 1

Angle = θ = ?

Since

$$2d\sin\theta = m\lambda$$

$$\sin\theta = \frac{m\lambda}{2d} = \frac{1\times3\times10^{-9}\text{m}}{2\times5\times10^{-9}\text{m}} = 0.3$$

$$\theta = \sin^{-1}(0.3)$$

$$\theta = 17^\circ$$

Text Book Exercises

Q.1 Select the correct answer of the following questions

(1) **The principle of Young's double slits experiment is based on the division of:**
(a) Amplitude (b) Frequency (c) Velocity (d) Wave front

(2) **Which one of the following properties proves the transverse wave nature of light.**
(a) Interference (b) Refraction (c) Polarization (d) Diffraction

(3) **Coloured fringes observed in soap bubbles are the examples of**
(a) Diffraction (b) Interference (c) Polarization (d) Reflection

(4) **During a sunny day we see the objects in a class room even when all the electric lights are off, due to**
(a) Reflection of light (b) Refraction of light (c) diffraction of light (d) Interference of light

(5) **The principle of Michelson interferometer is based on the division of**
(a) Wave front (b) Amplitude (c) Frequency (d) Speed of light

(6) **In the Young's double slit experiment the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is**
(a) Halved (b) Unchanged (c) Doubled (d) Quadrupled

(7) **Signal from a remote control to the device operated by it travels with the speed of:**
(a) Sound (b) Light (c) Ultrasonic (d) Supersonics

(8) **Light of wavelength λ is incident normally on a diffraction grating for which the slit spacing is equal to 3λ. What is the sine of the angle between the second order maximum and the normal?**
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

(9) **Which of the following gives three regions of the electromagnetic spectrum in order of increasing wavelength, visible radiation.**
(a) Gamma rays, microwaves, visible radiation (b) Radio waves, ultraviolet, X-rays
(c) Ultraviolet, infra-red microwaves (d) Visible radiation, gamma rays, radio waves

(10) **Two monochromatic radiations X and Y are incident normally on a diffraction grating. The second order intensity maximum for X coincides with the third order intensity maximum for Y. What is the ratio $\frac{\text{wavelength of X}}{\text{wavelength of Y}}$?**
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{2}{1}$

(11) **The tip of a needle does not give a sharp image. It is due to**
(a) Polarization (b) Interference (c) Diffraction (d) Refraction

No.	Option	ANSWER	EXPLANATION
1	d	(no answer from given options)	The correct option should be " wave front". YDSE is based on the principle of division of wave front.
2	c	polarization	
3	b	interference	
4	c	Diffraction of light	
5	b	amplitude	
6	d	quadrupled	$\Delta y = \frac{\lambda L}{d}$ .... (1) d → halved and L→ doubled then $\Delta y' = \frac{\lambda(2L)}{d/2}$ $\Delta y' = 4\left(\frac{\lambda L}{d}\right)$ .... (2) Putting value from equation 1 in 2 $\Delta y' = 4\Delta y$

7	b	light	
8	C	$\frac{2}{3}$	$d \sin\theta = m\lambda$ putting $d = 3\lambda$ and $m = 2$ $3\lambda \sin\theta = 2\lambda$ $\sin\theta = 2/3$
9	c	Ultraviolet, infra-red microwaves	
10	c	$\frac{3}{2}$	$d \sin\theta = m\lambda$ For x light wavelength λ_1, $m = 2$ So $d \sin\theta = 2\lambda_1$ ____(1) For y light wavelength λ_2, $m = 3$ So $d \sin\theta = 3\lambda_2$ ____(2) Comparing equations (1) and (2) we have $2\lambda_1 = 3\lambda_2$ $\frac{\lambda 1}{\lambda 2} = \frac{3}{2}$
11	c	diffraction	

Short Answers of the Exercise

Q.2 Write short answers of the following questions.

Q.1 A soap bubble looks black when it bursts, why?

Ans: When soap bubble is about to burst then its thickness is very small (almost zero).

- As one part of light rays is reflected from denser medium from upper side of soap bubble and have a path difference of $\lambda/2$.
- While a part of light rays is reflected from rare medium (air) lower side of soap bubble and have no path difference.
- Therefore the two reflected rays have path difference $\lambda/2$ due to which destructive interference produces and soap bubble looks dark.

Q.2 What is the difference between interference and diffraction?

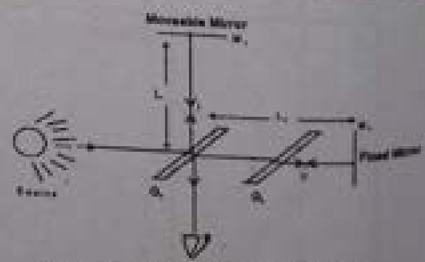
Ans:

Interference	Diffraction
(i) Interference is due to superposition of two waves coming from two different wave fronts.	(i) Diffraction is due to waves coming from different parts of the same wave front.
(ii) Interference bright fringes are of uniform high intensity.	(ii) Diffraction bright bands are of low intensity.
(iii) The dark fringe are perfectly dark.	(iii) The dark fringes are not perfectly dark.
(iv) Interference fringes are equally spaced.	(iv) The diffraction fringes are not equally spaced.

Q.3 In a Michelson interferometer a second glass plate is also used, why?

Ans:

- The ray-I have to travel through half silvered glass plate two times and it decreases speed of light two times.
- The glass plate G_2 cut from the same piece of glass as G_1 and is equal in thickness to G_1 is introduced in the path of beam II to decrease its speed of light two times.
- G_2 therefore equalizes the path length of the beam I and II in glass and is called compensating glass plate or compensator plate.
- The two beams having their different paths are coherent. They produce interference effects when they arrive at observer's eyes.



Schematic diagram of a Michelson's Interferometer

Q.4 How you can explain Brewster's law of polarization?

Ans:

- When un-polarized light falls on glass, water etc, the reflected light is in general partially plane polarized but at a certain angle of incidence called polarizing angle, the polarization is complete.
- At polarizing angle the reflected ray and the refracted ray in a transmitted medium are found to be at right angle to each other. The vibrations in the reflected ray are parallel to the surface, as shown, Applying Snell's law we have

$$n_1 \sin i_p = n_2 \sin r \qquad (1)$$

Where n_1 and n_2 are the absolute refractive indexes of the medium 1 and 2. Since from the Fig 9.24 we have

$$i_p + 90^\circ + r = 180^\circ$$

$$r = 90^\circ - i_p$$

Hence, putting this value in Eq. (1)

$$n_1 \sin i_p = n_2 \sin (90 - i_p)$$

$$n_1 \sin i_p = n_2 \cos i_p$$

$$\frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$

$$\frac{n_2}{n_1} = \tan i_p$$

This is known as Brewster's law.

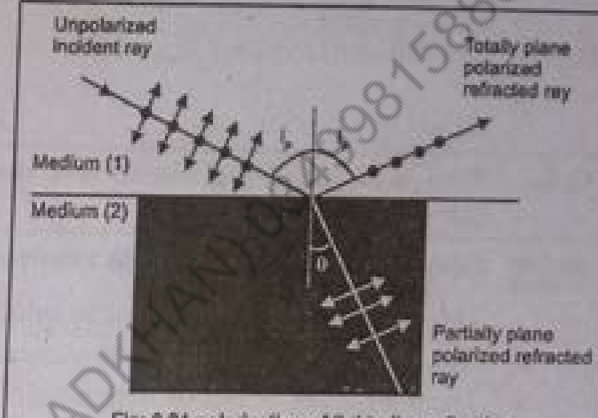


Fig: 9.24 polarization of light after reflection

Q.5 What is meant by the path difference with reference to the interference of two wave motion?

Ans: **Interference of Light Waves**

The effect produced by the superposition of light waves from two coherent sources passing through same region in same direction is called interference of light.

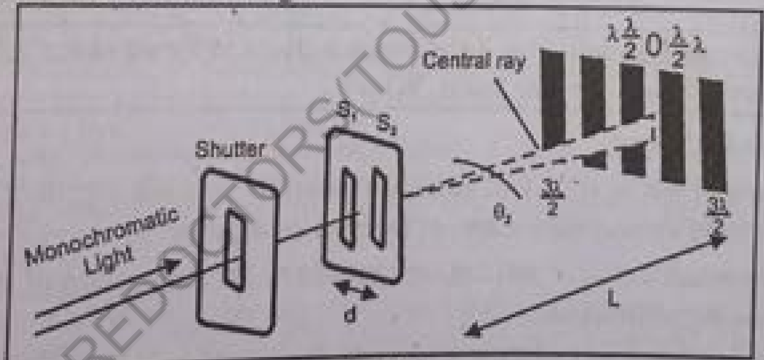


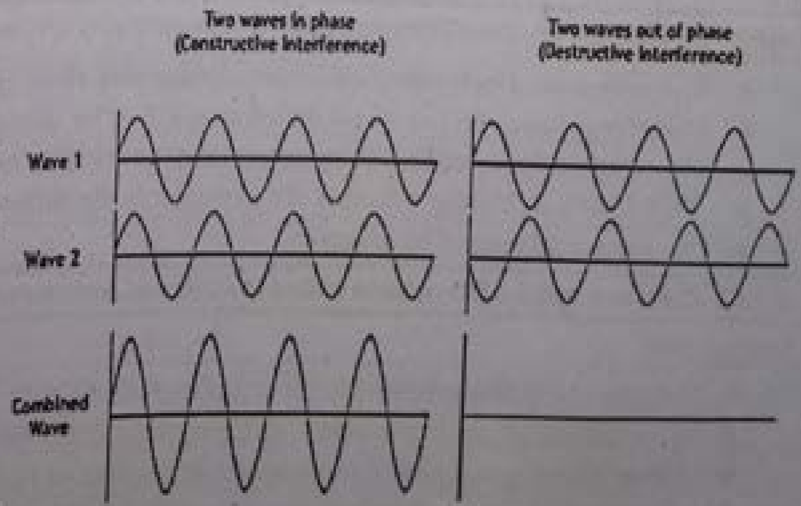
Fig:9.5

There are two types of interference:

(i) Constructive interference
(ii) Destructive interference

Constructive interference

If crest of one wave falls on the crest of the other wave, then they support each other. Such a interference in known as constructive interference.



- The path difference is the difference between the distances travelled by two waves meeting at a point.

For constructive interference path difference is either zero or integral multiple of wavelength λ.

Path difference = $d = 0, \lambda, 2\lambda, 3\lambda, 4\lambda, 5\lambda$..............

Path difference = $d = m\lambda$

Where $m = 0, 1, 2, 3$..........

(ii) Destructive interference

If crest of one wave falls on the trough of the other wave, then they cancel each other. Such a interference in known as destructive interference.

For destructive interference path difference is odd integral multiple of half wavelength (λ/2)

Path difference $= d = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, 7\frac{\lambda}{2}, 9\frac{\lambda}{2}, \ldots\ldots\ldots\ldots$

Path difference $= d = \left(m + \frac{1}{2}\right)\lambda,$

(OR) Path difference $= d = (2m+1)\frac{\lambda}{2}$

Where m = 0,1,2,3,.....

Q.6 Why it is not possible to see the interference where the light beams from the head lamps of a car overlap?

Ans: Conditions for detectable interference pattern:

The following condition must be fulfilled in order to observe the interference phenomenon;

- The interfering beams must be *monochromatic*.
- The interfering beams of light must be *coherent*.
- The sources should be *narrow* and very *close* to each other.
- The intensity of the two sources be *comparable*.
- The waves coming from the two separate head lamps differ in phase. Their phase difference not satisfies the conditions of interference.
- Also these sources are not monochromatic. Due to these reasons, it is not possible to see the interference where the light beams from the head lamps of a car overlap.

Q.7 A telephone pole clear shadow in the light from a distant head lamp of a car, but no such effect is noticed for the sound from the car horn. Why?

Ans: The sound and light both are waves.

- But the wavelength of sound waves is very large as compared to the wave length of light waves.
- Hence the sound waves bend around the corners of the pole so they are heard.
- The wave length of light wave is shorter and not comparable to the dimensions of the pole.
- Therefore light waves are not diffracted.
- Thus the light waves cannot bend around the corners and shadow of pole is observed.

Q.8 Why it is not possible to obtain the diffraction of X-rays by Young's double slits experiment?

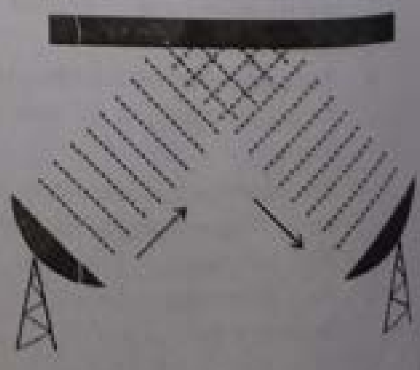
Ans: In Young's double slit experiment, the double slits are used to observe interference phenomenon for visible light.

- The x-rays are electromagnetic wave having very short wave length ($\approx 10^{-10}$ m) as compare to visible light.
- Therefore x-rays do not show diffraction effect by using ordinary diffracting objects like slits, diffraction grating etc. because their width is very larger than wavelength of X-rays.
- That is why in Young's double slit experiment the diffraction of x-rays is not possible.
- X-rays are diffracted by crystals.

Q.9 Can we apply Huygen's principle to radar waves?

Ans: Yes,

- We can apply Huygen,s principle to radar waves.
- RADAR waves are transverse electromagnetic radio waves .
- These waves propagate through space with speed of light.
- Therefore Huygen,s principle can be applied to RADAR wa ves like that of light waves.



Q.10 How would you justify that light waves are transverse?

Ans: The phenomenon of polarization of light proves that light waves are transverse waves.

- The light has electric and a magnetic field component which are perpendicular to each other and propagation of light is perpendicular to both electric and magnetic fields.
- The transverse waves can be polarized but longitudinal waves cannot be polarized. Therefore phenomenon of polarization of light confirms that light waves are transverse waves.

Experimental arrangement to show that light waves are transverse. The lines with arrows indicates electric vibrations of light waves.

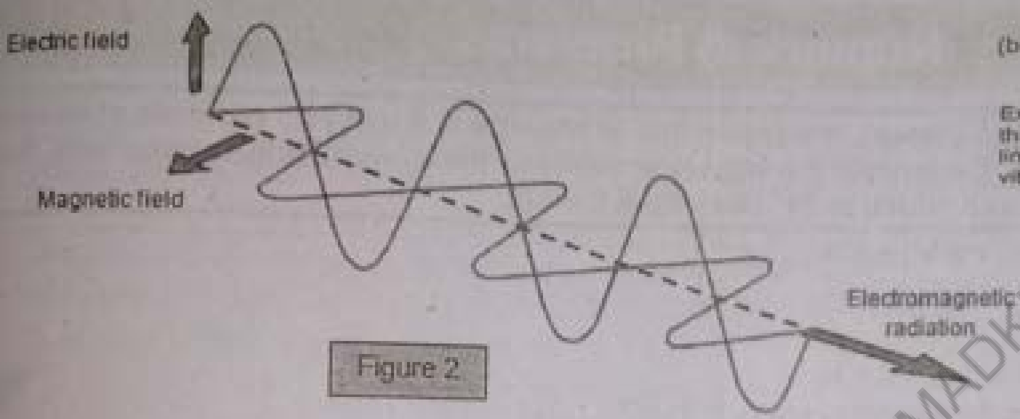


Figure 2

- As the wave propagates in (say) the z- direction, the electric field is oscillating in (say) the y - direction and the magnetic field is oscillating in the x -direction.

Since the x and y- axes are perpendicular to the z- axis, light is a transverse wave.

Comprehensive Questions

Q3. Give a short response to the following questions.

1. What is meant by the dual nature of light? Discuss the history about the nature of light in detail.

Ans: See Theory.

2. Explain the diffraction of X-rays by crystal and derive an expression for Bragg's law to find the wavelength of light used?

Ans: See Q.11 from book.

3. Describe the experimental arrangement for the production of interference fringes by Young's double slits method, and get an expression for the fringes space.

Ans: See Q.5 from book.

4. State and explain Huygen's principle. What is the difference between spherical and plane wavefronts?

Ans: See Q.2 from book.

5. Explain the interference effect produced by thin film.

Ans: See Q.6 from book.

6. What is the principle of interference of light? Discuss the necessary condition for interference of light.

Ans: See Q.4 from book.

7. What is diffraction grating? How can the wavelength of a beam of light be measured with it?

Ans: See Q.10 from book.

8. Describe the construction and working of Michelson's interferometer. How one can determine the wavelength of light used by this instrument?

Ans: See Q.7 from book.

9. What is meant by plane polarized light? How does this phenomenon decide that light waves are transverse in nature.

Ans: See Q.12 from book.

Numerical Problems

1. In a young double slit experiment, the separation of the slits is 1 mm and red light of wavelength 620 nm is falling on it. Determine the distance between the central bright band and the fifth bright fringe on the screen which is 3m away from the slit.

Data: Separation between two slits = d = 1mm = 1×10^{-3} m
Wavelength of red light = λ = 620nm = 620×10^{-9} m
For fifth bright fringe = m = 5
Distance between slits and screen = L = 3m
Distance of 5[th] bright fringe from center of screen = Y_5 = ?

Solution:

Formula $Y = \frac{m\lambda L}{d}$

Putting values

$$Y = \frac{5 \times 620 \times 10^{-9} \times 3}{1 \times 10^{-3}}$$

$$Y = 9.3 \times 10^{-3}\ \text{m}$$

$$\boxed{Y = 9.3\ \text{mm}}$$ **Answer**

2. Two parallel slits are illuminated by light of two wavelengths, one of which is 5.8×10^{-7} m. On the screen the fourth dark line of the known wavelength coincides with the fifth bright line of the light of unknown wavelength. Find the unknown wavelength.

Data: wavelength of 1[st] light = λ_1 = 5.8×10^{-7} m
For 4[th] dark fringe = m = 3
Wavelength of 2[nd] light = λ_2 = ?
For 5[th] bright fringe = m = 5

Solution:

Position of dark fringe is

$$Y_{dark} = \left(m + \frac{1}{2}\right)\frac{\lambda_1 L}{d}$$

(Putting m = 3 for 4[th] dark line)

$$(Y_{dark})_4 = \left(3 + \frac{1}{2}\right)\frac{\lambda_1 L}{d}$$

$$= \left(\frac{6+1}{2}\right)\frac{\lambda_1 L}{d}$$

$$(Y_{dark})_4 = \frac{7\lambda_1 L}{2d} \quad \text{------------- (1)}$$

Position of bright fringe is

$$(Y_{bright}) = \frac{m\lambda_2 L}{d}$$

Putting m = 5 for 5[th] bright fringe.

$$(Y_{bright})_5 = \frac{5\lambda_2 L}{d} \quad \text{---------------- (2)}$$

According to given condition 4^{th} dark line of 1^{st} light of known wavelength coincides with 5^{th} bright line of 2^{nd} light of unknown wavelength.

$$(Y_{dark})_4 = (Y_{bright})_5$$

Putting values from eq (1) and (2)

$$\frac{7\lambda_1 L}{2d} = \frac{5\lambda_2 L}{d}$$

$$\lambda_2 = \frac{7\lambda_1}{2 \times 5} \quad \text{Putting value of } \lambda_1$$

$$\lambda_2 = \frac{7 \times 5.8 \times 10^{-7}}{10}$$

$$\boxed{\lambda_2 = 4.1 \times 10^{-7} m}$$ **Answer**

3. When the movable mirror of a Michelson interferometer is moved 0.1mm. How many dark fringes pass through the reference point, if light of wavelength 580 nm is used?

Data: Distance moved by movable mirror = P = 0.1 mm = 0.1×10^{-3} m

Wavelength of light = λ = 580nm = 580×10^{-9} m

Number of fringes = m = ?

Solution:

Formula $$P = m\frac{\lambda}{2}$$

$$m = \frac{2P}{\lambda}$$

Putting values

$$m = \frac{2 \times 0.1 \times 10^{-3}}{580 \times 10^{-9}}$$

$$\boxed{m = 345 \text{ fringes}}$$ **Answer**

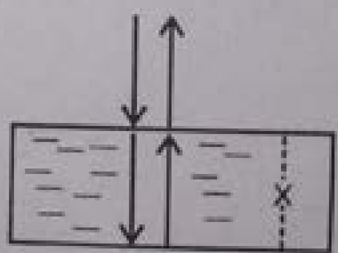
4. A soap film has a refractive index of 1.40. How thick must the film be, if it appears black, when mercury light of wavelength 546.1 nm falls on it normally?

Data: Refractive index = n = 1.40

Wavelength of light = λ = 546.1nm = 546.1×10^{-9} m

Thickness of film = x = ?

For 1^{st} order = m = 1



Solution:

Formula for destructive interference for thin film is

$2nx = m\lambda$

$$x = \frac{m\lambda}{2n}$$

$$\Rightarrow \quad x = \frac{546.1 \times 10^{-9}}{2(1.40)}$$

$$\Rightarrow \quad \boxed{x = 1.95 \times 10^{-7} m}$$ **Answer**

5. A diffraction grating has 5000 lines per centimeter. At what angle does the second order spectrum of the sodium yellow light of wavelength 589 nm occur?

Data: No. of lines = N = 5000 lines/cm

$N = 5000 \text{ lines}/(\frac{1}{100}m)$

$N = 5000 \times 100$ lines/m

$N = 5 \times 10^5$ lines/m

Gating element = $d = \frac{1}{N}$

$$d = \frac{1}{5 \times 10^5}$$

$$d = 2 \times 10^{-6}\ m$$

for 2nd order spectrum = m = 2

wavelength of light = λ = 589nm = 589×10^{-9} m

Angle = θ = ?

Solution:

Diffraction grating equation is

$$d \sin\theta = m\lambda$$

$$\sin\theta = \frac{m\lambda}{d} \quad \text{(Putting values)}$$

$$\sin\theta = \frac{2 \times 589 \times 10^{-9}}{2 \times 10^{-6}}$$

$$\sin\theta = 0.589$$

$$\theta = \sin^{-1}(0.589)$$

$$\boxed{\theta = 36^\circ}$$ **Answer**

6. Light is incident normally on a grating which has 250 lines/mm. Find the wavelength of spectral line for which the deviation in second order is 12°.

Given Data:

No. of lines = N = 250 lines/mm

$$N = 250\ \text{lines}/(\frac{1}{1000}\ m)$$

$$N = 250 \times 1000\ \text{lines/m}$$

$$N = 2.5 \times 10^5\ \text{lines/m}$$

$$d = \frac{1}{N} = \frac{1}{2.5 \times 10^5}$$

$$d = 4 \times 10^{-6}\ m$$

$$\theta = 12^\circ$$

For 2nd order = m = 2

wavelength = λ = ?

Solution:

Diffraction grating equation is

$$d \sin\theta = m\lambda$$

$$\lambda = \frac{d \sin\theta}{m}$$

$$\lambda = \frac{4 \times 10^{-6} \sin 12^\circ}{2}$$

$$\lambda = \frac{4 \times 10^{-6}(0.2079)}{2}$$

$$\boxed{\lambda = 4158 \times 10^{-10} m}$$ **Answer**

7. In a certain X-rays diffraction experiment the first order image is observed at an angle of 5° for a crystal plane spacing of 2.8×10^{-10} m. What is the wavelength of X-rays used?

Given Data:

for 1st order image = m = 1

Angle of diffraction = θ = 5°

Crystal plane spacing = d = 2.8×10^{-10} m

Wavelength of x-rays = λ = ?

Solution:

According to Bragg's equation

$$2d \sin\theta = m\lambda$$

$$\lambda = \frac{2d \sin\theta}{m}$$

Putting values

$$\lambda = \frac{2 \times 2.8 \times 10^{-10} \text{ Sin } 5^\circ}{1}$$

$$\lambda = \frac{2 \times 2.8 \times 10^{-10} (0.087)}{1}$$

$$\lambda = 0.488 \times 10^{-10} \text{ m}$$

$$\boxed{\lambda = 0.49 \times 10^{-10} \text{ m}}$$ **Answer**

8. An X-ray beam of wavelength 0.48×10^{-10} m is used to get Bragg reflection from a crystal at an angle of 20° for the first order maximum. What are the possible layer plane spacing which give rise to this maximum?

Given Data:
Wavelength of x-rays = $\lambda = 0.48 \times 10^{-10}$ m
Angle of diffraction = $\theta = 20^\circ$
For 1^{st} order maximum = m = 1
Crystal plane spacing = d = ?

Solution:

$$2d \text{ Sin } \theta = m\lambda$$

$$d = \frac{m\lambda}{2 \text{ Sin } \theta}$$

Putting values

$$d = \frac{1 \times 0.48 \times 10^{-10}}{2 \text{ Sin } 20^\circ}$$

$$d = \frac{0.48 \times 10^{-10}}{2 (0.342)}$$

$$\boxed{d = 0.70 \times 10^{-10} \text{ m}}$$ **Answer**

9. The spacing of one set of crystal planes in NaCl (table salt) is d= 0.282nm.A monochromatic beam of x-rays produces a Bragg maximum when its glancing angle with these planes is $\theta = 7^0$. Assuming that this is a first order maximum (n=1) .Find the wavelength of x-ray.

Given Data: Order of maximum = n =1
Crystal interplanar spacing = d = 0.282×10^{-9}m
Angle = $\theta = 7^\circ$

To find : Wavelength = λ = ?

Solution: $2 d \sin \theta = n \lambda$

$$\lambda = \frac{2d \text{ Sin } \theta}{n} = \frac{2 \times 0.282 \times 10^{-9} \times \sin 7^\circ}{1}$$

$$\lambda = 6.9 \times 10^{-11} \text{ m}$$

$$\lambda = 0.069 \times 10^{-9} \text{ m}$$

$$\boxed{\lambda = 0.069 \text{ nm}}$$ **Answer**

Additional Conceptual Short Questions With Answers

1. How can you increase the fringe width in Young's double slit experiment?

Ans. The fringe width is

$$\Delta y = \frac{\lambda L}{d}$$

The fringe width can be increased by increasing

(i) by increasing wavelength λ.

(ii) by increasing the distance between the coherent source L and
(iii) by decreasing the distance between the coherent source d.

2. The Young's double slit experiment apparatus is taken from air in to water. What will happen to the interference pattern?

Ans: The fringe width is

$$\Delta y = \frac{\lambda L}{d}$$

When Young's double slit apparatus is brought from air into water then wavelength λ of light decreases and the fringe width ΔY decreases.

3. What changes would you expect if monochromatic light is replaced by white light in single slit diffraction experiment?

Ans: The diffraction pattern is coloured. In higher order spectra due to higher dispersion there will be overlapping of colours and a diffused pattern is obtained.

4. How can we obtain the plane wave fronts (Parallel Rays) of light?

Ans: (i) As the wave fronts are the parts of spheres of very large radii. Then for large distances from the source, a part of the spherical wave front can be regarded as a plane wave front (parallel rays of light).

(ii) If the source of light is at the focus of convex lens. Then the rays coming out the lens will constitute the plane wave fronts.

5. If a class room door is open even just a small amount, you can hear sounds coming from the hallway. Yet you cannot see what is going on in the hallway. Why is there this difference?

Ans: The space between the slightly open door and the wall is acting as a slit for waves. The sound waves have wavelengths comparable with the slit width, so sound is diffracted effectively by the opening and sound spread out in the whole room. Light wavelengths are much smaller than the slit width, so there is a very little or no diffraction for the light. So they do not spread out in the room and we are not able to see what is going on in the hallway.

6. Why does a diamond sparkle more than a glass of the same shape and size?

Ans: The refractive index of diamond is very high and its critical angle is small (i.e., 24°). When a beam of light enters it, it is totally reflected number of times insides it and it emerges in random directions causing the sparkling of the diamond. But in case of glass refractive index is less and critical angle greater than diamond, so it does not sparkle like a diamond.

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **By using the transmitted light, the central spot of Newton's ring appears to be:**
(a) Bright (b) dark (c) coloured (d) all of these

2. **Longitudinal waves do not exhibit:**
(a) Reflection (b) refraction (c) diffraction (d) polarization

3. **The phase change of 180° is equal to the path difference of:**
(a) λ (b) $\lambda/2$ (c) 2λ (d) 3λ

4. **Bending of light around edges of an obstacle is called:**
(a) Refraction (b) polarization (c) interference (d) diffraction

5. **If 5000 lines per cm are ruled on diffraction grating, then slit spacing will be:**
(a) 5×10^{-3} A° (b) 0.02 m (c) 2×10^{-4} A° (d) 2×10^{4} A°

6. **Condition of constructive interference for first order image for diffraction grating is ...**
(a) $d\sin\theta = \lambda$ (b) $d\sin\theta = 0$ (c) $d\sin\theta = 2\lambda$ (d) $d\sin\theta = 3\lambda$

7. **Which one of the following wave cannot be polarized?**
(a) Radio wave (b) X-ray (c) longitudinal wave (d) transverse wave.

8. **What happened to the fringe spacing, when Young double slit experiment is performed in water instead of air:**
(a) Enlarge (b) Shrink (c) Disappear (d) No effect

9. **Soap film in sunlight appears coloured due to** (FBISE (ON) 2013)
(a) Scattering of light (b) interference of light (c) refraction of light (d) dispersion of light

10. **The blue of sky is due to**
(a) Diffraction of light (b) reflection of light (c) polarization of ling (d) scattering of light

11. **Optically active substances are those substances which:** (FBISE- 2017)
(a) Produce Polarized light (b) Produce double refraction
(c) Rotate the plane of polarization of polarized light
(d) Convert a plane polarized light into circulatory polarized light

12. **The refractive index of rarer medium with respect to a denser medium is:** (FBISE (ON) 2017)
(a) Zero (b) 1 (c) Greater than 1 (d) Smaller than 1

13. **A diffraction grating has 5000 lines per cm. Its grating element is:** (FBISE- 2018)
(a) 2×10^{-3} cm (b) 0.2×10^{-3} cm (c) 200×10^{-3} cm (d) 20×10^{-3} cm

14. **The Bragg equation is given as:** (FBISE(ON) 2018)
(a) $2d\sin\theta = n\lambda$ (b) $2n\sin\theta = d\lambda$ (c) $2\sin\theta = n\lambda$ (d) $n\sin\theta = d\lambda$

15. **In Michelson's interferometer, a fringe is shifted, each time the mirror is displaced through:**
(a) λ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{8}$ (d) $\frac{\lambda}{4}$ (FBISE(ON) 2018)

Answers Key

1.	a	2.	d	3.	b	4.	d	5.	d
6.	a	7.	c	8.	b	9.	b	10.	d
11	c	12	d	13	b	14	a	15	d

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 Choose the correct answer from the given options. (1 x 6 = 6)

SECTION – A

1. **The fringe spacing varies inversely with the:**
(A) distance between slits and screen (B) separation between the slits
(C) wavelength of light (D) none of these
2. **The blue colour of sky is due to:**
(A) diffraction of light (B) reflection of light (C) polarization of ling (D) scattering of light
3. **Soap film in sunlight appears coloured due to:**
(A)scattering of light (B) interference of white light
(C) refraction of light (D)dispersion of light
4. **If 5000 lines per cm are ruled on a diffraction grating, the slit spacing or grating element will be:**
(A) 2×10^{-2} cm (B) 2×10^{-3} cm (C) 2×10^{-4} cm (D) 2×10^{-5} cm
5. **Two monochromatic radiations X and Y are incident normally on a diffraction grating. The second order intensity maximum for X coincides with the third order intensity maximum for Y. What is the ratio $\frac{\text{wavelength of X}}{\text{wavelength of Y}}$**
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{1}$
6. **The tip of a needle does not give a sharp image. It is due to:**
(A) Diffraction of light (B) Interference of light (C) Polarization of light (D) Refraction of light

Question.No.2 Give short answers of followings: (3 x 7 =21)

SECTION – B

(i) Why is the fly wheel of an engine made heavy in the rim?
(ii) A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?
(iii) Why does the coasting rotating system slow down as water drops into the beaker?
(iv) Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles?
(v) Explain, why is there weightlessness in satellites?
(vi) At what speed (in km/h) is a bank angle of 45° required for aero-plane to turn on a radius of 60 m?
(vii) How artificial gravity is produced in spaceships? Explain

Question.No.3 Extensive Questions. (13)

SECTION – C

(a) Describe the Young's double slit experiment for demonstration of interference of light in detail. Derive an expression for fringe spacing. (8)

(b) Two parallel slits are illuminated by light of two wavelengths, one of which is 5.8×10^{-7} m. On the screen the fourth dark line of the known wavelength coincides with the fifth bright line of the light of unknown wavelength. Find the unknown wavelength. (5)

The End

CHAPTER 10

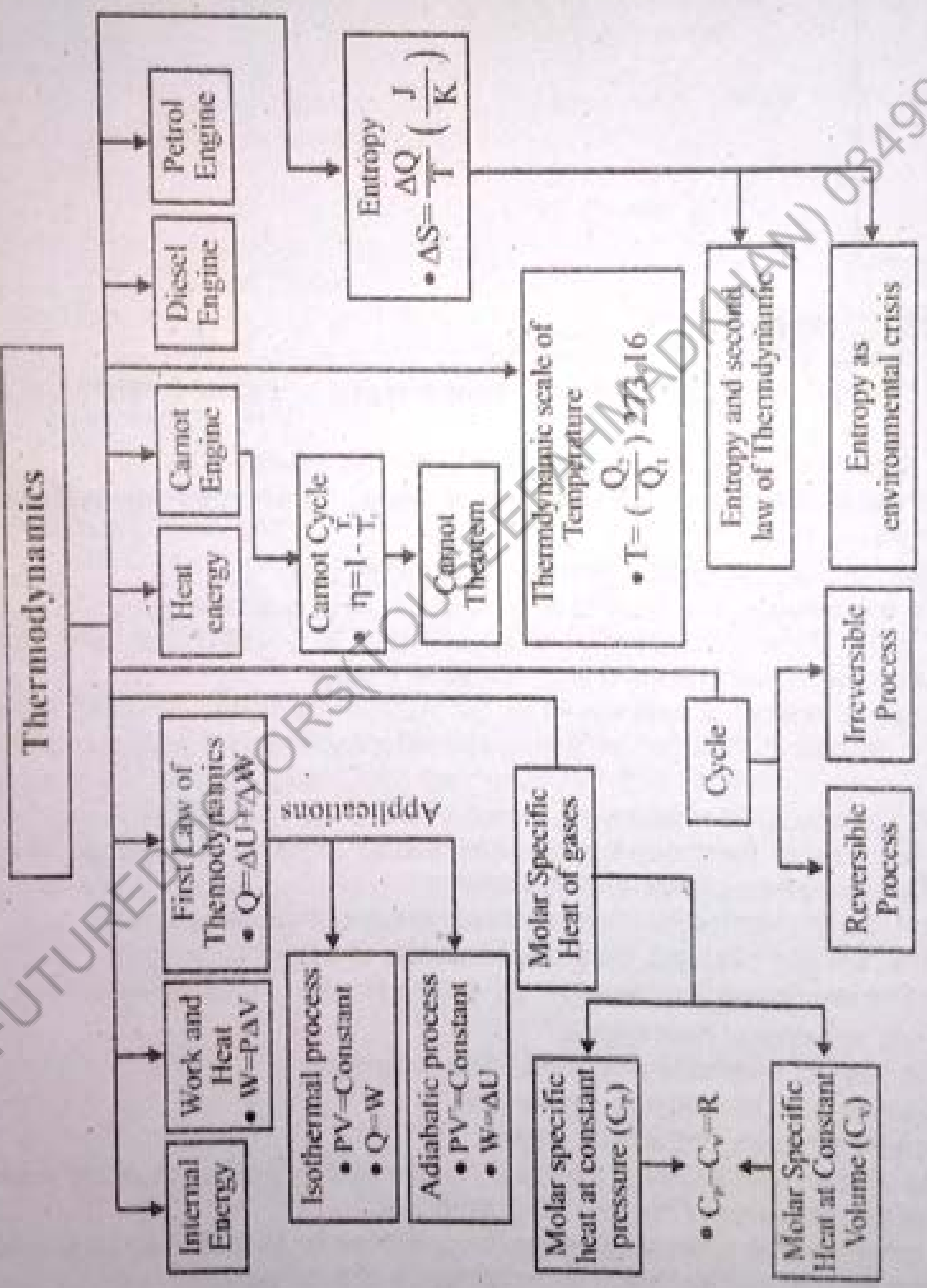
THERMODYNAMICS

Learning Objectives

- Describe that thermal energy is transferred from a region of higher temperature to a region of lower temperature.
- Describe that regions of equal temperatures are in thermal equilibrium.
- Describe that heat flow and work are two forms of energy transfer between systems and calculate heat being transferred.
- Define thermodynamics and various terms associated with it
- Relate a rise in temperature of a body to an increase in its internal energy.
- Describe the mechanical equivalent of heat concept, as it was historically developed, and solve problems involving work being done and temperature change.
- Explain that internal energy is determined by the state of the system and that it can be expressed as the sum of the random distribution of kinetic and potential energies associated with the molecules of the system.
- Calculate work done by a thermodynamic system during a volume change.
- Describe the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and work done on the system.
- Explain that first law of thermodynamics expresses the conservation of energy.
- Define the terms, specific heat and molar specific heats of a gas.
- Apply first law of thermodynamics to derive $C_p - C_v = R$.
- State the working principle of heat engine.
- Describe the concept of reversible and irreversible processes.
- State and explain second law of thermodynamics.
- Explain the working principle of Carnot's engine.
- Explain that the efficiency of a Carnot engine is independent of the nature of the working substance and depends on the temperatures of hot and cold reservoirs.
- Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine.
- Derive an expression for the coefficient of performance of a refrigerator.
- Describe that change in entropy is positive when heat is added and negative when heat is removed from the system.
- Explain that increase in temperature increases the disorder of the system.
- Explain that increase in entropy means degradation of energy.
- Explain that energy is degraded during all natural processes.
- Identify that systems tend to become less orderly over time.

Chapter No. 10

CONCEPT MAP



Thermodynamics:

"Thermodynamics is a combination of two words, "thermo" and "dynamics". The word thermo is related to heat while dynamics is related to the motion of particles".

Therefore, we define thermodynamics as the branch of physics that deals with the transformation of heat into other forms of energy such as mechanical, chemical and electrical energy and vice versa.

- Principally it is based on two laws of thermodynamics i.e. the first and second laws of thermodynamics.
- It is a practical subject that explains the working of heat engines, refrigerators and heat pumps etc.

For Your Information

The first thermodynamic textbook was written in 1859 by William Ranking, originally trained as a physicist and a civil and mechanical engineering professor at the University of Glasgow.

Role of Thermodynamics

- It plays a central role in technology.
- All the raw energy available for our use is liberated in the form of heat.

Q.1 What is thermal equilibrium?

Ans: **Thermal Equilibrium**

- When two objects at different temperatures are brought into contact with each other, energy is transferred from the hotter to the colder object until the bodies reach thermal equilibrium (that is, they are at the same temperature.
- These observations reveal that heat is energy transferred spontaneously due to a temperature difference. Figure 10.1 shows an example of heat transfer.

Figure 10.1 (a) Here, the soft drink has a higher temperature than the ice, so they are not in thermal equilibrium. (b) When the soft drink and ice are allowed to interact, heat is transferred from the drink to the ice due to the difference in temperatures until they reach the same temperature, T', achieving equilibrium.

- In fact, since the soft drink and ice are both in contact with the surrounding air and the bench, the ultimate equilibrium temperature will be the same as that of the surroundings.
- In other words, if two objects are in thermal equilibrium, they have the same temperature.

Point to Ponder

Q. Two bodies, one hot and the other cold, are kept in vacuum. What will happen to the temperature of the hot body after some time?

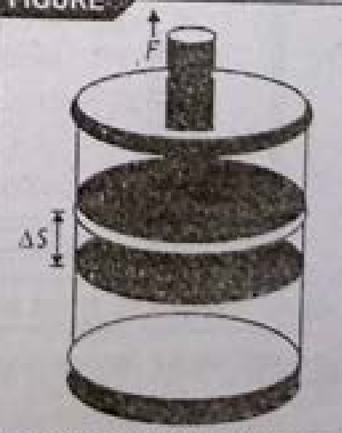
A. Due to thermal radiation, after some time the cold body will gain heat, and the hot body will lose heat. When the temperature of the two bodies become equal then bodies are said to be in themodynamic equilibrium.

Q.2 Explain work done in thermodynamics.

Ans: **WORK**

- Work W is defined as $\vec{F}.\Delta\vec{S}$
- In thermodynamics, work is said to be done by a system when the system as a whole expands.
- By means of the boundary, the system exerts a force on the surroundings and thereby displaces the surroundings,
- The sum of all $\vec{F}.\Delta\vec{S}$, S over the entire boundary of the system is the work done by a system on its surroundings.

FIGURE



Work done by a system

- Similarly, in the process of contraction of a system, work is done on the system by its surroundings.
- By convention the work done by a system is considered positive and the work done on the system is taken as negative.

QUIZ

Q. The temperature of a normal healthy person is 37°C. What will be the temperature of a dead person?

A. It depends on how long the person has been dead. Temperature of dead body slowly approaches that of the surrounding place (surrounding air).

Q.3 Define and explain internal energy.

Ans: **Internal Energy**

The sum of all the forms of molecular energies (such as kinetic and potential energy) of a substance is called internal energy. (OR)

The sum of the kinetic and potential energies associated with the random motion of the atoms of the substance is the internal energy of the substance.

The kinetic energy may be in the form of translational, rotational or vibrational kinetic energy.

- Atoms in solids vibrate back and forth about their equilibrium positions.
- Molecules in a liquid wander around among the other molecules, having frequent collisions with them and thus exchanging energy.
- In gases the molecules travel about at high speeds and have frequent elastic collisions with their neighbors.
- When we heat a substance, the random motion and the energy associated with it are increased: heat energy is converted into the internal energy of the substance.
- Similarly, work can be performed on a substance in such a way as to increase the random motion of the atoms.
- Since Temperature $\propto \langle K.E. \rangle$
- Therefore internal energy of an ideal gas is directly proportional to its temperature.
- The increase in temperature of the object indicates an increase in the internal energy.

Internal energy is a state function

- In thermodynamics, *internal energy is function of state.* Consequently, it does not depend on the path but depends on initial and final states of the system.

QUIZ

Q.1 Is the temperature of a normal healthy old man is less than that of a normal healthy young man?

A.1 When people get very old, blood circulation often becomes poor, resulting in cold hands and feet. The mean body temperatures of old man lower than the reference temperature

Q.2 A glass full of water contains ice cubes floating in it. What will happen to the water level when ice melts?

A.2 The water level remains the same when the ice cube melts.

According to the Archimedes principle , the floating substance displaces some liquid, so when the ice melts ,there will be no change in the water level as the melted ice will occupy the same volume as it was occupying earlier.

Q.4 What is heat and work?

Ans: **Heat, Work and internal energy**

Heat *is form of energy which flows from the hotter body to the colder body till the temperatures of the two bodies become equal .*

- To raise the temperature of some water we "heat" the water by placing the water pot on a flame.

- The earth is heated by the hot sun without a direct thermal contact.
- Thus heat is energy which is transferred between a substance and its surroundings or between one part of a substance to another as a result of temperature difference only.
- Heat is energy in transit.
- SI unit of heat is joule. Another common unit used for heat is calorie.

Calorie: Calorie is the heat energy required to change the temperature of one gram of water by 1°C Specifically 14.5 °C *to* 15.5 °C

Kilocalorie is the energy needed to change the temperature of 1 kg of water by 1°C

- Besides supplying heat to a body there is another very general method of raising the temperature (heating) of a body. The method is to perform work on the body. A few examples are as follows:
  - It is possible to warm up your hands by rubbing them together ;
  - When we hit a nail with a hammer into wood, the nail gets hot.
  - Pumping up a tyre with a hand pump heats up the pump.

In all these examples work is done on a body which warms up without heat supply.

- The conclusion is that heat and work both can increase the temperature of a body but they are mutually distinct forms of energy.
- The difference between heat and work is that work can be exchanged between an object and its surroundings by the over all displacement of the object without requiring the difference of temperature
- Whereas transfer of heat energy can be brought about by a temperature difference between two bodies.

QUIZ

Q. Bore with a small drill into a hard board. The drill becomes hot to touch. Why? What happens if hard board is replaced by a soft board?

A. A large amount of work has to be done in making the **bore** which produces a large amount of heat and the **drill becomes** too **hot to touch**. In case of the **soft board**, the friction is less, **small** work has to be done in making the **bore**. Hence heat produced is also **small**.

Q.5 What is mechanical equivalent of heat?

Ans: **Equivalence of Heat and Work**

- The equivalence between heat and work was established by Count Rumford, James Joule and others.
- In 1798, Count Rumford, an artillery engineer observed that a great deal of heat was given off in the process of boring a gun metal.
- He observed that heat could be produced by friction, i.e. work done against friction converts in to heat energy.
- This led Rumford to convince that heat is a form of energy.
- In 1845, Joule carried out experiments to measure very precisely the quantity of heat produced by a certain amount of work (mechanical energy) and observed that there is a definite relationship between the mechanical work done and the heat generated.
- He showed that when a given amount of work is done, the same amount of heat is always produced, no matter what may be the process of transformation.
- He showed that work (W) done, is proportional to the quantity of heat (Q)

$$W \propto Q$$
$$W = JQ$$

Where (J) is called "Mechanical Equivalent" of heat (Joule's constant) and it is defined as

- **The ratio of the work done in joules to the heat produced in calories (old unit of heat) is called the mechanical equivalent of heat (J).**
- The mechanical equivalent of heat may also be defined as the amount of mechanical energy or work required to produce a unit quantity of heat.

- The currently accepted value of mechanical equivalent of heat is J = 4.18 joule per calorie.
- Now if we use the SI units of work and energy, then both, work (W) and heat (Q), are measured in joules then the value of mechanical equivalent of heat is one.

QUIZ

Q. What is the significance of Joule's experiment for determining the value of J.

A. Joule's experiment proved that heat was actually a form of mechanical energy, so it was a step towards our modern understanding of the **conservation of energy**.

Brain Teaser

1. **Is the temperature of a normal healthy old man is less than that of a normal healthy young man?**

A. When people get very **old**, blood circulation often becomes poor, resulting in cold hands and feet. The mean body temperatures of old man lower than the reference temperature.

2. **A glass full of water contains ice cubes floating in it. What will happen to the water level when ice melts?**

A. **The water level remains the same when the ice cube melts.**

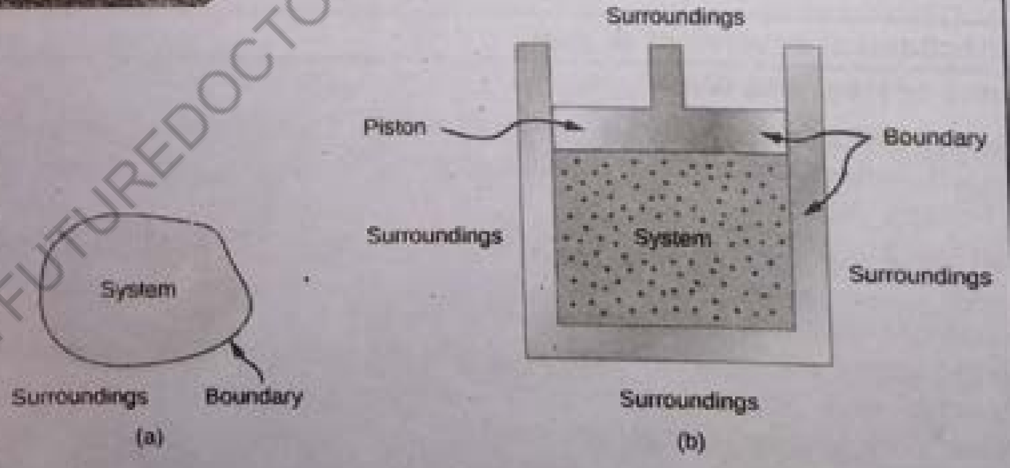
According to the Archimedes principle, the floating substance displaces some liquid, so when the ice melts, there will be no change in the water level as the melted ice will occupy the same volume as it was occupying earlier.

Q.6 Explain thermodynamic system and its types. Also discuss the thermodynamic state variable.

Ans: **Thermodynamic system**

- A thermodynamic system includes anything whose thermodynamic properties are of interest.
- It is enclosed in its surroundings or environment; it can exchange heat with, and do work on, its environment through a boundary, which is the imagined wall that separates the system and the environment as shown Figure.

FIGURE 10.2



(a) Illustration of a system (b) The burning gasoline in the cylinder of a car engine is an example of a thermodynamic system.

- In reality, the immediate surroundings of the system are interacting with it directly and therefore have a much stronger influence on its behavior and properties. For example, if we are studying a car engine, the burning gasoline inside the cylinder of the engine is the thermodynamic system; the piston, exhaust system, radiator, and air outside form the surroundings of the system. The boundary then consists of the inner surfaces of the cylinder and piston.

A system is described in terms of thermodynamics variables such as pressure (P), volume (V) and *temperature (T).*

- Any change or a series of changes in a thermodynamic system is called thermodynamic process.

We need to define the following technical terms which are essential for the study of thermodynamics.

System:

The quantity of matter or region of space under observation is called system.

For example a gas enclosed in a cylinder etc.

Surroundings of the system:

Everything other than the system in the universe is called the surroundings of the system.

Boundary of the system

- The system is separated from the surroundings by its boundary.
- In thermodynamics, the exchange of energy between the system and the surroundings can take place through the boundary by the performance of work or the flow of heat. Fig..There are several kinds of systems

Closed system:

The system in which heat energy can flow in to or out off the system but mass cannot enter or leave it, is called closed system.

For You Information

Q. An empty polythene bag burns or melts on a flame of a stove. However we can make a few cups of tea by placing water filled polythene bag on a flame of a stove.

A. The melting point of polythene bag is higher than water i.e. (100 °C) When water is inside the polythene bag, then by heating the polythene bag; it transfers heat to water and polythene bag does not keep enough heat to be ignited.

FIGURE. 10.3

(a) (b)

(a) This boiling tea kettle is an open thermodynamic system. It transfers heat and matter (steam) to its surroundings. (b) A pressure cooker is a good approximation to a closed system. A little steam escapes through the top valve to prevent explosion.

Ponit to Ponder

Q. An ink dot on a white porcelain dish appears dark. When the dish is raised to a very large temperature the dot appears brighter than the surroundings. Why?

A. When temperature of dish is raised to very high then it begin to emit radiation of different wavelengths. Due to this reason the ink dot start to appear brighter than its surrounding.

Open system:

The system in which both heat energy and mass can enter or leave it, is called open system.

- In an open system the transfer of heat energy can take place from the system to the surrounding or vice versa. Plants and animals are examples of open system because they exchange materials (food, oxygen, waste products) with the surroundings.

Isolated system:

The system in which there is no transfer of mass and heat energy across its boundary is called an isolated system.

- In an isolated system both mass and energy cannot enter or leave the system across its boundary. Tea contained in a well insulating thermo flask is a n example of an isolated system.

Thermodynamic state and thermodynamic state variables

- The particular condition when a system has specified values of pressure (P), volume (V) and temperature (T) etc. is called the state of the system.
- The variables or functions which determine the physical state of the system are called state variables and state functions of the system.

- If the system is homogeneous (i.e. it has the same composition every where) then it is usually enough to specify only three parameters, namely, volume, pressure and temperature.
  The mathematical relationship between these parameters is known as the equation of state of the system.
- In the case of ideal gas, the equation of state is $PV = nRT$ where n is number of moles. P, V and T are respectively the pressure, volume and temperature of the gas and R is the universal gas constant. P, V, T, are state variables.
- Other examples of the state variables are internal energy (U) and entropy (S).

Q.7 Write a note on Reversible and Irreversible Processes.

Ans: In thermodynamics, a process means a change in state of a system brought about by a change in the state variables.

A process occurs when system interacts and exchanges energy with its surrounding.

Reversible Process

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings.

Explanation

- In the reverse process, the working substance passes through the same stages as in the direct process, but thermal and mechanical effects at each stage are exactly reversed.
- If heat is absorbed in the direct process, it will be given out in the reverse process.
- If work is done by the substance in the direct process, work will be done on the substance in the reverse process.
- Hence, the working substance is restored to its original conditions.

Ponit to Ponder

Q. Why do heels crack in winter? What Affect does the application of lubricants have on the heels?

A. When the sensitive skin on the bottom of the feet and heels becomes too dry due to deficiency of moisture, it can split open, leaving painful cracks called fissures on your heels. The application of lubricants decreases the driness of skin.

Cycle

- A succession of events which bring the system back to its initial state is called a cycle.

Examples of Reversible Process

(1) The process of liquefaction and the evaporation of a substance performed **slowly** are reversible processes.

(2) Slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by decreasing the pressure on the piston to reverse the operation.

Irreversible Process

An irreversible process is one which can not be retraced in exactly reverse order, without producing any change in the surroundings.

Explanation

- All changes which occur **suddenly** or which involve **friction** or **dissipation** of energy through conduction, convection and radiation are irreversible.

Examples

(i) Explosion is an example of highly irreversible process.

(ii) Work done against friction.

Q.8 Explain the first law of Thermodynamics and its consequences?

Ans: **First Law of Thermodynamics**

"When the heat ΔQ is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work done ΔW by the system on its surroundings".

Mathematically

$$\boxed{\Delta Q = \Delta U + \Delta W} \quad \text{..........................(1)}$$

This is equation of first law of thermodynamics.

Explanation

- ΔQ is taken as positive when heat enters the system.
- ΔQ is taken as negative when heat leaves the system.
- ΔU is taken as positive when temperature of the system rises.
- ΔU is taken as negative when temperature of the system decreases.
- By convention the work done by a system is taken as positive
- The work done on the system is taken as negative.
- First law of thermodynamics obeys the law of conservation of energy.

When heat is added to a system; there is an increase in the internal energy from U_A to U_B due to the rise in temperature and an increase in pressure or change in the state. If at the same time, a substance is allowed to expand, then ΔW is the work done on its environment.

change in internal energy $\Delta U = U_B - U_A$ ....(2)

Where U_A is initial internal energy of system

U_B is final internal energy of system

From equation (1)

$$\Delta U = \Delta Q - \Delta W \quad \text{———— (3)}$$

Putting values from equation (2) in (3)

$$U_B - U_A = \Delta Q - \Delta W \quad \text{..........................(4)}$$

If system under goes cyclic process then $U_B = U_A$

Putting $U_B = U_A$ in equation (4)

$$U_A - U_A = \Delta Q - \Delta W$$

$$0 = \Delta Q - \Delta W$$

$$\Delta Q = \Delta W$$

QUIZ

Q. Two blocks of ice when pressed together, combine to form single piece. Explain how this happen?

A.
- When two blocks are pressed together little, heat is generated due to pressure and of friction, which just melts the outer layers and friction decreases.
- When pressure is decreased and due to the low temperature of both the ice blocks, the water just refreezes and blocks combine to form single piece. It's called Regelation.
- Regelation is the phenomenon of melting under pressure and freezing again when the pressure is reduced.

Q.9 Discuss applications of first law of thermodynamics.

Ans: Applications of First law of thermodynamics

(a) **Isochoric Process:**

The thermodynamics process in which the volume of the system remains constant is called isochoric process.

- We consider the gas contained in a cylinder having a conducting base, non-conducting walls and with a fixed piston at the end as shown in the Fig (10.4.a).
- Let heat (ΔQ) is supplied to the system at constant volume.
- The pressure of the gas increases from P_1 to P_2
- Temperature increases from (T_1) to (T_2).

Since the system neither expands nor contracts, work is neither done by the system nor on the system

$$\Delta W = 0$$

- According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Putting $\Delta W = 0$

$$\Delta Q = \Delta U + 0$$

$$\boxed{\Delta Q = \Delta U}$$

- The above equation shows that in an isochoric process the entire amount of heat supplied to the gas is converted to the internal energy of the gas. The pressure and temperature of the gas will increase.
- On the contrary, removal of heat from a system under isochoric condition will cause an equivalent decrease in the internal energy. The system will cool down and the pressure will fall.
- The graph of isochoric process is called an "isochor, which is a straight line, parallel to the pressure axis as shown in the Fig (10.4 b).

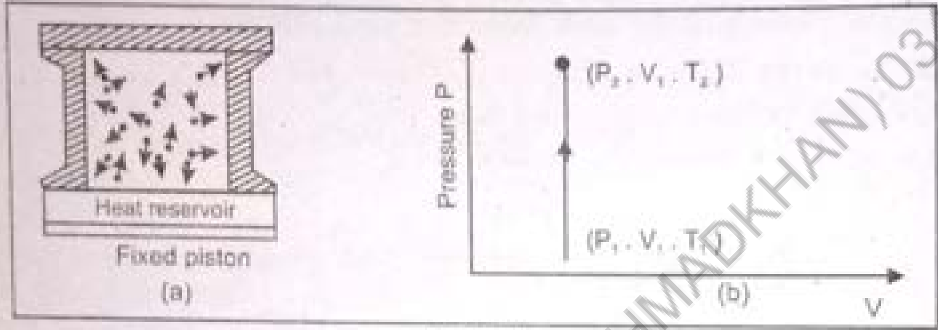


Fig 10.4

(b) Isobaric Process:

The thermodynamics process in which the pressure is kept constant is called an isobaric process.

- Isobaric expansion of a system is often used to convert heat into work.
- Practically all heat engines depend on the transformation of heat into work.
- We consider the gas contained in a cylinder having a conducting base and non-conducting walls and frictionless piston of cross-sectional area (A) as shown in the Fig (10.5a).
- Let ()(T_1) and (P) be the volume, temperature and pressure of the gas.
- When heat energy (ΔQ) is transferred into the system at constant pressure then gas expands and moves the piston outward.
- Volume of system changes from V_1 to V_2 ($V_2 > V_1$)
- Temperature of system changes from T_1 to T_2 ($T_2 > T_1$)
- If the piston moves through a small displacement (ΔY) in the upward direction, the work done by the gas against the environment is

$$\Delta W = (\text{Force})(\text{distance})$$

$$\Delta W = F.\Delta Y \qquad (1)$$

As we know that pressure is $P = \frac{F}{A}$

Where (F) is the force exerted on the piston during expansion: $F = PA$

Putting $F = PA$ in equation (1)

$$\Delta W = PA\,\Delta Y \text{ ....}(2)$$

Putting $A\,\Delta Y = \Delta V = V_2 - V_1$ in equation (2)

$$\Delta W = P(V_2 - V_1) = P\Delta V$$

- Where (V_1) is the initial volume, (V_2) is the final volume and (ΔV) is the increase in volume of the gas. Hence, the work done by the gas which expands at constant pressure is

$$\Delta W = P\Delta V \text{ -------------------------- } (3)$$

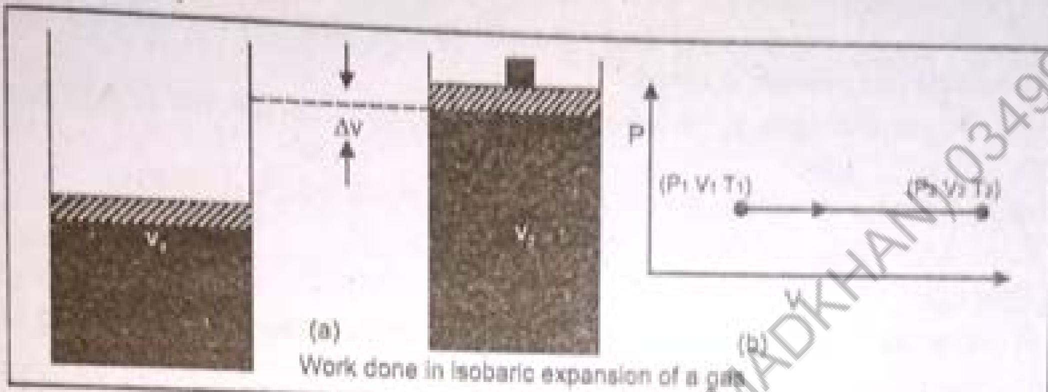
According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W \quad \ldots\ldots\ldots\ldots \quad (4)$$

Putting value of ΔW from equation (3) in above equation (4) we get

$$\Delta Q = \Delta U + P\Delta V \quad \ldots\ldots\ldots\ldots \quad (5)$$

- The work performed by the expanding or contracting gas comes from one or both sources: heat supplied to the gas and the internal energy of the gas.
- The graph of isobaric process is called an "isobar", which is a straight line, parallel to the volume axis as shown in the Fig (10.5b).



Work done in isobaric expansion of a gas

Fig: 10.5

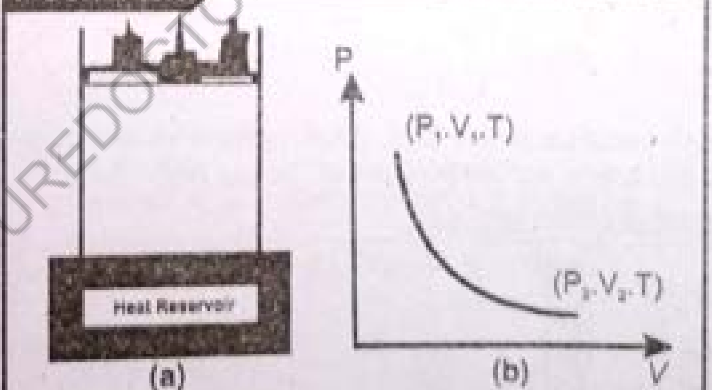
(c) **Isothermal Process**

A process in which the temperature of the system remains constant is called isothermal process.

- Let us consider the ideal gas is filled in a cylinder having a conducting base and non-conducting walls and with a movable piston at the end as shown in the Fig. (10.6a).
- The base of the cylinder is placed on a heat reservoir at temperature (T_1).

A reservoir is a body of large heat capacity that maintains the temperature of the gas at (T_1).

FIGURE. 10.6



Isothermal expansion of a gas.

- Pressure of the system (on piston) is decreased from P_1 to P_2. ($P_2 < P_1$)
- Volume increases from V_1 to V_2 ($V_2 > V_1$)
- Due to this expansion the temperature of the gas tends to fall but heat ΔQ is conducted from the heat reservoir to the gas so that the temperature of the system remains constant equal to the temperature of the reservoir.
- The whole process from an initial state (P_1), (V_1), (T) to a final state (P_2), (V_2), (T) is represented by a continuous curve which is called an " isotherm" as shown in the Fig (10.6. b).

- During isothermal expansion some work (ΔW) is done by the gas in pushing up the piston in the cylinder.
- Since the temperature remains constant, there is no change in the internal energy of the gas, that is,

$$\Delta U = 0$$

According to the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

Putting $\Delta U = 0$

$$\Delta Q = 0 + \Delta W$$

$$\Delta Q = \Delta W$$

- This shows that if the gas expands and does external work, an equal amount of heat has to be supplied in order to maintain its temperature constant.
- Conversely, if the gas contracts, work is being done on it and equal amount of heat has to be allowed to leave the gas.
- *In isothermal process the condition for the application of Boyle's Law is fulfilled.*

$$P_1V_1 = P_2V_2$$

(d) **Adiabatic Process**

A process in which no heat enters or leaves the system is called adiabatic process.

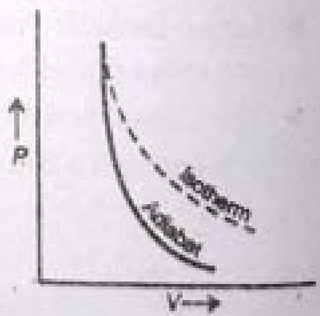
(OR)

The process in heat energy of the system remains constant is called adiabatic process.

Explanation

- In adiabatic process the walls,base and piston of cylinder are insulating so that no heat can enter or leave it.

Since in adiabatic process no heat enters or leaves the system i.e., $\Delta Q = 0$



- pressure decreases
- volume increases

According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Putting $\Delta Q = 0$

$$0 = \Delta U + \Delta W$$

OR $W = -\Delta U$

Adiabatic Expansion: Above equation **shows that** work is done by the system at the expense of internal energy of its molecules and hence, the temperature of the gas falls.

- Therefore adiabatic expansion causes cooling.

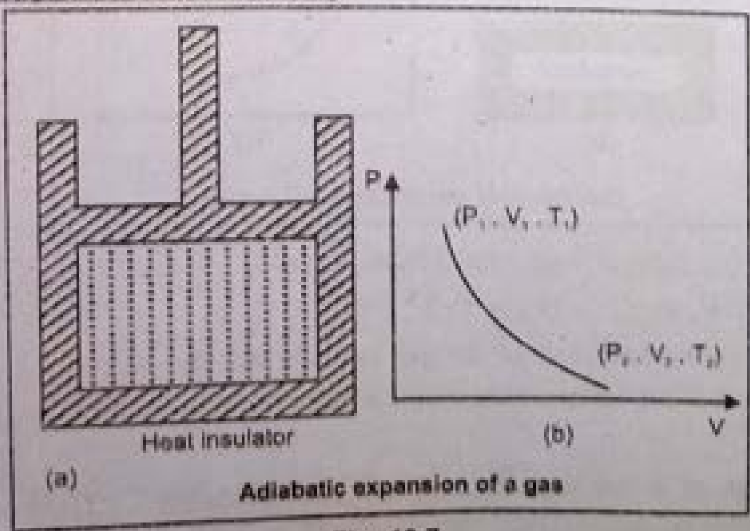


Fig: 10.7

Adiabatic compression.

- Pressure increases
- Volume decreases
- Work is done on the gas; it increases the temperature and internal energy of the gas.

$$-W = \Delta U$$

Condition for adiabatic change

- Adiabatic change occurs when the gas expands or is compressed rapidly. Particularly when the gas is contained in an isolated cylinder.

For adiabatic process PV^{γ} =constant

Where, γ is the ratio of the molar specific heat of the gas at constant pressure to the molar specific heat at constant volume. i.e.,

$$\gamma = \frac{C_p}{C_v}$$

Adiabat: The curve representing an adiabatic process is called an adiabat.
An adiabat is steeper than an isotherm.

Examples of Adiabatic Process

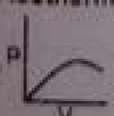
1. The rapid escape of air from a burst tyre.
2. The rapid expansion and compression of air through which a sound wave is passing.
3. Cloud formation in the atmosphere.

QUIZ

Q. The rate of formation of ice on ponds decreases gradually as more and more ice is formed. Why?

A. When ice forms on the water surface then ability to cool the water beneath decreases as ice is insulator.
If the ice sheet continues to grow thicker, it will act more and more as a heat insulator for the water. Water below the ice cannot transfer heat to the outer cool atmosphere. That's why temperature of denser water at 4°C does not further decrease i.e. it remains water.

MCQ's

1. **Which remains constant in adiabatic process?**
(A) Volume (B) Pressure (C) Entropy (D) Temperature
2. **Which one is true for the isothermal process?**
(A) Q = 0 (B) W = 0 (C) Q = W (D) None of these
3. **In case of adiabatic expansion process 1st law of thermodynamics is written as:**
(A) $W = \Delta U$ (B) $W = Q$ (C) $W = -\Delta U$ (D) $W = -\Delta Q$
4. **According to 1st law of thermodynamics the quantity which is conserved is:**
(A) Energy (B) Force (C) Momentum (D) Power
5. **A good example of first law of thermodynamics is:**
(A) Simple pendulum (B) Centripetal force (C) Bicycle pump (D) Doppler's effect
6. **Boyle's law is applicable to:**
(A) Isobaric process (B) Isochoric process (C) Isothermal Process (D) Adiabatic process
7. **Which is called the internal energy of an ideal gas?**
(A) Potential energy (B) Translational kinetic energy (C) Vibrational kinetic energy (D) All of these
8. **An isotherm is drawn as:**
(A) 

(B) 

(C) 

(D) 

9. **Cloud formation in atmosphere is an example of:**
(A) Isothermal process (B) Adiabatic process (C) Isobaric process (D) Isochoric process
10. **The measure of the degree of hotness or coldness of a substance is called:**
(A) Temperature (B) Heat (C) Internal energy (D) Energy

11. The formula connecting the pressure and volume of a gas undergoing an adiabatic change is:
(A) $P^{\gamma}V$ = constant (B) PV = constant (C) $PV^{\gamma-1}$ = constant (D) PV^{γ} = constant

12. At which of the following temperature a body has maximum internal energy?
(A) $-270^{0}C$ (B) 0 K (C) 273 K (D) -273 K

13. In thermodynamics system internal energy decreases by 100 J and 100 J of work is done on the system then heat lost will be:
(A) Zero (B) 100 J (C) 200 J (D) -200 J

14. A diatomic gas molecules has:
(A) Translation energy only (B) Rotational energy only (C) Vibrational energy only (D) All of these

15. For an ideal gas, the potential energy associated with its molecules is:
(A) Maximum (B) Zero (C) $\frac{1}{2}kx_0$ (D) $\frac{1}{2}kx_1$

16. For an ideal gas system, the internal energy is directly proportional to:
(A) Pressure (B) Volume (C) Mass (D) Temperature

17. Which of the following conditions is best for cooking purpose?
(A) Isothermal (B) Isobaric (C) Isochoric (D) Adiabatic

18. Which one is true for isothermal process:
(A) Q = W (B) Q = 0 (C) Q = 0 (D) $\Delta U = 0$

Answers Key

1. C	2. C	3. C	4. A	5. C	6. C	7. B	8. D	9. B	10. A	11. D	12. C
13. D	14. D	15. B	16. D	17. C	18. D						

Assignment 10.1:

An ideal has absorbs 5.00×10^3J of energy while doing 2.00×10^3J of work on the environment during a constant pressure process. (a) Compute the change in the internal energy of the gas. (b) If the internal energy now drops 4.50×10^3J and 7.50×10^3J is expelled from the system, find the change in volume, assuming a constant pressure process at 1.01×10^5 Pa.

Solution: Amount of heat energy supplied to the system = $\Delta Q = 5.00 \times 10^3$ J

Work done by the system =, $\Delta W = 2.00 \times 10^3$J

a) Change in internal energy ΔU=?

b) Change in volume = ΔV =?

$P = 1.01 \times 10^5$ Pa

$\Delta U = -4.50 \times 10^3$ J

$\Delta Q = -7.50 \times 10^3$J

a) Using the first law of thermodynamics;

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta U = 5.00 \times 10^3 J - 2.00 \times 10^3 J$$
$$\Delta U = (5.00 - 2.00) \times 10^3 J = 3.00 \times 10^3 J$$

b) Using the first law of thermodynamics;

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta W = \Delta Q - \Delta U = -7.50 \times 10^3 J - (-4.50 \times 10^3 J)$$
$$\Delta W = -7.50 \times 10^3 J + 4.50 \times 10^3 J$$
$$\Delta W = -3.00 \times 10^3 J$$

$\Delta W = P\Delta V$, hence;

$$P\Delta V = -3.00 \times 10^3$$
$$\Delta V = -3.00 \times 10^3 / P = -3.00 \times 10^3 / 1.01 \times 10^5$$
$$\Delta V = -2.97 \times 10^{-2} m^3$$

The negative sign in the equation shows that the volume of the system decreases

For Your Information

However, thermodynamics also applies to living systems, such as our own bodies. This forms the basis of the biological thermodynamics (Figure).

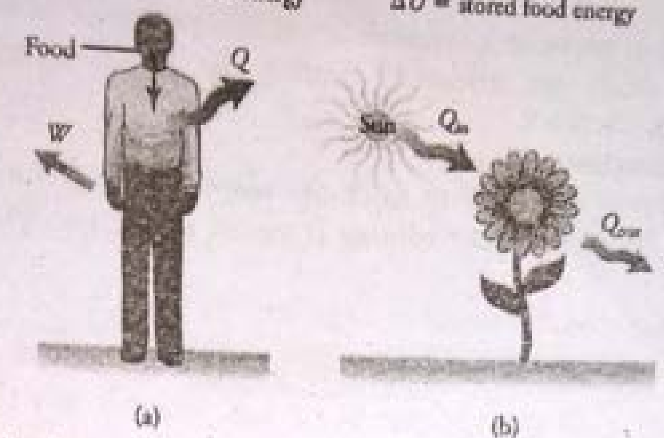


Figure (a) The first law of thermodynamics applies to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, whereas food intake replaces it. (Food intake may be considered work done on the body.) (b) Plants convert part of the radiant energy in sunlight into stored chemical energy, a process called *photosynthesis*.

Life itself depends on the biological transfer of energy. Through photosynthesis, plants absorb solar energy from the sun and use this energy to convert carbon dioxide and water into glucose and oxygen. Photosynthesis takes in one form of energy—light—and converts it into another form—chemical potential energy (glucose and other carbohydrates). *Human metabolism* is the conversion of food into energy given off by heat, work done by the body's cells, and stored fat. Metabolism is an interesting example of the first law of thermodynamics in action. Eating increases the internal energy of the body by adding chemical potential energy.

Q.10 (a) Define the following terms:
- **(i) molar specific heat**
- **(ii) molar specific heat at constant volume (C_v)**
- **(iii) molar specific heat at constant pressure (C_p)**

(b) Prove that $C_p - C_v = R$

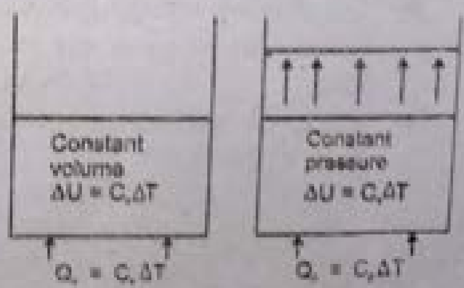
Ans: Specific heat

The amount of heat energy required to raise the temperature of one kilogram of a substance up to one Kelvin is called specific heat.

- One kilogram of different substances contains different number of molecules. Sometimes it is preferred to consider a quantity called mole. One mole of any substance contains same number of molecules.

Molar Specific Heat of a Gas

Molar specific heat of the substance is defined as the heat energy required to raise the temperature of one mole of a substance through 1K.



$$\Delta Q \propto n\Delta T$$

$$\Delta Q = c_M n\Delta T$$

- Where C_M is molar specific heat and n is number of moles

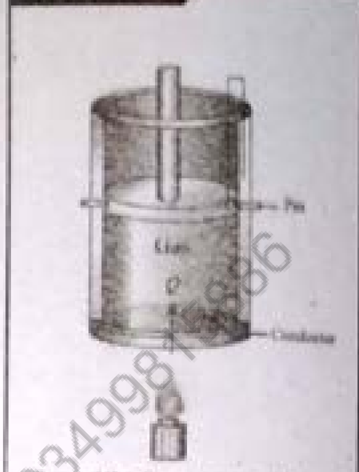
$$C_M = \frac{\Delta Q}{n\Delta T}$$

- S-I unit of molar specific heat is $J\ mol^{-1}\ K^{-1}$

Note

- In case of solids and liquids the change of volume and hence work done against external pressure during a change of temperature is negligibly small.
- But gases suffer variation in pressure as well as in volume with the rise in temperature. Hence, to study the effect of heating the gases, either pressure or volume is kept constant.

FIGURE: 10.9



Specific heat capacity at constant volume

(1) Molar specific heat at constant volume

The amount of heat energy required to raise the temperature of one mole of the gas though 1K at constant volume is called molar specific heat at constant volume.

$$\Delta Q_v = c_v\, n\Delta T$$

- Where C_v is molar specific heat at constant volume and its SI unit is J $mol^{-1}\,K^{-1}$

(2) Molar specific heat at constant pressure

The amount of heat energy required to raise the temperature of one mole of the gas through 1K at constant pressure is called molar specific heat at constant pressure.

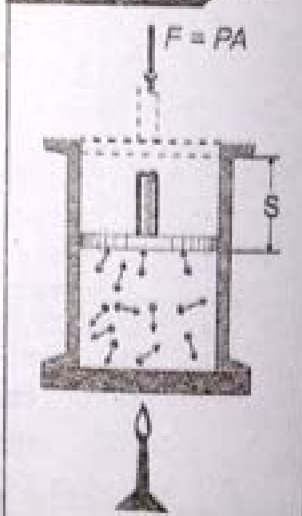
$$\Delta Q_P = c_P\, n\Delta T$$

- Where C_p is molar specific heat at constant pressure and its SI unit is J $mol^{-1}\,K^{-1}$

(3) Derivation of Cp – Cv = R

At constant volume

FIGURE: 10.10



If n moles of an ideal gas are heated at constant volume so that its temperature rises by ΔT then the heat transferred ΔQ_v is given by

$$\Delta Q_v = n\, C_v \Delta T \quad \text{-----------(1)}$$

Applying first law of thermodynamics.

$$\Delta Q_v = \Delta U + \Delta W$$

Putting value of ΔQ_v from equation (1)

$$nC_v\Delta T = \Delta U + \Delta W$$

Since volume remains constant (i.e $\Delta V = 0$), so work done by the system is zero. Thus the above equation becomes

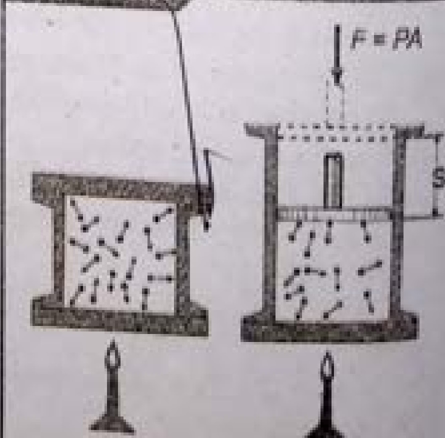
$$nC_v\Delta T = \Delta U + 0 \qquad [\because \Delta W = P\Delta V = P(0) = 0]$$

Hence $\quad nC_v\Delta T = \Delta U$

$$\boxed{\Delta U = nC_v\Delta T \quad \text{-----------(2)}}$$

At constant pressure

FIGURE: 10.10



Fixed piston · Moveable piston

- If n moles of an ideal gas are heated at constant pressure so that its temperature rises by ΔT then the heat transferred ΔQ_p is given by

$$\boxed{\Delta Q_p = n\, C_p\Delta T \quad \text{-----------(3)}}$$

- The internal energy increases by the same amount as at constant volume for the *same rise* in temperature ΔT.

Thus $\quad \Delta U = n\, Cv\Delta T \quad$ -----------(4)

- Since, the gas expands to keep the pressure constant, so the work done by the gas is

$$\Delta W = P\Delta V \quad \text{-----------(5)}$$

$$PV = nRT$$

And $\quad P\Delta V = nR\Delta T$

Putting value of $P\Delta V = nR\Delta T$ in equation (5)

$$\boxed{\Delta W = nR\Delta T} \quad \text{----------(6)}$$

According to first law of thermodynamics:

$$\Delta Q_p = \Delta U + \Delta W \quad \text{----------(7)}$$

Putting values from equations (2),(3) and (6) in equation (7)

$$nC_p\,\Delta T = nC_v\,\Delta T + nR\Delta T$$

$$nC_p\,\Delta T = n\,\Delta T(C_v + R)$$

OR

$$C_p = C_v + R$$

$$\boxed{C_p - C_v = R}$$

It is clear that $C_p > C_v$ by an amount equal to universal gas constant R.

Point to ponder

Q. A football in inflated in a warm room. It is used out of door on a cold day. What happened to ball. Why?

Ans: The volume of gas in the ball will decrease.

According to ideal due to gas law, PV = nRT

$$V = \frac{nRT}{P}$$

Where: V = Volume, n=no. of moles of gas, R = Gas constant, T = Temperature of gas, P = Pressure.

The temperature falls on cold day but the pressure of the air is kept constant, then the volume of gas in the ball will decrease.

MCQ's

1. **The difference between C_p and C_v is equal to:**
   (A) Planks constant (B) Universal gas constant (C) Molar gas constant (D) Boltzman constant
2. **If C_v is the molar heat capacity at constant volume and ΔT is the temperature then $C_v\,\Delta T$ gives:**
   (A) Area (B) Energy (C) Volume (D) Density
3. **The ratio of $\frac{C_p}{C_v}$ for a diatomic gas is equal to:**
   (A) 1.67 (B) 1.50 (C) 1.40 (D) 1.29
4. **For a diatomic gas $C_v = \frac{5R}{2}$ then Gamma "γ" for this gas is:**
   (A) $\frac{5}{7}$ (B) $\frac{4}{35}$ (C) $\frac{7}{5}$ (D) $\frac{35}{4}$
5. **What is the example of irreversible process?**
   (A) Explosion (B) Evaporation (C) Slow compression (D) Liquefaction
6. **Which one is an example of reversible process:**
   (A) Work done against friction (B) Heat produced by current (C) Melting of ice (D) None
7. **In reversible process the entropy of system:**
   (A) Remains constant (B) Decreases (C) Increases (D) Becomes zero

Answers Key

1. B	2. B	3. C	4. C	5. A	6. C	7. A

Q.11 Write a note on Heat Engine?

Ans: Heat Engine

Heat engine is a device which converts heat energy into mechanical work.

Introduction

- The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off, showing the ability to do work. This observation helped to develop a steam engine.

Do You Know?

The steam engine is a Thermodynamics system.

Construction

A heat engine consists of;

- **Hot reservoir or Heat source**

  It is very large reservoir of heat which can supply heat at high temperature T_1.It is also called high temperature reservoir (**HTR**)

- **Cold reservoir or Heat sink**

  It is very large reservoir at low temperature T_2 into which heat is rejected. It is also called low temperature reservoir (**LTR**).

- **Working substance**

  Normally a gas is used as working substance in heat engine .

  The working substance is taken through a cyclic process.

FIGURE: 10.12

Schematic representation of a heat engine

Working

- A heat engine is made cyclic to provide a continuous supply of work.
- Working substance absorbs heat Q_1 from heat source, converts some of it into work W by expansion and rejects the rest of the heat Q_2 to cold reservoir or sink.

After one complete cycle the change in internal energy is zero.

$$\Delta U = 0$$

Heat supplied(absorbed)= Q_1

Heat rejected to sink = Q_2

Change in heat energy= $\Delta Q = Q_1 - Q_2$

According to 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

Putting values of ΔQ and ΔU

$$Q_1 - Q_2 = 0 + \Delta W$$

OR $$\Delta W = Q_1 - Q_2$$

Efficiency of heat engine

- The efficiency of heat engine is defined as the ratio of work done (out put) to the heat supplied(input).

It is denoted by η.

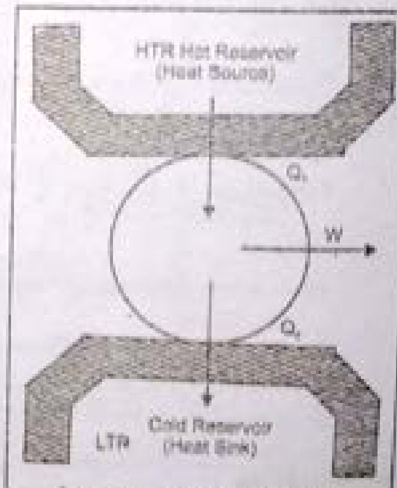
Thus $$\text{efficiency} = \frac{\text{work done}}{\text{heat absorbed}}$$

$$\eta = \frac{\Delta W}{Q_1}$$ Putting value of ΔW

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

OR $$\eta = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

$$\boxed{\eta = 1 - \frac{Q_2}{Q_1}}$$



Schematic diagram of heat engine.

Fig (10.11)

- If $Q_2 = 0$ i.e no heat were exhausted by engine so that all the heat Q_1 absorbed were converted to work

$$\eta = 1 - \frac{0}{Q_1} = 1 - 0 = 1$$

$$\eta\,\% = 100\,\%$$

- Practically it is not possible to construct 100% efficient engine because every heat engine reject some heat Q_2 to the sink. Therefore Q_2 can never be zero.
- **It is impossible to construct a heat engine which converts all the heat absorbed from a hot reservoir into work without rejecting any heat to sink.**

QUIZ

Q. What is function of spark plug in a petrol engine?

A. The function of spark plug is to ignite the air/fuel mixture.It delivers electric current from an ignition system to the combustion chamber of a spark-ignition engine to ignite the compressed fuel/air mixture by an electric spark. Electrical energy is transmitted through the spark plug, jumping the gap in the plugs firing end if the voltage supplied to the plug is high enough. This electrical spark ignites the gasoline/air mixture in the combustion chamber.

Q. There is no spark plug in diesel engine ,then how does fuel burn in it?

A. In diesel engines spark plugs are not needed. The air from the atmosphere is sucked into the cylinder of the engine and then the air is compressed to high pressure which eventually leads to the increase of temperature. when the diesel is supplied at end of compression stroke, the temperature developed is more enough to ignite the diesel. This high temperature makes the fuel to burn and then expansion of gases takes place from where the power stroke is obtained.

Assignment 10.2:

During one cycle, an engine extracts 2.00×103 J of energy from a hot reservoir and transfers 1.50× 103 J to a cold reservoir. (a) Find the thermal efficiency of the engine. (b) How much work does this engine do in one cycle? (c) What average power does the engine generate if it goes through four cycles in 2.50 s?

Given Data:
Heat supplied to the system = $Q_1 = 2.00 \times 10^3 J$
Heat energy rejected from the system = $Q_2 = 1.50 \times 10^3 J$
Time for four cycles = 2.50s
Hence time for one cycle is t = 2.50/4 = 0.625s
Thermal efficiency of the engine = η =?
Work done by the system in each cycle = ΔW =?
Average power delivered = P = ?

Solution:
Using the equation $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{1.50\times10^3}{2.00\times10^3} = 0.25$

$\eta\% = 0.25 \times 100 = 25\%$

Work done is: $\Delta W = \eta \times Q_1 = 0.25 \times 2.00 \times 10^3 J = 5.00 \times 10^2 J$

Power delivered is: $P = \frac{\Delta W}{\Delta t} = \frac{5.00 \times 10^3}{0.625} = 8.00 \times 10^2$ watt

Q.12 State and explain Second Law of Thermodynamics.

Ans: **Second Law of Thermodynamics**

According to **Lord Kelvin's statement** for working of heat engine,

"It is impossible to construct a heat engine operating in cycle which absorbs heat from a hot reservoir and converts it completely into work without rejecting any heat to sink".

OR

- There is no perfect heat engine
- The first law of thermodynamics is a generalization of law of conservation of energy.
- The second law of thermodynamics tells us how heat energy can be converted in to useful work.
- As a consequence of second law of thermodynamics two bodies at different temperature are essential for the conversion of heat energy into work.
- Heat cannot be completely converted into useful work during a complete cycle.
- Working substance absorbs heat Q_1 from heat source, converts some heat energy into work W by expansion and rejects the rest of the heat Q_2 to cold reservoir or sink.
- Therefore output is less than input and efficiency can never be 100% .
- Petrol engine converts roughly **25%** and diesel engine **35% to 40%** available heat energy into work.

Rudolf Clausius Statement

It is impossible to cause heat to flow from a cold body to a hot body without expenditure of work.

- The principle of the refrigerator is a good example of the second law.
- Work (electrical energy) has to be expended to transfer heat from inside the refrigerator to outside it, because by the second law, heat cannot of itself flow from inside to outside.

Q.13 What is Carnot's Engine? Explain its working and calculate its efficiency. Also state Carnot's theorem.

Ans: **Carnot's Engine**

A Carnot heat engine is a **hypothetical** engine that operates on the reversible **Carnot cycle.** Sadi Carnot in 1824 proposed this ideal engine using only isothermal and adiabatic process.

He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures would be the most efficient engine.

FOR YOUR INFORMATION

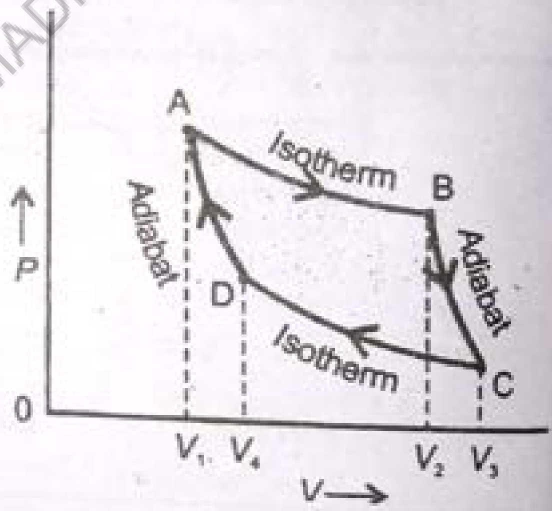
French Engineer Sadi Carnot. Carnot is pronounced as Karno.

Principle

- Carnot's engine works on the same principle as that of cyclic heat engine.
- It takes heat from hot body, convert a part of it into work and reject the remaining part to cold body.

Working

- A Carnot cycle using an ideal gas as a working substance is shown in P-V diagram. It consists of following four steps.



1. **Isothermal expansion**
   - Pressure decreases from P_1 to P_2
   - Volume increases from V_1 to V_2
   - The gas is allowed to expand isothermally at constant temperature T_1(high) absorbing heat Q_1 from the hot reservoir. The process is represented by curve AB.
2. **Adiabatic Expansion**
   - Pressure decreases from P_2 to P_3
   - Volume increases from V_2 to V_3
   - The gas is then allowed to expand adiabatically until its temperature drop to T_2(low). The process is represented by BC curve.
3. **Isothermal compression**
   - Pressure increases from P_3 to P_4
   - Volume decreases from V_3 to V_4
   - The gas at this stage is compressed isothermally at constant temperature T_2(low) rejecting heat Q_2 to the cold reservoir. The process is represented by curve CD.
4. **Adiabatic compression**
   - Pressure increases from P_4 to P_1
   - Volume decreases from V_4 to V_1
   - Finally the gas is compressed adiabatically to restore its initial state at temperature T_1. The process is represented by curve DA.
   - Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible.

Expression for Efficiency

- As the working substance returns to the initial state, there is no change in its internal energy i.e., $\Delta U = 0$.
- The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram. It can also be estimated from net heat ΔQ absorbed in one cycle.

$$\Delta Q = Q_1 - Q_2$$

From 1st law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad \text{\_\_\_\_\_\_\_\_ (1)}$$

Putting value of ΔQ and ΔU in equation (1), we get

$$Q_1 - Q_2 = 0 + \Delta W$$

OR $$W = Q_1 - Q_2$$

Efficiency η (eta) of heat engine is defined as

$$\eta = \frac{\text{output (work)}}{\text{Input (energy)}}$$

$$\eta = \frac{\Delta W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to Kelvin temperature.

i.e. Q_1 and Q_2 are proportional to Kelvin temperature T_1 and T_2 respectively.

Hence $$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

Thus eq. (2) becomes

$$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

The efficiency is usually taken in percentage.

$$\text{Percentage efficiency} = \eta = \left[1 - \frac{T_2}{T_1}\right] \times 100\,\%$$

For Your Information

The first successful petrol engine was invented by Nikolaus Otto in the year 1876 and the Diesel engine was invented by Rudolph Diesel in 1892.

Dependence of Efficiency

- Efficiency of a Carnot Engine depends on the **temperature of hot & cold reservoir.**
- It is independent of the nature of the working substance.
- The larger the temperature difference of the two reservoirs, the greater is the efficiency.
- In most practical cases the cold reservoir is near room temperature. So the efficiency can be increased by **raising the temperature of hot reservoir**

Can efficiency of heat engine be 100%?

- It can never be one or 100% unless cold reservoir is at absolute zero temperature. Such reservoirs are not available & hence maximum efficiency is always less than one.

Carnot's Theorem

Statement

- No real heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

Extended statement

- All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.

Note:

- All real heat engines are less efficient than Carnot engine due to friction & heat losses.

Q.14 Explain the working of refrigerator and discuss coefficient of performance of refrigerator.

Ans. Refrigerator

- The device used to maintain a body temperature below that of the surroundings is called refrigerator.
- Hence, heat must be made to flow from a body at low temperature to the surroundings at high temperature.
- The device in which the working substance performs cycle in a direction opposite to that of a heat engine is called refrigerator.
- The working substance used in it is called refrigerant.

Working:

- A refrigerator works in cycle in such a way that some amount of heat (Q_2) is removed from a low temperature source at temperature (T_2).
- A work (W) is performed by the compressor of the refrigerator on the working substance (refrigerant).
- The quantity of heat (Q_1) is rejected to the high temperature source (atmosphere) at temperature (T_1) by the radiator fixed at the back side of the refrigerator and Q_1 is given by

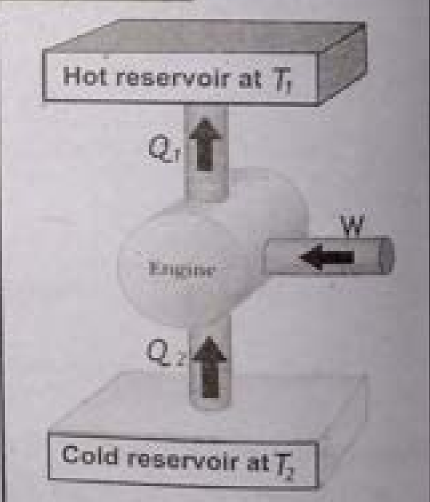
$$Q_1 = W + Q_2$$

(OR) $$W = Q_1 - Q_2 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

- The purpose of a refrigerator is to extract as much heat (Q_2) as possible from the cold reservoir with the expenditure of as little work as possible.

Coefficient of performance or energy ratio of refrigerator

FIGURE. 10.14



Refregirator

- Coefficient of performance is defined as the ratio of the amount of heat removed from the heat sink to the work required to do so.
- Coefficient of performance for cooling or cooling energy ratio is

$$E_{Cooling} = \frac{Q_2}{W}$$

As we know that $W = Q_1 - Q_2$

$$E_{Cooling} = \frac{Q_2}{Q_1 - Q_2}$$

Dividing the nominator and denominator by Q_2 we get

$$E_{Cooling} = \frac{\frac{Q_2}{Q_2}}{\frac{Q_1}{Q_2} - \frac{Q_2}{Q_2}} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

Thus $$E_{Cooling} = \frac{1}{\frac{T_1}{T_2} - 1} \qquad (\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2})$$

$$E_{Cooling} = \frac{T_2}{T_1 - T_2}$$

- Coefficient of performance for heating or heating energy ratio is given by

$$E_{Heating} = \frac{Q_1}{W}$$

Thus $$E_{Heating} = \frac{Q_1}{Q_1 - Q_2} \qquad (\because W = Q_2 - Q_1)$$

$$E_{Heating} = \frac{T_1}{T_1 - T_2}$$

- No cyclic device has ever been built that will extract heat (Q_2) from a cold reservoir and reject it entirely to a hotter reservoir without the expenditure of energy.
- This statement is referred to as the Clausius statement of the second law of thermodynamics.

Point to Ponder

1. Why is the freezer in the upper part of refrigerator?

Ans: **It follows phenomenon of convection in which warm air (less denser) rises up and cool air (denser air) moves downward.** Freezer is the source for the refrigerator's coldness. When the freezer is placed on top, the cold air produced from it is denser than the warmer air in the bottom. So cold air being dense moves down to other part of fridge and the warm air rises up to get cold in the freezer.

2. Why do we not keep bananas in the refrigerator?

Ans: Bananas grow in hot climates, so they are unused to the cold. If they're kept at a cold temperature, the enzymes that enable them to grew are inhibited. And as those enzymes become inactive, other enzymes operate more efficiently. Some cause cell damage, while others (browning enzymes) cause the skin to blacken.

3. Why does the refrigerator switch itself OFF intermittently with some noise?

Ans:
- The refrigerator is trying to maintain temperature. It does that by following the setting of it's thermostat, ON when called for, OFF when satisfied.
- The noise is due to the turning ON and turning OFF the compressor. Also noise is typically caused when the refrigerator enters the defrost cycle and the compressor shuts off. The compressor is under load at that point and will come to an abrupt stop that causes the motor to vibrate momentarily as it powers down.

Q.15 Define and explain the term entropy.

Ans: **Entropy**

Entropy is the measure of the molecular disorder, or randomness, of a system.

- Entropy is state variable of thermodynamically system.
- It was introduced by Rudolph Claudius in 1856.
- This gives quantitative basis or mathematical formula for second law of thermodynamics.

Change in entropy is denoted by ΔS

- If ΔQ is the amount of heat absorbed by the system at temperature T. Then change in entropy (state variable) of the system is,

$$\Delta S = \frac{\Delta Q}{T} \quad \Rightarrow \quad \Delta S = \frac{\pm \Delta Q}{T}$$

- Just like internal energy and potential energy, it is change in entropy which is more important than its absolute value.

Sigh Convention

→ The change in entropy is positive (means that entropy increases) when heat is added to a system.

→ Change of entropy is negative (means that entropy decreases)when heat is taken out of a system.

Unit: The SI unit of change of entropy or entropy is joule/Kelvin (JK^{-1}).

- Let T_1 and T_2 are the temperature of hot reservoir and cold reservoir respectively.
(i.e. $T_1 > T_2$)
- Q amount amount of heat flows through a conduction rod from hot to cold reservoir. Then

Decrease of entropy of HTR $= \frac{Q}{T_1}$

Increase of entropy of LTR $= \frac{Q}{T_2}$

Do You Know?

Approximate efficiencies of various devices

Device	Efficienc (%)
Electric generator	70-99
Electric motor	50-93
Dry cell battery	90
Domestic gas furnace	70-85
Storage battery	72
Hydrogen-oxygen fuel cell	60
Liquid fuel rocket	47
Steam turbine	35-46
Fossil-fuel power plant	30-40
Nuclear power plant	30-35
Nuclear reactor	39
Aircraft gas turbine engine	36
Solid-state laser	30
Internal combustion gasoline engine	20-30
Gallium arsenide solar cells	>20
Fluorescent lamp	20
Silicon solar cell	12-18
Steam locomotive	8
Incandescent lamp	5
Watt's steam engine	1

Net change in entropy $= \frac{Q}{T_2} - \frac{Q}{T_1} =$ positive

- As $T_1 > T_2$ so the sign of net change of entropy is +ve or we can say that net entropy of the system is increased.
- This proves that there is **net increase of entropy due to a natural process** (i.e. flow of heat from higher to lower temperature). This is also called another statement of second law of thermodynamics.

Second Law of Thermodynamics in terms of Entropy

"If a system undergoes a natural process, it will go in the direction that causes the entropy of system plus the environment to increase".

- It is observed that a natural process tends to proceed towards a state of greater disorder.
- For example, an irreversible heat flows from a hot body to a cold body to increases the disorder. So we can say that the entropy is increased.
- Addition of heat increases the disorder; hence the entropy is also increased.
- The free expansion of gas increases its disorder because molecules have greater randomness of position after expansion than before.
- The process in which entropy remains constant is a reversible process.
- For all irreversible process entropy of system increases.

Entropy as Unavailability of Mechanical Work (i.e., Degradation of energy):

- Let us consider two water tank of different temperature, so the average K.E of molecules in higher temperature water is greater than lower temperature. The two water tanks can be used as source and sink of a heat engine, which could be operated, between them and mechanical work can be obtained.
- *But if these two tanks are connected with a conducting rod then heat starts to flow from hot body towards the cold body until thermal equilibrium is reached. So no mechanical work is done due to the absence of heat engine, which results unavailability of mechanical work.*
- *It is also observed that in all natural process, energy tends to pass from more useful form to less useful form hence*, increase in entropy means the degradation of energy.
- Due to continuous increase in entropy and state of disorder of the universe the energy is continuously degrading for doing useful work.

Heat death of Universe

- When the entropy of the universe will reach at maximum value, everything will be at same temperature. All physical,chemical and biological process will have ceased and there will be no way to convert heat into useful work and this state of affairs is refered heat death of the universe.

MCQ's

1. **The efficiency of Carnot engine depends upon:**
(A) Sink temperature only (B) Source temperature only
(C) Both source and sink temperature (D) The working substance

2. **Which of the following is the efficiency of petrol engine?**
(A) 25-30% (B) 45-50% (C) 34-67% (D) 49-60%

3. **The efficiency of Carnot engine depends upon is:**
(A) Nature of the working substance (B) Size of engine
(A) Construction of engine (D) Temperature of hot and cold reservoirs

4. **If the temp of the sink is decreases, the efficiency of carnot engine:**
(A) Decrease (B) Increase
(C) Remains the same (D) First increases then decreases

5. **Absolute zero corresponds to:**
(A) – 460 °F (B) – 360 °F (C) 0°F (D) 460 °F

6. **A heat engine operates between the temperature 1000 *K* and 400 *K*. Its efficiency is:**
(A) 100 % (B) 70 % (C) 60 % (D) 50 %

7. **Which of the following is the expression for the change in entropy of a system?**
(A) $\Delta Q = \frac{\Delta S}{T}$ (B) $\Delta S = \frac{T}{\Delta Q}$ (C) $\Delta S = \frac{\Delta Q}{T}$ (D) $\Delta S = \Delta QT$

8. Which of the following is the latent heat of fusion of ice?
(A) $3.36 \times 10^5\ J\ kg^{-1}$ (B) $3.36 \times 10^6\ J\ kg^{-1}$ (C) $3.36 \times 10^7\ J\ kg^{-1}$ (D) $3.36 \times 10^8\ J\ kg^{-1}$

9. The carnot cycle can be shown by which graph:
(A) P – T graph (B) V – T graph (C) P – V graph (D) PV – T graph

10. No entropy change takes place in an:
(A) Isobaric process (B) Isothermal process (C) Adiabatic process (D) Isochoric process

11. Which of the following is the SI unit of entropy?
(A) JK^{-1} (B) $J^{-1}K$ (C) JK (D) $J^{-1}K^{-1}$

12. Which of the following are the Dimensions of entropy?
(A) $[ML^2T^2]$ (B) $[ML^2T^{-1}k]$ (C) $[ML^2T^{-2}K^{-1}]$ (D) $[ML^2T^3]$

13. The entropy of the universe with passage of time is:
(A) Increasing (B) Decreasing (C) Remains constant (D) Increases & decreases

14. Which of the following is the Triple point of water?
(A) 273.16 °F (B) 273.16 °C (C) 273.16 K (D) 373.16 K

15. Net change in entropy of a system after one complete Carnot cycle is:
(A) Positive (B) Negative
(C) Zero (D) Sometimes positive and sometimes negative

16. Which of the following is the temperature scale is independent of nature of substance?
(A) Theromodynamic scale (B) Centigrade scale (C) Fahrenheit scale (D) Regnault scale

17. What would be the approximate efficiency of a Carnot engine operating with boiling water as one reservoir and a freezing mixture of ice and water as the other reservoir?
(A) 60% (B) 26.8 % (C) 20 % (D) 12%

18. Change in entropy of reversible process is:
(A) Positive (B) Negative (C) 0 (D) Maximum

19. The increase in entropy means the increase in:
(A) Disorder (B) Unavailability of energy (C) Randomness (D) All of these

20. In reversible process, entropy of a system:
(A) Remains constant (B) Decreases gradually (C) Increases (D) First decreases then increases

Answers Key

1. C	2. A	3. D	4. B	5. A	6. C	7. C	8. A	9. C	10. C	11. A	12. C
13. A	14. C	15. C	16. A	17. B	18. C	19. D	20. A				

FOR YOU INFORMATION

Diesel fuel evaporates more slowly because it is heavier.

It contains more carbon atoms in longer chains than gasoline does (gasoline is typically C9H20, while diesel fuel is typically $C_{14}H_{30}$).

It takes less refining to create diesel fuel, which is why it used to be cheaper than gasoline.

Diesel fuel has a **higher energy density** than gasoline. On average, 1 gallon (3.8 L) of diesel fuel contains approximately 155 x106 joules (147,000 BTU), while 1 gallon of gasoline contains 132x106 joules (125,000 BTU).

This, combined with the improved efficiency of diesel engines, explains why diesel engines get better mileage than equivalent gasoline engines.

Assignment 10.3:

Find the change in entropy of 3.00 x 102g of lead when it melts at 327° C. Lead has a latent heat of fusion of 2.45 x 104 J/kg. (b) Suppose the same amount of energy is used to melt part of a piece of silver, which is already at its melting point of 961° C. Find the change in the entropy of the sitlver.

Given Data: Mass of lead = m = $3.00 \times 10^2 g = 0.3$ kg

Constant temperature at melting point for silver; T = 327°C = (327 + 273)K = 600K

Constant temperature at melting point for silver; T = 3961°C = (961 + 273)K = 1234K

Latent heat of fusion of Lead: $2.45 \times 10^4\ J\ kg^{-1}$

Change in entropy ΔS_{Lead} = ?

Change in entropy ΔS_{silver} = ?

Solution: Heat absorbed by the lead: $\Delta Q = mH_f = 0.3kg \times 2.45 \times 10^4 \, J\,kg^{-1} = 7350J$

Change in entropy of lead: $\Delta S_{Lead} = \frac{\Delta Q}{T} = \frac{7350 J}{600K} = 12.25 \, JK^{-1}$

Change in entropy of silver for same amount of heat is:

$$\Delta S_{silver} = \frac{\Delta Q}{T} = \frac{7350 J}{1234 K} = 5.956 \, JK^{-1} = 5.96 \, JK^{-1}$$

FORMULAE

Work done	$W = P\Delta V$		
First law of thermodynamics	$\Delta Q = \Delta U + W$	$\Delta Q = (U_2 - U_1) + W$	$\Delta U = \Delta Q - W$
Isothermal process	$\Delta Q = W$		
Adiabatic process	$W = -\Delta U$ (expansion)	$-W = \Delta U$ (compression)	
Relation between and C_p and C_v	$C_p - C_v = R$	$\gamma = \frac{C_p}{C_v}$	
Efficiency of heat engine	$\eta = \frac{W}{Q_1}$	$\eta = \frac{Q_1 - Q_2}{Q_1}$	$\eta = 1 - \frac{Q_2}{Q_1}$
Efficiency of carnot's engine	$\eta = \frac{T_1 - T_2}{T_1}$	$\eta = 1 - \frac{T_2}{T_1}$	
Entropy	$\Delta S = \frac{\Delta Q}{T}$		

Key Points

- **Thermodynamics:** The branch of physics which deals with the laws of transformation of heat into other forms of energy and vice versa is called thermodynamics.
- **Internal energy:** The sum of the kinetic and potential energies associated with the random motion of the atoms of the substance is called the internal energy of the substance.
- **First law of thermodynamics:** This law states that if an amount of heat energy ΔQ is supplied to a system a part of it may increase in internal energy by an amount ΔU while the remaining part may be used up as the external work ΔW by the system. $\Delta Q = \Delta U + \Delta W$.
- **Molar specific Heat:** The quantity of heat required to raise the temperature of one mole of the substance (gas) by 1°C or 1 K is called molar specific heat or molar specific heat capacity of that substance.
- **Reversible Process:** A process is said to be reversible if it can be retraced exactly in reverse order without producing any change in the surroundings.
- **Irreversible Process:** A process which cannot be retraced in the backward direction by reversing the controlling factors is said to irreversible.
- **Heat engine:** A heat engine is a device for converting heat energy into mechanical work.
- **Reversible heat engine:** The engine in which the process can be retraced at any stage of its operation by reversing the boundary conditions is called a reversible heat engine.
- **Heat Source:** A body of infinite heat capacity which is capable of absorbing or rejecting an unlimited quantity of heat with out any change in its temperature is called heat source or heat reservoir.
- **Principle of increase of entropy:** Any process taking place within a thermally isolated system, the entropy of the system either increases or remains constant.
- **Degradation of energy:** The conversion of heat energy from high ordered state to a less ordered state is called degradation of energy.

Solved Examples

Example 10.1: A heat engine operates with 65.0 kcal of heat supplied and exhausts 40.0 kcal of heat. How much work did the engine do?

Given: Heat input $Q_H = 65.0$ kcal

Heat rejected QL = 40.0 kcal

Mechanical equivalent of heat 1 kcal = 4, 184 J

Required: Work done = W = ?

Solution: The relationship between these quantities is found in equation,

$W = J(Q_H - Q_L)$

Putting values

$= 4{,}184$ (J/kcal) 65.0 kcal – 40.0 kcal)

$= 4{,}184$ (J/kcal) (25.0 kcal)

$= 4{,}184 \times 25.0$ (J. kcal/kcal)

$= 104{,}600$ J = 105 kJ

$\boxed{W = 105\ kJ}$

Example 10.2: In a certain process, 400J of heat energy is supplied to a system and at the same time 150J of work is done by the system. What is the increase in internal energy of the system?

Given: Heat energy supplied to the system, $\Delta Q = 400J$

Work done by the system, $\Delta W = 150J$

Required: Increase in internal energy of the system ΔU = ?

Solution: Using the first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

$\Delta U = \Delta Q - \Delta W$

$\Delta U = 400\ J - 150\ J = 250\ J$

$\boxed{= 250\ J}$

Example 10.3: What is the change in internal energy of 200g of nitrogen as it is heated from 10°C to 30°C at constant volume? (For nitrogen gas Cv = 20.815 J mole^{-1} K^{-1}.

Given:

Mass of nitrogen gas $m = 200g$

Initial temperature of nitrogen gas $T_i = 10°C$

Final temperature of nitrogen gas $T_f = 30°C$

Change in temperature $\Delta T = T_f - T_i = 30°C - 10°C$

$= 20°C = 20K$

Required: Change in internal energy ΔU = ?

Solution: Molecular mass of nitrogen gas M = 28g per mole

Number of moles, $n = \frac{m}{M} = \frac{200g}{28g \text{ per mole}} = 7.143$ mole

The heat added in converted entirely into the internal energy of the nitrogen gas.

$\Delta U = \Delta C_v \Delta T$

$\Delta U = 7.143 \text{ mole} \times 20.815 \text{ J mole}^{-1} \text{K}^{-1} \times 20K$

$\boxed{\Delta U = 2973.6\ J}$

Example 10.4: A reversible engine works between two temperatures whose difference is 100 °C. If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.

Solution: Temperature of the source = T_1 = ?

Temperature of the sink = T_2 = ?

Difference between temperatures,

$T_1 - T_2 = 100\ ^\circ C = 100K$

Heat absorbed, $Q_1 = 746\ J$

Heat rejected, $Q_2 = 546\ J$

The efficiency of a reversible heat engine is given by the formula:

$$\left(\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}\right)$$

Substitute the given values in the above equation, we get:

$$\eta = \frac{746J - 546J}{746J} = \frac{100K}{T_1}$$

$$\Rightarrow \quad \frac{200J}{746J} = \frac{100K}{T_1}$$

$$\Rightarrow \quad T_1 = \frac{746\ K}{2} = 373\ K = (373 - 273)^\circ C$$

$$T_1 = 100\ ^\circ C$$

Since, $T_1 - T_2 = 100^\circ C$

$$\Rightarrow \quad T_2 = T_1 - 100^\circ C = 100^\circ C - 100^\circ C$$

$$T_2 = 0\ ^\circ C$$

Example 10.5: **A Carnot heat engine has a maximum efficiency of 55%. It takes certain amount of heat from a source, converts a part of it into work and rejects the remaining heat towards the heat sink. If the temperature of the heat source is 600 K then find the temperature of the heat sink.**

Solution: Temperature of the heat source $T_1 = 600\ K$

Maximum efficiency of Carnot heat engine

$\eta = 55\% = 0.55$

Temperature of the heat sink $T_2 = ?$

Using the equation

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{600\ K} = 1 - 0.55$$

$$T_2 = 600\ K \times 0.45 = 270\ K$$

Example 10.6: **A refrigerator has a coefficient of performance 8. If the temperature in the freezer is −23°C. What is the temperature at which it rejects heat?**

Solution: The coefficient of performance, $E = 8$

Temperature of the heat sink, $T_2 = -23\ ^\circ C = -23 + 273 = 250\ K$

Temperature of the heat source, $T_1 = ?$

Using the equation

$$E = \frac{T_2}{T_1 - T_2}$$

$$8 = \frac{250K}{T_1 - 250K}$$

$$T_1 - 250 = \frac{250}{8}$$

$$T_1 - 250 = 31.25$$

$$T_1 = 281.25\ K = 281.25 - 273 = 8.2\ °C$$

Example 10.7: **What is the change in entropy of 30 g of water at 0 °C as it is changed into ice at 0 °C? Take the latent heat of fusion of ice = 336000 J kg^{-1}**

Solution: Mass of water: $m = 30\ g = 0.03\ kg$

Constant temperature at fusion point $T = 0\ °C = 273\ K$

Latent heat of fusion of ice $H_f = 3360000\ J\ kg^{-1}$

Heat removed from water $= \Delta Q = mH_f = 0.03 \times 336000\ J = 10080\ J$

Change in entropy: $\Delta S = \Delta S = \frac{-\Delta Q}{T} = \frac{-10080\ J}{273\ K} = -36.92\ JK^{-1}$

Text Book Exercises

Q.1 Select the correct answer of the following questions.

(1) Assume we can change the equilibrium state of a system via two different process. Assume that the initial and the final state are the same. Which of the quantities $\Delta U, \Delta Q, \Delta W$ and ΔT must be the same for the two process?

(a) Only ΔQ and ΔW (b) Only ΔU and ΔT (c) Only ΔQ and ΔT (d) Only ΔU and ΔW

(2) In any process the maximum amount of mechanical energy that can be converted to heat:

(a) can be converted to heat

(b) Depends upon the amount of friction

(c) Depends upon the intake and exhaust temperature

(d) Depends upon whether kinetic or potential energy is involved

(e) Is 100%

(3) In an isothermal change, internal energy:

(a) Decreases (b) Increases (c) Becomes zero (d) Remains constant

(4) A thermos bottle containing hot coffee is vigorously shaken. Consider coffee as the system, then its temperature:

(a) Increases (b) Decreases below than 0°C

(c) Remains the same (d) Decreases

(5) Maximum work can be obtained in the process called:

(a) Cyclic (b) Isothermal (c) Adiabatic (d) Isochoric

(6) A heat engine takes in 800J of heat at 1000K and exhausts 600J of heat at 400 K. What is the actual efficiency of this engine?

(a) 25% (b) 40% (c) 50% (d) 75%

(7) If the temperature of the heat source is increased, the efficiency of a Carnot's engine

(a) Increases (b) Decreases

(c) Remains constant (d) First increases and then becomes constant

(8) Triple point of water is:

(a) 273.16 °C (b) 372.16 K (c) 273.16 °F (d) 273.16 K

(9) A real gas can be approximated to an ideal gas at: **(FBISE-2019)**

(a) Low density (b) High pressure (c) High density (d) Low temperature

(10) If the volume of the gas is to be increased by 4 times, then:

(a) Temperature and pressure must be double

(b) At constant P the temperature must be increased by four times

(c) At constant T the pressure must be increased by four times

(d) It cannot be increased

(11) In which of the system listed below is the entropy decreasing?

(a) A gas is cooled (b) A plate is shattered
(c) An egg is scrambled (d) A drop of dye diffuses in a cup of water

(12) If the temperature of source and sink of a Carnot engine having efficiency η are each decreased by 100K, then the efficiency η:

(a) Remain constant (b) Become 1 (c) Increases (d) Decreases

No.	Option	ANSWER	EXPLANATION
1	(b)	Only ΔU and ΔT	$U \propto T$ When final state of system is equal to initial state then final temperature of system is equal to initial temperature and final internal energy of system is equal to initial internal energy. Therefore only ΔU and ΔT are equal.
2	(b)	Depends upon the amount of friction	
3	(d)	Remains constant	Internal energy is directly proportional to temperature. In isothermal process temperature remains constant therefore internal energy remains constant.
4	(a)	increases	$T \propto \langle K.E \rangle$ While shaking $\langle K.E \rangle$ of molecules increases and temperature increases
5	(b)	Isothermal	Area of P-V graph is equal to work done by the system. In given options area of isothermal P-V graph is geseter than other options.
6	(a)	25%	$\eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{600}{800} = 1 - 0.75 = 0.25 \times 100\% = 25\%$
7	(a)	increases	
8	(d)	273.16 K	
9	(a)	Low density	
10	(b)	At constant P the temperature must be increased by four times	The volume of the given mass of a gas is directly proportional to the absolute temperature when the pressure is kept constant; $V \propto T$.
11	(a)	A gas is cooled	
12	(c)	Increases	Let $T_1 = 400K$ T_2 200K Since $\eta = 1 - \frac{T_2}{T_1}$ Now $\eta = 1 - \frac{200}{400} = 1 - \frac{1}{2} = 0.5 = 50\%$ When temperatures are decreased by 100 K, then; $\eta = 1 - \frac{100}{300}$ $\eta = 1 - 0.333 = 0.67$ $\eta = 67\%$

Short Answers of the Exercise

Q.2 Write short answers of the following questions.

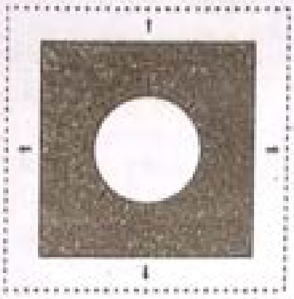
Q.1 Why is the earth not in thermal equilibrium with the sun?

Ans: The two objects are said to be in thermal equilibrium if they are at the same temperature. The distance from the Earth to the Sun is roughly149.6 million km.

- The earth is not in thermal equilibrium with sun because while earth is being warmed by absorbing radiant energy, it is also losing heat in various ways e.g re-emission of radiation,
- The Earth and the Sun are not in contact with each other. Usually things reach thermal equilibrium if their particles can directly interact with each other.
- But the only way for energy to move from the Sun to the Earth is via EM radiation, because the space between the Earth and the Sun is mostly a vacuum.
- Also very small percentage of sun energy can reach earth.

Q.2 When a block with a hole in it is heated, why does not the material around the hole expand into the hole and make it small?

Ans:
- If you heat the ring it expands, making the circumference bigger. When a block is heated its temperature increases
- The block will expand on heating and its all dimensions increase .
- Therefore diameter (size) of hole also increases.



Q.3 A thermometer is placed in direct sunlight. Will it read the temperature of the air, or of the sun, or of something else?

Ans:
- When thermometer is placed in direct sunlight it will read the temperature of its surroundings or temperature of air surrounding the glass bulb.
- when the temperature of the thermometric substance becomes equal to the temperature of surrounding then thermal equilibrium is reached.
- A normal thermometer cannot be used to measure temperature of sun.
- Temperature of sun can be estimated by studying its spectrum.

Q.4 The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen. Why?

Ans: According to Graham's law of diffusion, the rate of diffusion of a certain gas is inversely proportional to square root of its molecular mass. $\frac{r_1}{r_2} = \sqrt{\frac{M2}{M1}}$

- The smaller the molecular mass of a gas the greater will be the rate of diffusion and vice versa.
- As the hydrogen is lighter than oxygen i-e its molecular mass and density is less than that of oxygen therefore rate of diffusion of hydrogen gas is greater than oxygen.
- That is why the pressure in a gas cylinder containing hydrogen will leak more quickly than the gas cylinder containing oxygen.

Q.5 What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room?

Ans: The temperature of the room will not decrease but it increases slightly .

Reason

- Air conditioner absorbs heat from the room as well as rejects heat in the same room at the same rate.
- During compression of gas some heat is produced due to friction in the compressor .This heat is also expelled in the same room.
- Therefore the temperature of the room will increase slightly.

Q.6 Why does the pressure of the air in automobile tyre increases if the automobile is driven for a while?

Ans: When the automobile is driven for a while then heat produce due to friction between road and tyre i-e the work done against friction appears as heat energy.

- This heat is absorbed by the gas molecules.
- It increases the temperature and average kinetic energy of gas molecules.

As pressure $P \propto \langle K.E. \rangle$,

- When $\langle K.E. \rangle$ of gas molecules increases then number of collision of gas molecules with walls of tyre increases per unit time therefore pressure of gas inside tyre also increases.

Q.7 On removing the valve, the air escaping from a cycle tube cool. Why?

Ans: The air inside cycle tube is at high pressure and high temperature.

- When the valve is removed the air rushes out from the tube.
- The outside atmospheric pressure is less than the air pressure inside the tube.
- Therefore adiabatic expansion produces which causes cooling.
- As a result, the temperature of air falls down and hence becomes cool.

Q.8 Write the limitations of first law of thermodynamics.

Ans:

- First law of thermodynamics does not provide a clear idea about the direction of absorption or evolution of heat.
- It does not tells that under what condition and in which direction transformation of heat and work takes place .
- It does not tell about the entropy of system. If all energy is conserved, entropy can never be achieved; if entropy is achieved, conservation has failed.

Q.9 Is it possible, according to the second law of thermodynamics, to construct a heat engine that is free from thermal pollution?

Ans: No, it is not possible to construct a heat engine that will not expel heat into the atmosphere.

Reason:

- According to 2nd law of thermodynamics Lord Kelvin's statement 'It is impossible to construct a heat engine operating in cycle which absorbs heat from a hot reservoir and converts it completely into work without rejecting any heat to sink'
- Every heat engine expels heat energy to the surrounding and produce thermal polution.
- If it is possible, then it will be violation of second law of thermodynamics.

Q.10 Can specific heat of a gas be zero or infinity? Can specific heat be negative?

Ans: (a) Molar specific heat can be zero in adiabatic process.

As we know that

$\Delta Q = C\, n\Delta T$

Where C is molar specific heat.

$C = \frac{\Delta Q}{n\Delta T}$ ---------------------- (1)

- In adiabatic process no heat enters or leaves the system. i-e $\Delta Q = 0$

Putting $\Delta Q = 0$ in equation (1)

$C = \frac{0}{n\Delta T}$ (or) $C = 0$

(b) Molar specific heat can be infinity in isothermal process.

- In isothermal process the temperature of the system remains constant.

And $\Delta T = 0$

Putting $\Delta T = 0$ in equation (1)

$C = \frac{\Delta Q}{n(0)}$ (OR) C = Infinity

(c) Molar specific heat of a gas is always positive. It can never be negative.

Q.11 An inventor claims to have developed a heat engine, working between 27 °C and 227 °C having an efficiency of 45%. Is the claim valid? Why?

Ans: Temperature of heat source = $T_1 = 227\,^0C = 227 + 273 = 500$ K
Temperature of sink = $T_2 = 27\,^0C = 27 + 273 = 300$ K
Efficiency = η = ?

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{300}{500}$$

$$\eta = 1 - 0.6 = 0.4$$

$$\eta\,\% = 0.4 \times 100 = 40\,\%$$

- The inventor of engine claimed its efficiency 45% therefore his claim is not valid.

Comprehensive Questions

Q3. Give a short response to the following questions.

1. Explain, briefly, the following terms used in thermodynamics: System, Surroundings, Boundary and State variables.

Ans: See Q.5 from book.

2. Distinguish among the three forms of energy: work; heat and internal energy.

Ans: See Q.2 and Q.4 from book.

3. State and explain the first law of thermodynamics.

Ans: See Q.6 from book.

4. In the light of the first law of thermodynamics describe the processes:
(a) Isochoric process (b) Isobaric process
(c) Isothermal process and (d) Adiabatic process

Ans: See Q.7 from book.

5. Define the molar heat capacities C_p and C_v for a gas. Show that, for a mole of an ideal gas, $C_p - C_v = R$

Ans: See Q.8 from book.

6. Explain with examples reversible and irreversible processes.

Ans: See Q.9 from book.

7. What is meant by a heat engine? What is its main purpose? How is its efficiency defined?

Ans: See Q.10 from book.

8. State the second law of thermodynamics in its alternative forms. Discuss the assertions of the first and second laws about heat and work energies.

Ans: See Q.11 from book.

9. What were the basic questions that led Carnot to invent Carnot engine?

Ans: See Q.12 from book.

10. What is meant by Carnot cycle and by Carnot engine?

Ans: See Q.12 from book.

11. State Carnot Theorem about the characteristics of a Carnot engine.

Ans: See Q.12 from book.

12. What do you mean by a refrigerator? How does it function? Derive an expression for the Coefficient of performance of a refrigerator.

Ans: See Q.13 from book.

13. Explain the concept of entropy. Mention its major properties. How is the second law of thermodynamics expressed in terms of entropy?

Ans: See Q.14 from book.

Numerical Problems

1. Water at 20°C falls from a height of 854 m. If the whole energy is used in increasing the temperature, find out the final temperature. Specific heat of water is 4200 JK^{-1} kg^{-1}.

Data: Initial temperature of water = $T_1 = 20\ ^0C = 20 + 273 = 293$ K

Final temperature of water = T_2 = ?

Height from which water falls = h = 854 m

Specific heat of water = c = 4200 J kg^{-1} k^{-1}

Solution:

When water falls on ground then its P.E

Converts into heat energy.

$$\text{Loss of P.E} = \text{gain of heat energy}$$

$$mgh = \Delta Q$$

$$\cancel{m}gh = C\cancel{m}\Delta T$$

$$gh = C\Delta T$$

$$\Delta T = \frac{gh}{C}$$

$$T_2 - T_1 = \frac{gh}{C}$$

$$T_2 - T_1 = \frac{gh}{C}$$

$$T_2 = T_1 + \frac{gh}{C} \quad \text{Putting values}$$

$$T_2 = 293 + \frac{9.8 \times 854}{4200}$$

$$T_2 = 293 + 2 = 295 \text{ K}$$

$$T_2 = 295 - 273 = 22\ ^0C$$

Alternate method:

Final velocity of water just before hitting the ground is $v = \sqrt{2\ gh}$

$$v = \sqrt{2 \times 9.8 \times 854}$$

$$v = 129.4 \text{ m/s}$$

K.E of water converts into heat energy on hitting the ground

Loss of K.E of water = Hent energy

$$\frac{1}{2}\cancel{m}v^2 = c\cancel{m}\Delta T$$

$$\Delta T = \frac{v^2}{2c}$$

$$T_2 - T_1 = \frac{v^2}{2c}$$

$2\ gh = v_f^2 - v_i^2$
$v_i = 0$
$v_f^2 = 2\ gh$
$v_f = \sqrt{2\ gh}$ or
$v = \sqrt{2\ gh}$

$$T_2 = \frac{v^2}{2c} + T_1$$

$$T_2 = \frac{(129.4)^2}{2 \times 4200} + 293$$

$$T_2 = 2 + 293$$

$$T_2 = 295\ K$$

$$T_2 = 295 - 273$$

$$\boxed{T_2 = 22\ ^0C}$$

2. 25200 J of heat is supplied to the system while the system does 6000 J of work. Calculate the change in internal energy of the system.

Data: Heat supplied = $\Delta Q = 25200\ J$

Work done by system = $\Delta W = 6000\ J$

Change in internal energy = $\Delta U = ?$

Solution:

According to 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = \Delta Q - \Delta W$$

$$\Delta U = 25200 - 6000 = 19200\ J$$

3. A sample of ideal gas is uniformly heated at constant pressure. If the amount of 180 J of heat is supplied to the gas, calculate the Change in internal energy of the gas and Work done by the gas. Take $\gamma = 1.41$.

Given Data: Heat supplied at constant pressure = $\Delta Q_p = 180\ J$

$\gamma = 1.41$

(a) Change in internal energy = $\Delta U = ?$

(b) Work done by system = $\Delta W = ?$

Solution: (a)

$$\gamma = \frac{C_P}{C_V}$$

$$C_P = \gamma C_V$$

$$\Delta Q_p = C_p n\Delta T \quad \text{(Putting } \Delta Q_p = 180\ J\text{)}$$

$$C_p n\Delta T = 180 \quad (C_P = \gamma C_V)$$

$$\gamma C_V n \Delta T = 180$$

$$C_V n \Delta T = \frac{180}{\gamma} \quad \text{(Putting } C_V n \Delta T = \Delta U\text{)}$$

$$\Delta U = \frac{180}{\gamma}$$

$$\Delta U = \frac{180}{1.41}$$

$$\boxed{\Delta U = 127.66\ J}$$

(b)

$$\Delta W = \Delta Q_p - \Delta U$$

$$= 180 - 127.66$$

$$\boxed{\Delta W = 52.345\ J}$$

4. Find the efficiency of a Carnot's heat engine working between the steam and ice points?

Data: High temperature = $T_1 = 100\ ^\circ C = 100 + 273 = 373\ K$

Low temperature = $T_2 = 0\ ^\circ C = 0 + 273 = 273\ K$

Efficiency = $\eta = ?$

Solution:

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{273}{373}$$

$$= 1 - 0.7319$$

$$\eta = 0.2681$$
$$\eta\% = 0.2681 \times 100 = 26.8\%$$

5. A Carnot heat engine absorbs 2000 J of heat from the source of heat engine at 227 °C and rejects 1200 J of heat during each cycle to sink. Calculate efficiency of engine temperature of sink and amount of work done during each cycle.

Data:

Heat absorbed from heat source = $Q_1 = 2000$ J

High temperature = $T_1 = 227°C = 227 + 273 = 500$ K

Heat rejected to sink = $Q_2 = 1200$ J

(a) Efficiency = η = ?

(b) Temperature of sink = T_2 = ?

(c) Work done = ΔW = ?

Solution:

(a) Efficiency is given by

$$\eta = 1 - \frac{Q_2}{Q_1}$$
$$\eta = 1 - \frac{1200}{2000}$$
$$\eta = \frac{2000 - 1200}{2000}$$
$$\eta = \frac{800}{2000}$$
$$\eta = 0.4$$
$$\eta\% = 0.4 \times 100 = 40\%$$

(b) Also

$$\eta = 1 - \frac{T_2}{T_1}$$
$$0.4 = 1 - \frac{T_2}{500}$$
$$\frac{T_2}{500} = 1 - 0.4$$
$$\frac{T_2}{500} = 0.6$$
$$T_2 = 0.6 \times 500$$
$$T_2 = 300 \text{ K}$$
$$\boxed{T_2 = 300 - 273 = 27°C}$$

(c)

$$\Delta W = Q_1 - Q_2$$
$$\Delta W = 2000 - 1200$$
$$\Delta U = 800 \text{ J}$$

6. What is the least amount of work that must be performed to freeze one gram of water at 0 °C by means of a refrigerator? Take the temperature of the surrounding as 37 °C. How much heat is passed on to the surrounding during this process?

Given Data:

Mass = m = 1 g

Low temperature = $T_2 = 0$ °C = 273 K

High temperature = $T_1 = 37$ °C = 37 + 273 = 310 K

Heat of fussion = $H_f = 336$ J g^{-1}

Amount of heat removed = $Q_2 = mH_f$

$$Q_2 = 1 \times 336 = 336 \text{ J}$$

Heat rejected to surrounding = Q_1 = ?

Work done = ΔW = ?

Solution:

Coefficient of performance of refrigerator is

$$E = \frac{T_2}{T_1 - T_2}$$

$$E = \frac{273}{310 - 273}$$

$$E = \frac{273}{37}$$

$$\boxed{E = 7.378}$$

Also $$E = \frac{Q_2}{\Delta W}$$

$$\Delta W = \frac{Q_2}{E}$$

$$\Delta W = \frac{336}{7.378}$$

$$\boxed{\Delta W = 45.54\ J}$$

$$\Delta W = Q_1 - Q_2$$

$$Q_1 = \Delta W + Q_2$$

$$= 45.54 + 336$$

$$\boxed{Q_1 = 381.54\ J}$$

7. Calculate the change in entropy when 10 kg of water is heated from 90 °C to 100 °C? (Specific heat of water is 4180 J mol^{-1} K^{-1})

Given Data: mass = m = 10 kg
Initial temperature = T_1 = 90°C = 90 + 273 = 363 K
Final temperature = T_2 = 100°C = 100 + 273 = 373 K
Change in temperature = ΔT = 373 − 363 = 10 K
Specific heat of water = C = 4180 J mol^{-1} K^{-1}
Change in entropy = ΔS = ?

Solution:

Average temperature = $$T = \frac{T_1 + T_2}{2}$$

$$T = \frac{363 + 373}{2}$$

$$T = 368\ K$$

Heat supplied to water

$$\Delta Q = C\,m\,\Delta T$$

$$= 4180 \times 10 \times 10$$

$$\boxed{\Delta Q = 418000\ J}$$

Change in entropy

$$\Delta S = \frac{\Delta Q}{T}$$

$$\Delta S = \frac{418000}{363}$$

$$\boxed{\Delta S = 1135.8\ JK^{-1}}$$

8. A system absorbs 1176 J of heat and at the same time does 352.8 J of external work. Find the change in internal energy of the system? Find the change in internal energy in the system when it absorbs 1050 J of heat while 84 J of work is done? What will be the change in internal energy of the gas if 210 J of heat is removed at constant volume?

Given Data:

(a) Heat supplied = ΔQ = 1176 J
Work done = ΔW = 352.8 J
Change in internal energy = ΔU = ?

(b) Heat supplied = ΔQ = 1050 J
Work done = ΔW = 84 J
ΔU = ?

(c) Heat removed = ΔQ_v = −210 J
At constant volume = ΔW = 0
ΔU = ?

Solution:

(a) According to 1st law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta U = \Delta Q - \Delta W$$
$$= 1176 - 352.8$$
$$\boxed{\Delta U = 823.2\ J}$$

(b) Formula

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta U = \Delta Q - \Delta W$$
$$= 1050 - 84$$
$$\boxed{\Delta U = 966\ J}$$

(c) Formula

$$\Delta Q_v = \Delta U + \Delta W$$
$$\Delta U = \Delta Q_v - \Delta W$$
$$= -210 - 0$$
$$\boxed{\Delta U = -210\ J}$$

9. An ideal gas at 20.0°C and a pressure of 1.50×10^5 Pa is in a container having a volume of 1.00L.
(a) Determine the number of moles of gas in the container.
(b) The gas pushes against the piston, expanding to twice the original volume, while pressure falls to atmospheric pressure. Find the final temperature.

Data:
Temperature of ideal gas = T_1 = 20°C = 273 + 20 = 293K
Pressure = $P_1 = 1.50 \times 10^5$ Pa
Initial Volume = V_1 = 1.00 L = $\frac{1.00}{1000}m^3 = 10^{-3}\ m^3$
Universal gas constant = R = 8.315 J/mol K
Final Volume = $V_2 = 2V_1$
(a) Number of moles of gas = n = ?
(b) Final temperature = T_2 = ? When P_2 = 1 atm = 1.01325×10^5 $V_2 = 2V_1$

Solution:
(a) From ideal gas equation;

$$P_1V_1 = nRT_1$$
$$1.50 \times 10^5 \times 10^{-3} = n \times 8.315 \times 293$$
$$1.50 \times 10^2 = n\,(2436.295)$$
$$n = \frac{1.50\times10^2}{2436.295}$$
$$n = 6.16 \times 10^{-2} \text{moles}$$

(b) From ideal gas equation;

$$P_2V_2 = nRT_2$$
$$P_2 \times 2V_1 = nRT_2$$
$$1.01 \times 10^5 \times 2 \times 10^{-3} = 6.16 \times 10^{-2} \times 8.315 \times T_2$$
$$T_2 = \frac{2.02\times10^2}{6.16\times10^{-2}\times8.315} = 395K$$

10. A block of ice at 273K is put in thermal contact with container of steam at 373 K, converting 25 g of ice to water at 273 K while condensing some of the steam to water at 373 K.
(a) Find the change in entropy of the ice.
(b) Find the change in entropy of the steam.
(c) Find the change in entropy of the universe.

Given Data: Temperature of ice = T_{ice} = 273K
Temperature of stream = T_{stream} = 373 K
Mass of ice block = m = 25.0g = $\frac{25.0}{1000}$ kg = 0.025kg
Latent heat of fusion of ice is H_f = 336000 J/kg
Change in the entropy of ice = ΔS_{ice} = ?
Change in the entropy of stream = ΔS_{stream} = ?
Change in the entropy of ice = $\Delta S_{universe}$ = ?

Solution:

The heat absorbed from the stream is;
$\Delta Q = mH_f = 0.025\text{kg} \times 336000\text{ J/kg} = 8400\text{J}$
The change in the entropy of ice ΔS_{ice}:

$$\Delta S_{ice} = \frac{\Delta Q}{T_{ice}} = \frac{8400}{273} = 30.76\text{ J/K} = 30.8\text{ J/K}$$

Change in the entropy of ice $\Delta S_{universe}$:

$$\Delta S_{steam} = -\frac{\Delta Q}{T_{steam}} = -\frac{8400}{373} = -22.52\text{ J/K} = -22.5\text{ J/K}$$

Negative sign shows a decrease in the entropy of the steam.
Change in the entropy of ice $\Delta S_{universe}$:

$$\Delta S_{universe} = \Delta S_{ice} + \Delta S_{steam} = 30.8\text{ J/K} + (-22.5\text{ J/K})$$

$$\Delta S_{universe} = -22.5\text{ J/K} = 8.27\text{ J/K} = 8.3\text{ J/K}$$

Additional Conceptual Short Questions With Answers

1. Why the food is cooked quicker in pressure cooker?

Ans: In pressure cooker, due to increase in pressure the boiling point of the food rises so the food is cooked quicker.

2. A person is painting the ceiling, and drop of paint from the brush falls onto an operating incandescent light bulb. The bulb breaks. Why?

Ans: An incandescent light bulb has glass that receives light from the bulb filament. The glass heats up by radiations of filament and by convection through gas filled in glass. Thus, glass becomes very hot.
When drop of paint falls onto the glass, that portion of glass suddenly becomes cold, and the contraction of this region creates thermal stresses that causes the glass to break.

3. Write down values of molar specific heat at constant volume and at constant pressure and find their ratio constant γ for mono, di and polyatomic gases?

Ans:

	CV	C_P	$\gamma = \frac{C_p}{C_v}$
Mono-atomic gas (Ideal gas)	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic gas	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.4$
Polyatomic gas	7R/2	9R/2	1.29

4. For a mono-atomic gas if $\gamma = \left(\frac{5}{3}\right)$, then find value of C_p and C_v?

Ans: We know that

$$C_p - C_v = R$$

Dividing above eq. by C_v

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\left(\frac{C_p}{C_v} - 1\right) = \frac{R}{C_v} \qquad \text{putting } \frac{C_p}{C_v} = \gamma$$

$$(\gamma - 1) = \frac{R}{C_v}$$

$$\Rightarrow \quad C_v = \left(\frac{R}{\gamma - 1}\right)$$

$$\Rightarrow \quad C_v = \frac{R}{\left(\frac{5}{3} - 1\right)}$$

$$\Rightarrow \quad C_v = \frac{R}{\left(\frac{5-3}{3}\right)} = \frac{R}{\left(\frac{2}{3}\right)}$$

$$\boxed{C_v = \frac{3}{2}R}$$

Similarly, $\frac{C_p}{C_v} = \gamma$

$$C_p = \gamma C_v$$

$$C_p = \frac{5}{3} \cdot \frac{3}{2}R$$

$$\boxed{C_p = \frac{5}{2}R}$$

5. If one mole of a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, then what is value of γ for the mixture?

Ans: We know that

$$(C_v)_{mono} = \frac{3}{2}R$$

and $(C_v)_{di} = \frac{5}{2}R$

So $$(C_v)_{mixture} = \frac{\left(\frac{3}{2}R + \frac{5}{2}R\right)}{2} = \left(\frac{3+5}{2(2)}\right)R$$

$$= \frac{8}{4}R$$

$$\boxed{(C_v)_{mixture} = 2R}$$

From the relation

$$C_p - C_v = R$$
$$C_p - 2R = R$$
$$C_p = 2R + R$$
$$\boxed{C_p = 3R}$$

Since, $\gamma = \frac{C_p}{C_v} = \frac{3R}{2R} = \frac{3}{2} \Rightarrow \boxed{\gamma_{mix} = \frac{3}{2}}$

6. If PV^{γ} = constant, prove that $TV^{\gamma-1}$ = constant

Ans:

$$PV^{\gamma} = \text{Constant} \quad \text{......(1)}$$

Also $PV = nRT$

For one mole n = 1

$$PV = RT$$
$$P = \frac{RT}{V} \quad \text{......(2)}$$

Putting value from eq. (2) in (1)

$$\frac{RTV^{\gamma}}{V} = \text{Constant}$$
$$RTV^{\gamma-1} = \text{Constant}$$
$$TV^{\gamma-1} = \frac{\text{Constant}}{R}$$

As R is also a constant

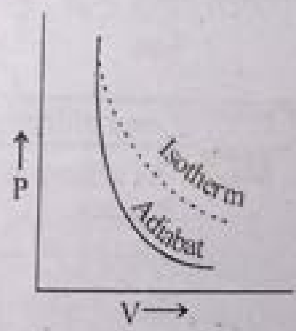
$$TV^{\gamma-1} = \text{Constant}$$

7. Explain why adiabatic is steeper than an isotherm?

Ans:

- If an isotherm and an adiabatic are drawn on the same graph, it can be seen that adiabatic is steeper than the isotherm. This steepness is due to the reason that in an adiabatic expansion, the system does work at the cost of its own internal energy. While in an isothermal expansion energy is supplied by the heat reservoir.
- Adiabatic process is faster than isothermal process.

Due to these reasons adiabat is more steeper than isotherm.



(Alternate Method)

For an adiabatic process $PV^{\gamma} = k$

$$P = k / V^{\gamma}$$
$$\frac{d}{dV}(P) = \frac{d}{dV}(k / V^{\gamma})$$
$$\frac{dP}{dV} = k\frac{d}{dV}(V)^{-\gamma}$$
$$\frac{dP}{dV} = k\left[-\gamma V^{-\gamma-1}\left(\frac{dV}{dV}\right)\right]$$
$$\frac{dP}{dV} = -\gamma k\,(1 / V^{\gamma+1})\,(1)$$
$$\frac{dP}{dV} = -\frac{\gamma}{V}(k / V^{\gamma}) \quad \text{putting } k / V^{\gamma} = P$$
$$\frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$$

Slope of adiabatic process $= \frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$ ---------------- (1)

For isothermal process $PV = k$

$$P = \frac{k}{V}$$

Differentiating

$$\frac{d}{dV}(P) = \frac{d}{dV}\left(\frac{k}{V}\right)$$

$$\frac{dP}{dV} = k\frac{d}{dV}(V)^{-1}$$

$$\frac{dP}{dV} = k\left[-1\, V^{-1-1}\left(\frac{dV}{dV}\right)\right]$$

$$\frac{dP}{dV} = -\frac{k}{V}\left(\frac{1}{V}\right)(1) \qquad \text{putting } \frac{k}{V} = p$$

$$\frac{dP}{dV} = -\frac{P}{V}$$

Slope of an isothermal process $= \frac{dP}{dV} = -\frac{P}{V}$ ----------------(2)

As we know that $\gamma > 1$

From equation (1) and (2) it is clear that the slope of the adiabatic process is γ times more than the slope of the isothermal process therefore adiabat is more steeper than isotherm.

8. Specific heat of a gas at constant pressure is greater than specific heat at constant volume. Why?

Ans: **Reason**

When gas is heated at constant pressure then;

(i) a part of heat is used to do ***work on piston***

(ii) rest of heat is used to ***increase the temperature*** through 1K

When gas is heated at constant volume then all the heat absorbed is used to increase temperature through 1K. As piston is fixed therefore no work is done by system.

That is why the molar specific heat at constant pressure is greater than molar specific heat at constant volume.

Also $C_P = C_v + R$

$(C_p > C_v)$.

9. Does entropy of a system increases or decreases due to friction?

Ans: The entropy of a system ***increases*** due to friction.

Reason

Due to friction, some mechanical energy is ***converted into heat*** which increases the entropy of system.

As $\Delta S = \frac{\Delta Q}{T}$

When ΔQ increase then ΔS also increases.

10. Give an example of a process in which no heat is transferred to or from the system but the temperature of the system changes.

Ans: In ***adiabatic process*** no heat enters or leaves the system take place but temperature of the system changes.

During adiabatic expansion temperature falls.

During adiabatic compression temperature increases.

Examples

- ***Rapid escape of air*** from a burst tyre
- ***Rapid expansion and compression*** of air through which sound wave is passing.
- *Cloud formation* **in the atmosphere**

MCQ's From Past FBISE Papers
(FEDERAL BOARD)

1. **For a mono atomic gas C_v =3R/2, therefore gamma 'γ' for this gas is:**
(a) 3/5 (b) 5/3 (c) 4/15 (d) 15/4

2. **In thermodynamic process the equation ΔQ=ΔU represents process:**
(a) Isothermal (b) Isobaric (c) Adiabatic (d) Isochoric

3. **An isothermal process is represented by:**
(a) Charles' law (b) Gay-Lussac law (c) Ideal gas law (d) Boyle's law

4. **In an isothermal process, internal energy of the system:**
(a) Increases (b) Decreases then increase (c) Decreases (d) Remains constant

5. **If the temperature of the source increases, the efficiency of a Carnot engine.**
(a) Decreases (b) Increases than decreases (c) Increases (d) Remains same

6. **The efficiency of a Carnot engine working between higher and lower temperatures T_1 & T_2 respectively is given by:**
(a) $\eta = \frac{T_2 - T_1}{T_1}$ (b) $\eta = \frac{T_1 - T_2}{T_1}$ (c) $\eta = \frac{T_2 - T_1}{T_2}$ (d) $\eta = \frac{T_1}{T_1 - T_2}$

7. **The expression for pressure exerted by the gas on container on any side is**
(a) $P = \frac{1}{2}\rho\langle v^2\rangle$ (b) $P = \frac{3}{2}\rho\langle v^2\rangle$ (c) $P = \frac{1}{3}\rho\langle v^2\rangle$ (d) $P = \frac{1}{3}\rho\langle v\rangle$

8. **Maximum efficiency of Carnot engine is always**
(a) greater than one (b) less than one (c) equal to one (d) none of these

9. **In reversible process, the entropy**
(a) remains constant (b) increases (c) decreases (d) initially increases then decreases

10. **The triple point of water is**
(a) 0K (b) 100K (c) 373.16K (d) 273.16K

11. **What would be the efficiency of a Carnot engine operating with boiling water as one reservoir and a freezing mixture of ice and water as the other reservoir?** (FBISE- 2017)
(a) 27% (b) 67% (c) 12% (d) 100%

12. **In a reversible cycle, the entropy of the system:** (FBISE- 2017)
(a) First increases and then decreases (b) Increases
(c) Decreases (d) Does not change

13. **A frictionless heat engine can be 100% efficient only if its exhaust temperature is:** (FBISE (ON) 2017)
(a) Zero kelvin (b) Equal to its input temperature
(c) Kelvin temperature (d) Zero ^{0}C temperature

14. **The temperature at which a system undergoes a reversible isothermal process without transfer of heat is called as;** (FBISE(ON) 2017)
(a) Reversible temperature (b) Critical temperature
(c) Kelvin temperature (d) Absolute zero temperature

15. **The entropy of universe always:** (FBISE - 2018)
(a) Increases and decreases simultaneously (b) Remains constant
(c) Increases (d) Decreases

16- **The efficiency of diesel engine is about:** (FBISE(ON) 2018)
(a) 30% to 35% (b) 25% to 30% (c) 45% to 50% (d) 35% to 40%

Answers Key

1.	b	2.	d	3.	d	4.	d	5.	c
6.	b	7.	c	8.	b	9.	a	10.	d
11	a	12	d	13	a	14	d	15	c
16.	d								

SELF – ASSESSMENT PAPER

Total Mark: 40

Question.No.1 Choose the correct answer from the given options. (1 x 6 = 6)

SECTION – A

1. **For a mono-atomic gas $C_V = \frac{3R}{2}$ then γ is:**
   (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) $\frac{4}{15}$ (D) $\frac{15}{4}$
2. **Which of following is correct for 1 calorie = ________?**
   (A) 420 J (B) 10 J (C) 40 J (D) 4.2 J
3. **Efficiency of carnot engine can be:**
   (A) 100% (B) Maximum (C) Infinite (D) Zero
4. **Temperature would be same on °F and °C at ________.**
   (A) – 40° (B) 40° (C) 30° (D) –30
5. **In an isothermal change, internal energy:**
   (A) Decreases (B) Increases (C) Becomes zero (D) Remains constant
6. **A real gas can be approximated to an ideal gas at:** (FBISE-2019)
   (A) Low density (B) High pressure (C) High density (D) Low temperature

Question.No.2 Give short answers of followings: (3 x 7 =21)

SECTION – B

(i) Why does the pressure of gas in a car tyre increase when it is driven through some distance?
(ii) prove that area of P-V graph is equal to work done by the system.
(iii) Can specific heat of a gas be zero or infinity? Can specific heat be negative?
(iv) An inventor claims to have developed a heat engine, working between 27 °C and 227 °C having an efficiency of 45%. Is the claim valid? Why?
(v) What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room?
(vi) Differentiate between reversible and irreversible process.
(vii) During one cycle, an engine extracts 2.00×10^3 J of energy from a hot reservoir and transfers 1.50×10^3 J to a cold reservoir. (a) Find the thermal efficiency of the engine. (b) How much work does this engine do in one cycle? (c) What average power does the engine generate if it goes through four cycles in 2.50 s?

Question.No.3 Extensive Questions. (13)

SECTION – C

(a) What is Carnot's Engine? Explain its working and calculate its efficiency. Also state Carnot's theorem. (6)
(b) A Carnot heat engine absorbs 2000 J of heat from the source of heat engine at 227 °C and rejects 1200 J of heat during each cycle to sink. Calculate efficiency of engine temperature of sink and amount of work done during each cycle. (4)
(c) What is mechanical equivalent of heat? (3)

The End

THE END

FEDERAL PHYSICS HSSC – I

Time: 25 Minutes | SECTION-A | Marks: 17

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Choose the correct answer i.e. A/B/C/D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1. **In a cricket match 500 spectators are counted one by one. How many significant figures will be there in final result?**
   A. 3 B. 1 C. 2 D. 0
2. **The SI unit of solid angle is:**
   A. degree B. radian C. steradian D. revolution
3. **If $\overline{A}.\overline{B}=\overline{B}.\overline{A}$ the scalar product is:**
   A. additive B. associative C. commutative D. multiplicative
4. **The rectangular components of a force of 5N are:**
   A. 3 and 4N B. 2.5 and 25N C. 1 and 2N D. 2 and 3N
5. **Distance covered by free falling body in 2 second is:**
   A. 4.9 m B. 19.6 m C. 39.2 m D. 44.1 m
6. **If momentum is increased by 20% then K.E increases by:**
   A. 44% B. 55% C. 66% D. 77%
7. **The consumption of energy of 60 watt bulb is:**
   A. 30 J B. 8 J C. 12 J D. 16 J
8. **SI unit of moment of inertia is:**
   A. Kg/m B. Kg/m^2 C. Kgm^2 D. Kgm
9. **Who gave the inverse square law for gravity?**
   A. Einstein B. Galileo C. Newton D. Plank
10. **Pressure will be low where the speed of the fluid is:**
    A. Zero B. High C. Low D. Constant
11. **The displacement of particle having amplitude 'a' in SHM in one time period is:**
    A. zero B. a C. 2a D. 4a
12. **The distance covered by a body in one complete linear vibration is 20 cm. What is the amplitude of body?**
    A. 10 cm B. 5 cm C. 15 cm D. 7 cm
13. **Which waves are used in Sonography?**
    A. microwaves B. X-rays C. ultrasonic waves D. Material waves
14. **Which is not optically active?**
    A. Sugar B. Tartaric acid C. Water D. Milk
15. **The triple point of water is:**
    A. 273 K B. OK C. 273.16K D. 37K
16. **A heat engine absorbs 50J of energy and give 45J of work. Its efficiency will be:**
    A. 60% B. 70% C. 80% D. 90%
17. **Velocity of the efflux is measured by the relation:**
    A. $\sqrt{gh}$ B. $\sqrt{2gh}$ C. $\sqrt{\frac{1}{2}gh}$ D. $\sqrt{\frac{4}{3}gh}$

FEDERAL PHYSICS HSSC – I

Time allowed: 2:35 Minutes — **Total Marks Section B, C and D: 68**

Note: Section 'B' 'C' and 'D' comprise pages 1-2 and questions therein are to be answered on the separately provided Answer Book. Use supplementary answer sheet i.e., sheet B if required. Write your answers neatly and legibly.

SECTION – B (Marks 21) (Chapters 1 to 5)

NOTE: Please write answer in no more than FIVE / SIX lines.

Q.2 Answer any SEVEN parts. All parts carry equal marks. **(7 × 3 = 21)**

(i) Show that one radian = 57.3°.
(ii) How are uncertainties measured in final result of power factor and volume?
(iii) Prove that if the vector $\overline{A}$ & $\overline{B}$ are parallel to each other, then $\overline{A}\cdot\overline{B} = \pm AB$
(iv) If $(\overline{A}\times\overline{B}) = 0$, then either of the two vectors is a null vector or vector $\overline{A}$ and $\overline{B}$ are parallel to each other. Prove it.
(v) Why is it helpful to wear a helmet while riding?
(vi) Does a moving abject have impulse? Explain.
(vii) Name the various non-conventional sources of energy and write down their characteristics.
(viii) Why are energy savers used instead of normal bulbs?
(ix) Prove that $r_0 = \left[\frac{GMeT^2}{4\pi^2}\right]^{1/2}$ of geostationary satellite.
(x) Why does the coasting rotating system slowdown as water drops into the breaker?

SECTION – C (Marks 21) (Chapters 6 to 10)

Q.3 Answer any SEVEN parts. All parts carry equal marks. **(7 × 3 = 21)**

(i) What is an aerofoil? Explain its working in accordance with Bernoulli's equation.
(ii) When water falls from a tap, why does the cross-sectional area of droplet decreases as it comes down?
(iii) What is mean t by damped Oscillation? Show it graphically.
(iv) In relation to SHM, explain the equations: a. $Y = A\,\mathrm{Sin}(wt + \varphi)$ b. $a = -w^2x$
(v) Write down the applications of Doppler Effect.
(vi) How can you generate ultrasonic waves and how can you detect them?
(vii) What do you know about grating spectrometer?
(viii) Why is ordinary light not polarized?
(ix) Two blocks of ice are pressed together in order to form a single piece. Explain how this happens.
(x) What is the function of spark plug in a petrol engine?

SECTION – D (Marks 26)

Q.3 Answer any TWO questions. All parts carry equal marks. **(2 × 13 = 26)**

Q.4 a. Derive a relation for the time period of a simple pendulum using dimensional analysis. The various possible factors on which the time period 'T' may depend are: (8)
(i) length of pendulum
(ii) mass of the bob
(iii) angle O which the thread makes with the vertical
(iv) acceleration due to gravity

b. Consider a ladder weighting 20N vesting against a smooth wall such that it makes an angle of 60° with the horizontal. Find the reaction on the ladder due to the wall and ground. (5)

Q.5 a. What is Bernoulli's equation? Show that how it is based on law of conservation of energy and name the three applications of Bernoulli's equation. (8)

b. Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any inslasat t is given by $x = 0.25\cos\left(\frac{\pi}{8}\right)t$. Find the displacement of the object after 2 second. (5)

Q.6 a. State Carnot theorem and the characteristics of a Carnot engine. Explain Carnot cycle and Carnot engine. (8)

b. Find the polarizing angle for a glass of refractive index of 1.55. (5)

Federal Board HSSC-I Examination

Physics Model Question Paper

SECTION – A

Version Number

Time allowed: 25 Minutes | **Marks: 17**

NOTE: Section-A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Center Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer sheet according to the instructions given there. Each part carries one mark.

1. **What is to ratio 1μm/1Gm?**
   (A) 10^{-3} (B) 10^{-9} (C) 10^{-12} (D) 10^{-15}

2. **For which angle the equation $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ is correct?**
   (A) 30° (B) 45° (C) 60° (D) 90°

3. **What is the angle between $\vec{A}$ and $\vec{B}$ for which $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$?**
   (A) 30° (B) 45° (C) 60° (D) 90°

4. **What does NOT change when force is applied on a body?**
   (A) Mass (B) Velocity (C) Position (D) Acceleration

5. **A projectile is thrown so that it travels a maximum range of 100m. How high will it rise?**
   (A) 500 m (B) 250 m (C) 400 m (D) 1000 m

6. **If momentum is increased by 20% then K.E increases by:**
   (A) 44% (B) 55% (C) 66% (D) 77%

7. **The atmosphere is held to the earth by:**
   (A) winds (B) gravity (C) clouds (D) rotation of earth is

8. **The angular speed in rad/hrs for daily rotation of our earth is:**
   (A) 2π (B) 4π (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

9. **Artificial satellite moves around:**
   (A) Moon (B) Sun (C) Stars (D) Earth

10. **The pressure will be low where the speed of the fluid is:**
    (A) Zero (B) High (C) Low (D) Constant

11. **The periods of the pendulum at Karachi (T_k) and at Murree (T_m) are related as:**
    (A) $T_k > T_m$ (B) $T_k < T_m$ (C) $T_k = T_m$ (D) $2T_k = 3T_m$

12. **Which one of the following factors has no effect on the speed of sound in a gas?**
    (A) Humidity (B) Pressure (C) Temperature (D) Density

13. **Which one of the following properties is NOT exhibited by the longitudinal waves?**
    (A) Reflection (B) Interference (C) Diffraction (D) Polarization

14. **The tip of a needle does NOT give a sharp image. It is due to**
    (A) Polarization (B) Interference (C) Diffraction (D) Refraction

15. **The Principle of Michelson Interferometer is based on the division of:**
    (A) Wave front (B) Amplitude (C) Frequency (D) Speed of light

16. **In an isothermal change, internal energy of a system:**
    (A) Decreases (B) Increases (C) Becomes Zero (D) Remains Const

17. **Triple point of water is:**
    (A) 273.16° F (B) 372.16 K (C) 273.16°C (D) 273.16 K

PHYSICS HSSC-I

Time allowed: 2:35 Minutes **Total Marks Section B, C and D: 68**

Note: Section 'B' 'C' and 'D' comprise pages 1-2 and questions therein are to be answered on the separately provided Answer Book. Use supplementary answer sheet i.e., sheet B if required. Write your answers neatly and legibly.

SECTION – B (Marks 21)

(Chapters 1 to 5)

Q.2 Answer any SEVEN parts. All parts carry equal marks. **$(7 \times 3 = 21)$**

(i) Show that the famous Einstein's equations $E = mc^2$ is is dimensionally consistent.

(ii) Give the drawbacks to use the period of pendulum as a time standard.

(iii) Under which circumstances would a vector shall have components of same magnitude?

(iv) Explain briefly why buses and heavy trucks have large steering wheels?

(v) What is Head-on Collision? Explain briefly with an example.

(vi) What is the angle of projection for a projectile for which the maximum height reached and corresponding range are equal?

(vii) Does a hydrogen-filled balloon possess P.E? Explain briefly.

(viii) A bucket is taken to bottom of a well; does the bucket possess any P.E? Explain briefly.

(ix) Why the fly wheel of an engine is made heavy at the rim?

(x) A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?

SECTION – C (Marks 21)

(Chapters 6 to 10)

Note: Answer any SEVEN parts. All parts carry equal marks. **$(7 \times 3 = 21)$**

(i) Why a sports car has oblong shape design?

(ii) Describe the working of an engine carburetor.

(iii) Give two applications from daily lives of phenomenon in which resonance plays an important role.

(iv) What will be the frequency of a simple pendulum if its length is 1m?

(v) What are the conditions for constructive and destructive interference?

(vi) How one can locate the position of nodes and anti-nodes in a vibrating string?

(vii) Can we apply Huygens Principle to radar waves? Explain briefly.

(viii) How you can explain Brewster's Law of Polarization?

(ix) What are the conditions for a process to be reversible?

(x) Entropy has often called as "times arrow". Explain briefly.

SECTION – D (Marks 26)

Note: Attempt any TWO questions. All questions carry equal marks. **$(2 \times 13 = 26)$**

Q.4 (a) Prove that Absolute $P.E = \frac{GmM_e}{R_e}$ (8)

(b) A man whose mass is 70 kg walks up to the third floor of a building which is 12 m above the ground is 20 Sec. Find his power in watts and hp. (5)

Q.5 (a) Derive equation for kinetic and potential energy of a body executing S.H.M for a mass-spring system. (8)

(b) What should be the length of a simple pendulum whose time period is one second? What is its frequency? (5)

Q.6 (a) Describe the experimental arrangement for the production of interference fringes by young's double slit method, and get and get an expression for the fringe spacing. (8)

(b) In a certain X-rays diffraction experiment the first order image is observed at an angle of 5° for a crystal plane spacing of 2.8×10^{-10} m. What is the wave length of X-ray used? (5)

⊙⊙⊙⊙⊙⊙⊙⊙

PHYSICS HSSC-I (New Course)
OBJECTIVE KEY

Serial #	V # 3081	V # 3082	V # 3083	V # 3084
1)	C	FC	A	A
2)	D	D	FC	B
3)	B	A	A	FC
4)	B	C	B	B
5)	C	C	D	C
6)	FC	D	D	A
7)	A	FC	A	A
8)	C	B	FC	B
9)	C	D	C	FC
10)	D	D	A	D
11)	B	A	A	B
12)	C	C	B	B
13)	A	D	D	C
14)	B	B	A	A
15)	B	C	C	B
16)	C	C	D	D
17)	FC	D	D	A

SOLUTION PHYSICS HSSC-I (NEW COURSE)
SECTION B

Q#2

(i) **Given Data:**

Einstein equation $E = mc^2$

To Prove:

Einstein equation $E = mc^2$ is dimensionally consistent

Calculations:

As $E = mc^2$

Where E is the energy in joules

Dimensions of L.H.S of equation $= [E] = [ML^2T^{-2}]$ ... (1)

Dimensions of R.H.S of equation $= mc^2 = [M][LT^{-1}]^2$

$= [ML^2T^{-2}]$ ... (2)

Thus L.H.S = R.H.S **(03)**

(ii) In microwave oven, we use electromagnetic waves, called microwaves, to heat food. Micro means small, because these waves are smaller in wavelength than other radio waves. It does not show that its wavelength is in micrometer.

Wavelength of microwaves used in microwave oven is 12 cm and their frequency is 2450 MHz. **(03)**

(iii) **Steps for addition of vectors**

a) Find the x and y-components of all given vectors.

b) Add x-components of all the vectors to find the x-component R_x of the resultant vector.

c) Add y-components of all the vectors to find the y-component R_y of the resultant vector.

d) Find the magnitude of resultant vector $\vec{R}$ by using

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}}$$

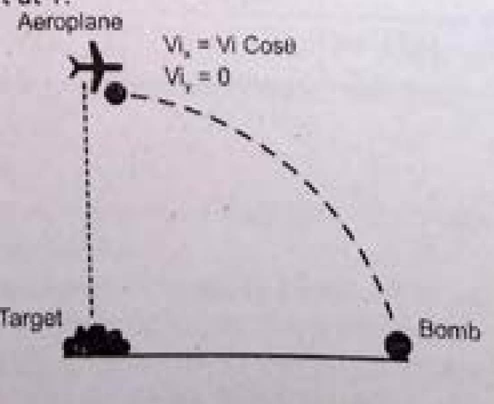
e) Find the direction of resultant vector $\vec{R}$ by using

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

(03)

(iv) Cranes are made heavy at its bottom which lowers its C.G. and they are constructed such that their line of actions of forces lies within its base area (their base area is made large). That is why the cranes mostly do not topple over after lifting load. **(03)**

(v) The bomb has the same velocity as that of the aero-plane when it is dropped. If the bomb is dropped when the aero-plane is vertically above the target, it will strike a point ahead of the target due to constant horizontal velocity component and inertia, the bomb misses the target. The bomb moves like a projectile as shown in fig. It will not hit the target but it will hit at T. **(03)**



(Figure not compulsory)

(vi) **Relation**

$$\vec{J} = \vec{F} \times \Delta t$$

As force is the time rate of change of momentum So

$$\vec{F} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

or $\quad \vec{F} \times \Delta t = m\vec{v}_f - m\vec{v}_i$

Putting $\vec{F} \times \Delta t = \vec{J}$

$$\vec{J} = \vec{P}_f - \vec{P}_i$$

$$\vec{J} = \Delta \vec{P}$$

So the impulse is equal to the ***change in momentum*** of the body.

(vii) **Work Done by a Variable Force**

In many cases, the force is not constant, but it varies in magnitude or direction or in both e.g.

- Force of gravity acting on a rocket moving away from earth.
- Force exerted by spring increases by the amount of stretch.

Let us consider the path of particle in xy-plane from point a to b as shown in figure.

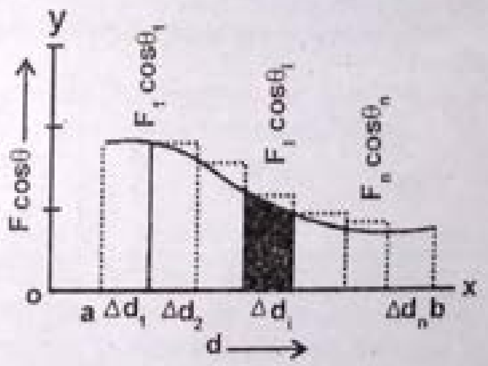
Divide the path into n short intervals of displacements $\Delta\vec{d}_1, \Delta\vec{d}_2, \ldots\ldots, \Delta\vec{d}_n$

The forces acting during these intervals are $\vec{F}_1, \vec{F}_2, \ldots\ldots, \vec{F}_n$ respectively.

The force is considered to be approximately constant for each interval of displacement.

So work done for the first interval is,

$$\Delta W_1 = \vec{F}_1 . \Delta\vec{d}_1 = F_1\cos\theta_1 \Delta d_1$$



Similarly,

$$\Delta W_2 = \vec{F}_2 . \Delta\vec{d}_2 = F_2\cos\theta_2 \Delta d_2$$

and up to nth interval

$$\Delta W_n = \vec{F}_n . \Delta\vec{d}_n = F_n\cos\theta_n \Delta d_n$$

Now the total work done in moving the body from point a to b is.

$$W = (\Delta W_1 + \Delta W_2 + \ldots\ldots + \Delta W_n)$$

OR $\quad W = (F_1\cos\theta_1\Delta d_1 + F_2\cos\theta_2\Delta d_2 + \ldots\ldots + F_n\cos\theta_n\Delta d_n)$

OR $\quad W = \sum_{i=1}^{n} F_i\cos\theta_i \Delta d_i$

OR Graphical Method **(03)**

(viii) **Correct any three differences**

Solar Energy:

Solar energy is the radiant light and heat from the Sun. Solar radiation along with secondary solar resources such as wind and wave power, hydroelectricity and biomass account for most of the available renewable energy on Earth.

There are two methods to use solar energy, depending on the way they capture, convert and distribute sunlight.

Active solar techniques include the use of photovoltaic panels and solar thermal collectors (with electrical or mechanical equipment) to convert sunlight into useful outputs.

Passive solar techniques include orienting a building to the Sun, selecting materials with favorable thermal mass or light dispersing properties, and designing spaces that naturally circulate air.

Wind Energy:

The non-uniform distribution of heat due to solar energy in different regions causes the movement of hot and cold air over the earth's surface, winds blow from area of high pressure to areas of low pressure. At sea the winds are even stronger than on the land surface.

In Pakistan places like Chitral, Gilgit, coastal areas and high mountain valleys are suitable for the exploitation of wind power. **(03)**

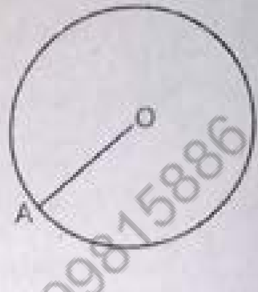
(ix) **Relation between Angular and Linear velocities**

Consider a particle that is moving in a circle of radius 'r' with center at O. Let particle moves from point "A" to point "B" in a circle such that it

If we take 'θ' in radians, then $S = r\theta$ ----------(1)

Similarly, in linear, motion, when a body moves with uniform velocity $\vec{v}$, in time 't', its linear displacement will be:

$$S = vt \quad \text{----------(2)}$$



Comparing the above equations, we can derive

$$vt = r\,\theta$$

$$v = r\,(\theta/t)$$

Putting $\frac{\theta}{t} = \omega$

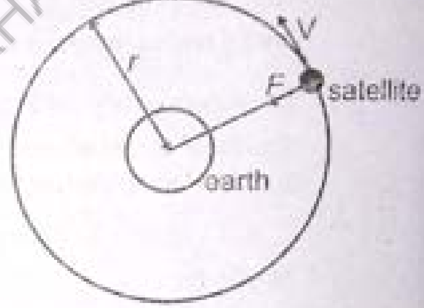
$$v = r\,\omega$$

In vector form, we can write $\vec{v} = \vec{\omega} \times \vec{r}$ **(03)**

(x) **Expression for Orbital Velocity**

Consider a satellite of mass m_s moving with orbital velocity v around the earth of mass M. If r is the radius of the orbit then centripetal force F can be expressed as

$$F_c = \frac{m_s v^2}{r} \quad \text{\_\_\_\_\_\_ (1)}$$



This force is provided by gravitational force of attraction between earth and satellite and is given by

$$F = G\frac{Mm_s}{r^2} \quad \text{\_\_\_\_\_\_ (2)}$$

Equating equation (1) and (2), we get

$$\frac{m_s v^2}{r} = G\frac{Mm_s}{r^2}$$

Or $$v^2 = \frac{GM}{r}$$

Or $$v = \sqrt{\frac{GM}{r}} \quad v \propto \frac{1}{\sqrt{r}}$$ **(03)**

SECTION (C)

Q # 3

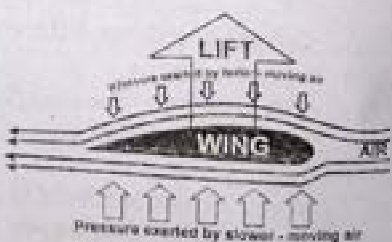
(i) **Lift of an aero-plane (Aerofoil) (Figure is not compulsory)** **(01 + 02)**

The lift of an aero-plane is due to the effect, *where speed of fluid is high, its pressure will be low.*

Explanation

The design of wing deflects the air in such a way that

Stream lines are ***closer*** together above the wing than lower side. Air moves faster at the ***upper side*** of the wing than the lower side. Pressure is ***lower*** at the top of the wing. Hence, the wing experiences a net ***upward*** force.



(ii) Each time your heart beats, it generates a high pressure pulse of blood into the arteries. Counting these pulses gives you an accurate measurement of a person's heart rate in Beats per Minute.

When heart pumps the blood in the vessels, the pressure of blood in the vessels varies accordingly. **The increase in pressure inflated the vessels and decrease in pressure squeezes the vessels.** These changes in the pressure can be felt in the form of pulsations in the pulse.

Checking your pulse is a simple way to get information about your health. **(03)**

(iii) **Free Oscillations** **(03)**

A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force.

For example

A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum.

Forced Oscillations

A body is said to be executing forced vibrations if it oscillates with the interference of an external force.

For example

If the mass of vibrating pendulum is struck repeatedly, then forced vibrations are produced.

The vibrations of factory floor caused by the running of heavy machinery is another example.

(iv) **Applications of Resonance (any two applications)** (03)

- **a.** Radio and Resonance
- **b.** Magnetic Resonance Image (M.R.I)

(v) If E be the elastic modulus and ρ be the density of the medium then speed of sound waves is

$$v \quad \sqrt{\frac{E}{\rho}}$$

Although the density of solids is greater than that of gases but the modulus of elasticity of solids is *much greater* than that of gases ($E_{solids} >> E_{gases}$).

Therefore $\left[\frac{E}{\rho}\right]$ solids $>$ $\left[\frac{E}{\rho}\right]$ gases

Also molecules are closer in solids than in the gases, so they respond more **quickly** to a disturbance. That is why speed of sound is greater in solids than the gases. (03)

(vi) **Any THREE differences.** (03)

	Progressive waves		Stationary waves
1.	The disturbance produced in the medium travels onward, it being handed over from one particle to the next. Each particle executes the same type of vibration as the preceding one, though not at the same time.	1.	There is no onward motion of the disturbance as no particle transfers its motion to the next. Each particle has its own characteristic vibration.
2.	The amplitude of each particle is the same but the phase changes continuously,	2.	The amplitudes of the different particles are different, ranging from zero at the nodes to maximum at the antinodes. All the particles in a given segment vibrate in phase but in opposite phase relative to the particles in the adjacent segment.
3.	No particle is permanently at rest. Different particles attain the state of momentary rest at different instants,	3.	The particles at the nodes are permanently at rest but other particles attain their position of momentary rest simultaneously.
4.	All the particles attain the same maximum velocity when they pass through their mean positions.	4.	All the particles attain their own maximum velocity at the same time when they pass through their mean positions.

(vii) The ray-I have to travel through half silvered glass plate two times and it decreases speed of light two times. The glass plate G_2 cut from the same piece of glass as G_1 and is equal in thickness to G_1 is introduced in the path of beam II to decrease its speed of light two times. G_2 therefore equalizes the path length of the beam I and II in glass and is called compensating glass plate or compensator plate. (03)

(viii) The Dual nature of light

- Sometimes it behaves like a particle (called a photon), which explains how light travels in straight lines
- Sometimes it behaves like a wave, which explains how light bends (or diffracts) around an object.
- Scientists accept the evidence that supports this dual nature of light *(even though it intuitively doesn't make sense to us!)* (03)

(ix) **Reversible Process**

- **A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings.**

- In the reverse process, the working substance passes through the same stages as in the direct process, but thermal and mechanical effects at each stage are exactly reversed.
- **Example of Reversible Process (any one)**
  The process of liquefaction and the evaporation of a substance performed **slowly** are reversible processes.

Irreversible Process

- **An irreversible process is one which can not be retraced in exactly reverse order, without producing any change in the surroundings.**
- All changes which occur **suddenly** or which involve **friction** or **dissipation** of energy through conduction, convection and radiation are irreversible.

Example (any one):

- Explosion is an example of highly irreversible process (03)

(x) a) First law of thermodynamics does not provide a clear idea about the direction of absorption or evolution of heat.

b) The informations provided by the first law of thermodynamics are not enough to predict the spontaneity or feasibility of a process.

c) It does not tell about the entropy of system. If all energy is conserved, entropy can never be achieved; if entropy is achieved, conservation has failed. (03)

SECTION (D)

Q # 4

a. Definition and figure of projectile motion **(01 + 01)**

Derive: Maximum height + time of flight + range of projectile **(02 + 02 + 02)**

b. Numerical: **(0.5 + 1.5 + 1.5 + 1.5)**

Given Data: Mass of ball = m = 100gm = 0.1kg

Speed of ball = v = 25 m/sec.

To Find: (a) Height to which the ball would reach = h =? (ans : 31.9 m)

(b) If height = h' = 25m, then

(i) work done = ? (6.7 J) (ii) Force of friction = F = ? (0.3 N)

Q # 5

a. Derivation of Kinetic and Potential Energies in terms of S.H.M and also draw graph **(03 + 03 + 01)**

b. Numerical: **(03 + 03)**

Solution:

Frequency of sound = f = 500 Hz

Speed of sound = v = 340 ms^{-1}

Speed of car = a = 20m/s

(a) The apparent frequency when the car approaches = f' = ? : $f' = \left(\frac{v}{v-a}\right)f$ (Ans: 531 Hz)

(b) The apparent frequency when receding the car = f'' = ? : $f'' = \left(\frac{v}{v+a}\right)f$ (Ans: 472 Hz)

Q # 6

(a) Explain the diffraction of X-Rays by crystal and derivation Bragg's Law and also finds the wavelength of Light. **(03 + 04 + 01)**

(b) Numerical **(01 + 01 + 03)**

Data (E = 8, T_2 = -23°C = 250 K) + formula ($E = \frac{T_2}{T_1 - T_2}$) + calculation and result

(T_1 = 281.25 K = 281.25 − 273 = 8.2°C)

THE END